Chain of Kamal Transform and Homotop Perturbation Method for Solving Burger's Equation

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Abstract-In this article, a chain form of the Kamal transform method with homotopy perturbation method is proposed to solve Burger's equation. This method is called the Kamal Homotopy Perturbation Method (KHPM).The results of Burger's equations have been obtained in terms of convergent series with easily computable components.The nonlinear terms can be easily handled by the use of Homotopy Perturbation Method(HPM).with thefact that theproposed technique solves nonlinear problems without using Adomian's decomposition method.

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Key words- Kamal transform, Homotopy Perturbation Method, Burger's equation.

I. INTRODUCTION

Nonlinear marvels have critical impact on connected Mathematics, Physics and Engineering and also over the world. Numerous a times, we get the essential issues of Physics and Mathematics by transforming them into nonlinear partial differential equation to locate their correct and closest arrangements. There have been numerous basic change techniques in the years to come. They have an imperative place in the field of Mathematics. Utilizing these fundamental changes in straightforward and time-trap ways, we get the correct and rough arrangement of the issue.
The concept of Kamal transform [3] was proposed by Abdelilah Kamal and Homotopy perturbation method by He[8-9] who, by merging the standard homotopy and perturbation method,solved various physical problems.
In these years, the authors have used several methods for solution of nonlinear partial differential equation. Some analytical methods are perturbation technique [10-11], Hirota bilinear method [12] andvariational iteration method. One of which is also called Adomian decomposition method[2].This method was introduced by George Adomian in 1980s. This is a semi- analytical method for solving varied types of differential and integral equation, both linear and non-linear including partial differential equations. Its solution procedure is simple, but the calculations of Adomian polynomials arecomplicated. We will be implementing a new technique for this calculation in order to remove the complexity.
We will be applying a new approach using He's polynomials. The propose method is capable of reducing the volume of the computational work as compared to the classical method. The aim of this article is to show the applicability and efficiency of the combination of Kamal transform and Homotopy perturbation method. It will help us to solve Burger's equation in $(2+1)-$ dimensional, $(3+1)-$ dimensional and$(n+1)-$ dimensional with boundary conditions [1].

1. Kamal transform and Homotopy perturbation method

A. Kamal transform

The Kamal transformis denoted by operator K (.) and Kamal transform of $\Im(t)$is defined by the integral equation:

$$\mathbb{K}\big(\Im(t)\big) = \mathbb{E}(\vartheta) = \int_0^\infty \Im(t)\, e^{\frac{-t}{\vartheta}}\, dt \quad , t \geq 0, \quad and \quad \delta_1 \leq \vartheta \leq \delta_2 . \tag{1}$$

in a set A the function is defined in the form

$$A = \left\{\Im(t) \colon \exists \mathbb{M}, \delta_1, \delta_2 > 0 \,.\, |\Im(t)| < \mathbb{M} e^{\frac{|t|}{\delta_j}}, if\, t \epsilon (-1)^j \times [0,\infty)\right\} , \tag{2}$$

where δ_1 and δ_2 may be finite or infinite and the constant $\mathbb{M}$ must be finite number.For existence of Kamal transform $\Im(t)$is essential for $t \geq 0$ a piece wise continuous and of exponential order is required , else it does not exist.

Remark : The reader can be read more about the Kamal transform in[3-6].

B. Derivative of Kamal transform

Let function $\Im t)$ be the derivative of $\Im(t)$ with respect to $'t'$ and the n^{th} order derivative of the same with respect to$'t'$. Then Derivative of Kamal transform is given by

$$\mathbb{K}[\Im^n(t)] = \frac{1}{\vartheta^n} \mathbb{E}(\vartheta) - \sum_{k=0}^{n-1} \vartheta^{k-n+1} \Im^k(0). \tag{3}$$

If we put $n = 1,2,3 \ldots$ in equation (3), then the Kamal transform of first and second derivative of $\Im(t)$ with respect to "t":

$\mathbb{K}[\mathfrak{I}'(t)] = \frac{1}{\vartheta}\mathbb{E}(\vartheta) - \mathfrak{I}(0)$

$\mathbb{K}[\mathfrak{I}''(t)] = \frac{1}{\vartheta^2}\mathbb{E}(v) - \frac{1}{\vartheta}\mathfrak{I}(0) - \mathfrak{I}'(0).$

Lemma 2.1.[Fundamental properties of a Kamal transform of partial derivatives] Let $\mathbb{E}(\mathfrak{x},\vartheta)$ be a Kamal transform of $\mathfrak{I}(\mathfrak{x},t)$ .To obtain Kamal transform of partial derivative, we use integral by parts, which is [4],

a) $\mathbb{K}\left[\frac{\partial\mathfrak{I}(\mathfrak{x},t)}{\partial t}\right] = \frac{1}{\vartheta}\mathbb{E}(\mathfrak{x},\vartheta) - \mathfrak{I}(\mathfrak{x},0)$

b) $\mathbb{K}\left[\frac{\partial^2\mathfrak{I}(\mathfrak{x},t)}{\partial t^2}\right] = \frac{1}{\vartheta^2}\mathbb{E}(\mathfrak{x},\vartheta) - \frac{1}{\vartheta}\mathfrak{I}(\mathfrak{x},0) - \frac{\partial\mathfrak{I}(\mathfrak{x},0)}{\partial t}$

c) $\mathbb{K}\left[\frac{\partial\mathfrak{I}(\mathfrak{x},t)}{\partial \mathfrak{x}}\right] = \frac{d}{d\mathfrak{x}}(\mathbb{E}(\mathfrak{x},\vartheta))$

d) $\mathbb{K}\left[\frac{\partial^2\mathfrak{I}(\mathfrak{x},t)}{\partial \mathfrak{x}^2}\right] = \frac{d^2}{d\mathfrak{x}^2}(\mathbb{E}(\mathfrak{x},\vartheta))$

e) $\mathbb{K}\left[\frac{\partial^n\mathfrak{I}(\mathfrak{x},t)}{\partial \mathfrak{x}^n}\right] = \frac{d^n}{d\mathfrak{x}^n}\left(\mathbb{E}(\mathfrak{x},\vartheta)\right).$

C. Homotopy Perturbation Method: The homotopy perturbation method provides an alternative approach to introduce an expanding parameter. If we study the homotopy analysis method, we find that homotopy perturbation method is considered as special case of homotopy analysis method. Let $\mathbb{X}$and$\mathbb{Y}$be the topological spaces. If $\mathfrak{I}(\mathfrak{x})$ is homotopic to $\mathfrak{G}(\mathfrak{x})$ and if there is continuous map $G: \mathbb{X}\times[0,1]\rightarrow\mathbb{Y}$ such that ,$G(\mathfrak{x},0) = \mathfrak{I}(\mathfrak{x})$ and $G(\mathfrak{x},1) = \mathfrak{G}(\mathfrak{x})$ for each $\mathfrak{x}\in\mathbb{X}$, the map G is called homotopy between $\mathfrak{I}(\mathfrak{x})$ and $\mathfrak{G}(\mathfrak{x})$.

Now, to explain the homotopy perturbation method, we consider a general equation of the type,

$$A(w) = 0. \quad (4)$$

Where A is any differential operator. We define a convex homotopy $\mathcal{H}(w,\mathcal{P})$ by

$$\mathcal{H}(w,\mathcal{P}) = (1-\mathcal{P})G(w) + \mathcal{P}A(w). \quad (5)$$

Where$G(w)$ is a functional operator with known solution v_0, which can be obtained easily. It is clear that for $\mathcal{H}(w,0) = G(w)$ and $\mathcal{H}(w,\mathcal{P}) = 0$. we have $\mathcal{H}(w,1) = A(w) = 0$, which are the linear and non linear original equations respectively.In topology, this shows that $\mathcal{H}(w,\mathcal{P})$ continuously traces an implicitly define curve from a starting point $\mathcal{H}(v_0,0)$to a solution function $\mathcal{H}(\mathfrak{I}(\mathfrak{x}),1)$.The HPM uses the embedding parameter$\mathcal{P}$ [13] as a small parameter for $0\le\mathcal{P}\le 1$,and write the solution as a power series :

$$w(\mathfrak{x},t) = w_0(\mathfrak{x},t) + \mathcal{P}w_1(\mathfrak{x},t) + \mathcal{P}^2 w_2(\mathfrak{x},t) + \cdots . \quad (6)$$

If $\mathcal{P}\rightarrow 1$, then equation (6) corresponds to equation (5) and becomes the approximate solution of the form,

$$\mathfrak{I}(\mathfrak{x}) = \lim_{p\rightarrow 1} w(\mathfrak{x},t) = \sum_{i=0}^{\infty} w_i(\mathfrak{x},t) . \quad (7)$$

The embedding parameter$\mathcal{P}$, monotonically increases from zero to unit, so $\mathcal{P}\in[0,1]$ as the trivial problem$A(w) = 0$. It is well known that the series (6) is convergence dependingon $A(w) = 0$.We assume that equation (6) has a unique solution. The comparison of power of $\mathcal{P}$ gives solution to various orders.

2. The solution of Burger's equation by Kamal Homotopy Perturbation Method(KHPM)

The general form of Burger's equation with velocity w and viscosity coefficient ψ, is given by

$$\frac{\partial w}{\partial t} + (w.\nabla)w - \psi\nabla^2 w = 0 , \quad (8)$$

with the initial condition $w(\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3,\ldots,\mathfrak{y}_n,0) = \sum_{i=1}^{n}\mathfrak{y}_i$.

Where $w = w(\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3,\ldots,\mathfrak{y}_n,0), \nabla = \sum_{i=1}^{n}\frac{\partial}{\partial\mathfrak{y}_i}$and $\nabla^2 = \sum_{i=1}^{n}\frac{\partial^2}{\partial\mathfrak{y}_i^2}$.

Lemma 3.1 (Convolution theorem for Kamal transform)

If $\mathfrak{I}(t)$ and $\mathfrak{G}(t)$ are the two functions then Kamal transform of convolution theorem of two function is given by

$$\mathbb{K}(\mathfrak{I}*\mathfrak{G}) = \frac{1}{\vartheta}\mathbb{K}(\mathfrak{I})\mathbb{K}(\mathfrak{G}).$$

Theorem 3.2: let $\mathcal{P}$ is given by

$$w(Y,t) = \sum_{n=0}^{\infty}\mathcal{P}^n w_n(Y,t).$$

Then the solution of Burger's equation is given by

$$w(Y,t) = \sum_{n=0}^{\infty} w_n(Y,t),$$

where

$$\mathcal{P}^0: w_0(Y,t) = \mathfrak{y}_1 + \mathfrak{y}_2 + \mathfrak{y}_3 + \cdots + \mathfrak{y}_n$$

$\mathcal{P}^n: w_n(Y,t) = -\mathbb{K}^{-1}\{\vartheta\,\mathbb{K}[\mathcal{H}_{n-1}(w)]\}$ for all natural number n.

Proof: Taking Kamal transform on the equation (8),and using the differential property of Kamal transform with initial condition, we obtained

$$\mathbb{K}[w(Y,t)] = \vartheta \sum_{i=1}^{\infty} \mathfrak{y}_i - \vartheta \mathbb{K}[(w(Y,t).\nabla) w(Y,t) - \psi \nabla^2 w(Y,t)] \quad (9)$$

where$\mathfrak{y}_i = \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3, \dots, \mathfrak{y}_n$.

Presently, taking converse Kamal transform at both side of condition (9).we touch base at

$$w(Y,t) = \sum_{i=1}^{n} \mathfrak{y}_i - \mathbb{K}^{-1}\{\vartheta \mathbb{K}[(w(Y,t).\nabla) w(Y,t) - \psi \nabla^2 w(Y,t)]\}. \quad (10)$$

We apply the homotopy perturbation method

$$w(Y,t) = \sum_{n=0}^{\infty} \mathcal{P}^n w_n(Y,t). \quad (11)$$

and by using the lemma 3.1, we get

$$(w(Y,t).\nabla) w(Y,t) - \psi \nabla^2 w(Y,t) = \sum_{n=0}^{\infty} \mathcal{P}^n \mathcal{H}_n(w) \quad (12)$$

here $\mathcal{H}_n(w)$ is He's polynomials, which is given by

$$\mathcal{H}_n(w_1, w_2, w_3, \dots, w_n) = \frac{1}{n!} \frac{\partial^n}{\partial \mathcal{P}^n} \left[\sum_{i=0}^{\infty} \mathcal{P}^i w_i \right], \quad (13)$$

using equation (11) and equation (12) in to the equation (10),with $n = 0,1,2, \dots$. we get

$$\sum_{n=0}^{\infty} \mathcal{P}^n w_n(Y,t) = (\mathfrak{y}_1 + \mathfrak{y}_2 + \mathfrak{y}_3 + \cdots + \mathfrak{y}_n) - \mathcal{P}\left[\mathbb{K}^{-1} \left[\vartheta\, \mathbb{K} \left[\sum_{n=0}^{\infty} \mathcal{P}^n \mathcal{H}_n(w) \right] \right] \right]. \quad (14)$$

This is called chain of Kamal transform and Homotopy perturbation method. Presently by contrasting the coefficient of intensity of $\mathcal{P}$. the accompanying approximations are gotten:

$$\mathcal{P}^0 : w_0(Y,t) = \mathfrak{y}_1 + \mathfrak{y}_2 + \mathfrak{y}_3 + \cdots + \mathfrak{y}_n$$

$$\mathcal{P}^n : w_n(Y,t) = -\mathbb{K}^{-1}\{\vartheta\, \mathbb{K}[\mathcal{H}_{n-1}(w)]\},$$

for all natural number n .the solution is given by

$$w(Y,t) = \sum_{n=0}^{\infty} w_n(Y,t). \quad (15)$$

3. Utilization: Utilization of the chain of Kamal transform and Homotopy Perturbation Method (HPM) is utilized for unraveling Burger's equations. In this segment, we apply the Kamal transform and homotopy perturbation method keeping in mind the end goal to get the arrangement .The accompanying cases delineate the utilization of this new coupling method to take care of certain underlying worth issues portrayed by dimensional Burger's equations as nonlinear partial differential equations.

Example 4.1- Consider the $(2+1)$ $-$ dimensional Burger's equation.

$$\frac{\partial w}{\partial t} + \left(w \frac{\partial w}{\partial \mathfrak{x}} + w \frac{\partial w}{\partial \mathfrak{y}} \right) - \psi \left(\frac{\partial^2 w}{\partial \mathfrak{x}^2} + \frac{\partial^2 w}{\partial \mathfrak{y}^2} \right) = 0, \quad (16)$$

subject to the initial condition $w(\mathfrak{x}, \mathfrak{y}, 0) = \mathfrak{x} + \mathfrak{y}$.

Solution: Taking Kamal transform on both side of equation (16), we obtained

$$\mathbb{K}\big(w(\mathfrak{x},\mathfrak{y},t)\big) = \vartheta(\mathfrak{x}+\mathfrak{y}) - \vartheta \mathbb{K}\left\{ \left(w \frac{\partial w}{\partial \mathfrak{x}} + w \frac{\partial w}{\partial \mathfrak{y}} \right) - \psi \left(\frac{\partial^2 w}{\partial \mathfrak{x}^2} + \frac{\partial^2 w}{\partial \mathfrak{y}^2} \right) \right\}.$$

Next , we take the inverse Kamal transform to obtain

$$w(x,\mathfrak{y},t) = (\mathfrak{x}+\mathfrak{y}) - \mathbb{K}^{-1}\left[\vartheta \mathbb{K}\left\{ \left(w \frac{\partial w}{\partial \mathfrak{x}} + w \frac{\partial w}{\partial \mathfrak{y}} \right) - \psi \left(\frac{\partial^2 w}{\partial \mathfrak{x}^2} + \frac{\partial^2 w}{\partial \mathfrak{y}^2} \right) \right\} \right].$$

Now, applying the homotopy perturbation method we get:

$$\sum_{n=0}^{\infty} \mathcal{P}^n w_n(\mathfrak{x},\mathfrak{y},t) = \mathfrak{x}+\mathfrak{y} - \mathcal{P}\left[\mathbb{K}^{-1}\left[\vartheta\,\mathbb{K}\left[\sum_{n=0}^{\infty}\mathcal{P}^n\mathcal{H}_n(w)\right]\right]\right]. \quad (17)$$

$$\mathcal{P}\left(ww_{\mathfrak{x}} + ww_{\mathfrak{y}}\right) - \psi\mathcal{P}\left(w_{\mathfrak{x}\mathfrak{x}} + w_{\mathfrak{y}\mathfrak{y}}\right) = 0,$$

$$w = w_0 + \mathcal{P}w_1 + \mathcal{P}^2 w_2 + \cdots. \quad (18)$$

Then equation (18) can be written as

$$\mathcal{P}[w_0 + \mathcal{P}w_1 + \mathcal{P}^2 w_2 + \cdots][w_{0\mathfrak{x}} + \mathcal{P}w_{1\mathfrak{x}} + \mathcal{P}^2 w_{2\mathfrak{x}} + \cdots] + \mathcal{P}[w_0 + \mathcal{P}w_1 + \mathcal{P}^2 w_2 + \cdots]$$

$$\left[w_{0\mathfrak{y}} + \mathcal{P}w_{1\mathfrak{y}} + \mathcal{P}^2 w_{2\mathfrak{y}} + \cdots\right] - \psi\mathcal{P}[w_{0\mathfrak{x}\mathfrak{x}} + \mathcal{P}w_{1\mathfrak{x}\mathfrak{x}} + \mathcal{P}^2 w_{2\mathfrak{x}\mathfrak{x}} + \cdots] -$$

$$\psi\mathcal{P}\left[w_{0\mathfrak{y}\mathfrak{y}} + \mathcal{P}w_{1\mathfrak{y}\mathfrak{y}} + \mathcal{P}^2 w_{2\mathfrak{y}\mathfrak{y}} + \cdots\right] = 0\ . \quad (19)$$

On comparing the first few components, of He's polynomials ,we get

$$\mathcal{H}_0(w) = w_0 w_{0\mathfrak{x}} + w_0 w_{0y} - \psi w_{0\mathfrak{x}\mathfrak{x}} - \psi w_{0\mathfrak{y}\mathfrak{y}}\,,$$

$$\mathcal{H}_1(w) = w_0 w_{1\mathfrak{x}} + w_1 w_{0\mathfrak{x}} + w_0 w_{1\mathfrak{y}} + w_1 w_{0\mathfrak{y}} - \psi w_{1\mathfrak{x}\mathfrak{x}} - \psi w_{1\mathfrak{y}\mathfrak{y}}\,,$$

$$\cdots$$

again looking at the coefficient of a similar intensity of $\mathcal{P}$, we arrive,

$\mathcal{P}^0: w_0(\mathfrak{x},\mathfrak{y},t) = \mathfrak{x}+\mathfrak{y}\ ,\mathcal{H}_0(w) = 2(\mathfrak{x}+\mathfrak{y})$

$\mathcal{P}^1: w_1(\mathfrak{x},\mathfrak{y},t) = -\mathbb{K}^{-1}\{\vartheta\mathbb{K}[\mathcal{H}_0(w)]\} = 2t(\mathfrak{x}+\mathfrak{y}),$

$$\mathcal{H}_1(w) = -8t(\mathfrak{x}+\mathfrak{y})$$

$\mathcal{P}^2: w_2(\mathfrak{x},\mathfrak{y},t) = -\mathbb{K}^{-1}\{\vartheta\mathbb{K}[\mathcal{H}_1(w)]\} = 4t^2(\mathfrak{x}+\mathfrak{y})$

$\mathcal{P}^3: w_3(\mathfrak{x},\mathfrak{y},t) = -8t^3(\mathfrak{x}+\mathfrak{y})$

$\mathcal{P}^4: w_4(\mathfrak{x},\mathfrak{y},t) = 16t^4(\mathfrak{x}+\mathfrak{y})$

$$\cdots$$

In conclusion, the arrangement $w(\mathfrak{x},\mathfrak{y},t)$ is given by

$$w(\mathfrak{x},\mathfrak{y},t) = w_0(\mathfrak{x},\mathfrak{y},t) + w_1(\mathfrak{x},\mathfrak{y},t) + w_2(\mathfrak{x},\mathfrak{y},t) + \cdots$$

$$= (\mathfrak{x}+\mathfrak{y}) - 2t(\mathfrak{x}+\mathfrak{y}) + 4t^2(\mathfrak{x}+\mathfrak{y}) - 8t^3(\mathfrak{x}+\mathfrak{y}) + 16t^4(\mathfrak{x}+\mathfrak{y}) + \cdots$$

$$= (\mathfrak{x}+\mathfrak{y})[1 - 2t + 4t^2 - 8t^3 + 16t^4 + \cdots],$$

In series form, and

$$w(\mathfrak{x},\mathfrak{y},t) = \frac{\mathfrak{x}+\mathfrak{y}}{1+2t}.$$

In finished up shape.

Example 4.2 Let us consider $(3+1)-$ dimensional Burger's equation

$$\frac{\partial w}{\partial t} + \left(w\frac{\partial w}{\partial \mathfrak{x}} + w\frac{\partial w}{\partial \mathfrak{y}} + w\frac{\partial w}{\partial z}\right) - \psi\left(\frac{\partial^2 w}{\partial \mathfrak{x}^2} + \frac{\partial^2 w}{\partial \mathfrak{y}^2} + \frac{\partial^2 w}{\partial z^2}\right) = 0\,, \quad (20)$$

subject to the initial condition $w(\mathfrak{x},\mathfrak{y},z,0) = \mathfrak{x}+\mathfrak{y}+z$.

Solution. By the same method in example 4.1, we apply homotopy perturbation method. After taking Kamal transform and the inverse Kamal transform of the equation (20), we get

$$\sum_{n=0}^{\infty}\mathcal{P}^n w_n(\mathfrak{x},\mathfrak{y},t) = \mathfrak{x}+\mathfrak{y}+z - \mathcal{P}\left[\mathbb{K}^{-1}\left[\vartheta\mathbb{K}\left[\sum_{n=0}^{\infty}\mathcal{P}^n\mathcal{H}_n(w)\right]\right]\right], \quad (21)$$

where

$$\mathcal{H}_0(w) = w_0 w_{0\mathfrak{x}} + w_0 w_{0\mathfrak{y}} + w_0 w_{0z} - \psi(w_{0\mathfrak{x}\mathfrak{x}} + w_{0\mathfrak{y}\mathfrak{y}} + w_{0zz}),$$

$$\mathcal{H}_1(w) = w_0 w_{1\mathfrak{x}} + w_1 w_{0\mathfrak{x}} + w_0 w_{1\mathfrak{y}} + w_1 w_{0\mathfrak{y}} + w_0 w_{1z} + w_1 w_{0z} - \psi(w_{1\mathfrak{x}\mathfrak{x}} + w_{1\mathfrak{y}\mathfrak{y}} + w_{1zz}),$$

looking at the coefficients of similar forces of $\mathcal{P}$, we acquired

$$\mathcal{P}^0: w_0(\mathfrak{x},\mathfrak{y},z,t) = \mathfrak{x}+\mathfrak{y}+z \ ; \mathcal{H}_0(w) = 3(\mathfrak{x}+\mathfrak{y}+z)$$

$$\mathcal{P}^1: w_1(\mathfrak{x},\mathfrak{y},z,t) = -\mathbb{K}^{-1}\{\vartheta K[\mathcal{H}_0(w)]\} = -3t(\mathfrak{x}+\mathfrak{y}+z),$$

$$H_1(w) = -18t(\mathfrak{x}+\mathfrak{y}+z)$$

$$\mathcal{P}^2: w_2(\mathfrak{x},\mathfrak{y},z,t) = -\mathbb{K}^{-1}\{\vartheta K[\mathcal{H}_1(w)]\} = 9t^2(\mathfrak{x}+\mathfrak{y}+z)$$

$$\mathcal{P}^3: w_3(\mathfrak{x},\mathfrak{y},z,t) = -27t^3(\mathfrak{x}+\mathfrak{y}+z)$$

...

At long last the arrangement $w(x,\mathfrak{y},z,t)$ is given by

$$w(\mathfrak{x},\mathfrak{y},z,t) = w_0(\mathfrak{x},\mathfrak{y},z,t) + w_1(\mathfrak{x},\mathfrak{y},z,t) + w_2(\mathfrak{x},\mathfrak{y},z,t) + \cdots$$

$$= (\mathfrak{x}+\mathfrak{y}+z) - 3t(\mathfrak{x}+\mathfrak{y}+z) + 9t^2(\mathfrak{x}+\mathfrak{y}+z) - 27t^3(\mathfrak{x}+\mathfrak{y}+z) + \cdots$$

$$= (\mathfrak{x}+\mathfrak{y}+z)[1 - 3t + 9t^2 - 27t^3 + \cdots],$$

In series form, and

$$w(\mathfrak{x},y,z) = \frac{\mathfrak{x}+\mathfrak{y}+z}{1+3t}.$$

In concluded form.

Example-4.3 We would like to consider$(n+1)$ −dimensional Burger's equation

$$\frac{\partial w}{\partial t} + \left(w\frac{\partial w}{\partial \mathfrak{y}_1} + w\frac{\partial w}{\partial \mathfrak{y}_2} + \cdots + w\frac{\partial w}{\partial \mathfrak{y}_n}\right) - \psi\left(\frac{\partial^2 w}{\partial {\mathfrak{y}_1}^2} + \frac{\partial^2 w}{\partial {\mathfrak{y}_2}^2} + \cdots + \frac{\partial^2 w}{\partial {\mathfrak{y}_n}^2}\right) = 0\,, \qquad (22)$$

with the initial condition

$$w(\mathfrak{y}_1,\mathfrak{y}_2,\dots,\mathfrak{y}_n,0) = \mathfrak{y}_1 + \mathfrak{y}_2 + \cdots + \mathfrak{y}_n.$$

Solution. By using the same method in example 4.2,we apply homotopy perturbation method .After taking Kamal transform and the inverse Kamal transform of the equation (22), we get

$$\sum_{n=0}^{\infty}\mathcal{P}^n w_n(\mathfrak{y}_1,\mathfrak{y}y_2,\dots,\mathfrak{y}_n,t) = (\mathfrak{y}_1+\mathfrak{y}_2+\cdots+\mathfrak{y}_n) - \mathcal{P}\left[\mathbb{K}^{-1}\left[\vartheta\mathbb{K}\left[\sum_{n=0}^{\infty}\mathcal{P}^n\mathcal{H}_n(w)\right]\right]\right],$$

where

$$\mathcal{H}_0(w) = \sum_{i=0}^{n} w_0 w_{0\mathfrak{y}i} - \psi\sum_{i=o}^{n} w_{0\mathfrak{y}i\mathfrak{y}i}\,,$$

$$\mathcal{H}_1(w) = \sum_{i=0}^{n}(w_0 w_{1\mathfrak{y}i} + w_1 w_{0\mathfrak{y}i}) - \psi\sum_{i=o}^{n} w_{1\mathfrak{y}i\mathfrak{y}i},$$

taking a gander at the coefficients of comparative powers of $\mathcal{P}$, we get

$$\mathcal{P}^0: w_0(Y,t) = \sum_{i=1}^{n}\mathfrak{y}_i\,, \mathcal{H}_0(w) = n\sum_{i=1}^{n}\mathfrak{y}_i$$

$$\mathcal{P}^1: w_1(Y,t) = -nt\sum_{i=1}^{n}\mathfrak{y}_i \ \ ,\mathcal{H}_1(w) = -2n^2t\sum_{i=1}^{n}\mathfrak{y}_i$$

$$\mathcal{P}^2: w_2(Y,t) = n^2t^2\sum_{i=1}^{n}\mathfrak{y}_i$$

$$\mathcal{P}^3: w_3(Y,t) = -n^3t^3\sum_{i=1}^{n}\mathfrak{y}_i$$

...

where $Y = \mathfrak{y}_1,\mathfrak{y}_2,\dots,\mathfrak{y}_n$.

Consequently, we obtain

$$w(\mathfrak{y}_1,\mathfrak{y}_2,\dots,\mathfrak{y}_n,t) = \sum_{i=1}^{n}\mathfrak{y}_i\,[1 - nt + n^2t^2 - n^3t^3 + \cdots]\,,$$

In series form, and

$$w(\mathfrak{y}_1,\mathfrak{y}_2,\dots,\mathfrak{y}_n,t) = \frac{\sum_{i=1}^{n}\mathfrak{y}_i}{1+nt}.$$

In finished up shape.
Which is a correct arrangement of equation (22).

II. CONCLUSION

Kamal homotopy perturbation method has been illustrated. The arrangement of Burger's equation by blend of Kamal transform and homotopy perturbation method are presented. This method has been effectively utilized to acquire the estimated solution of Burger's equation. The outcome affirms that the Kamal transform strategy is a basic and great instrument. We contrasted the outcome got and Elzaki Homotopy Perturbation Method (EHPM) [7] and saw that they are the same. It is in this manner demonstrated that the legitimacy of Kamal Homotopy Perturbation Method (KHPM) is dependable.

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