An Analysis of Mortgage point Payment, Breakeven duration of Ownership, cost of point Payment and Personal Marginal Tax Rate

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Abstract

We analytically proved that the breakeven duration of ownership for any mortgage point payment is independent of the principal amount. Using a 6% per annum traditional 30-year fixed rate mortgage, repeated using 3% per annum, and assuming each point payment reduces the mortgage rate by 12.5 basis points from the 6% or 3% fixed rate, we proved numerically that, *ceteris paribus*, the breakeven ownership duration is inversely related to the marginal tax rate, but the breakeven ownership duration is positively related to the points paid, and to the cost of capital that the homebuyer faces to finance the point payment.

Keywords: Mortgage, marginal tax rate, financing

I. Introduction

In a typical home mortgage, the lender often offers "point" option to the borrower in exchange for lower rate of interest. *x* points mean the borrower pays the lender *x* percent of the mortgage's principal amount on the closing day. Typically, one point lowers the interest rate by one-eighth of a percent, and mortgages with 1 or 2 points are most common while those of 3, 4 or 5 points are not unseen. Other than receiving a lower interest rate after making point payment, borrowers are also motivated to pay for points on the closing day because the dollar amount of points paid is tax-deductible in the fiscal year when the closing takes place.

In this study, we attempt to analyze the minimum number of years a borrower needs to own the house in order to breakeven the point payment made at closing after considering her personal marginal tax rate and the opportunity cost of capital to finance the point payment. In order to make the results of the analysis of practical values, we co-vary the marginal tax rate and the opportunity cost of point payment of the borrower over their respective reasonable domains to find the arrays of breakeven duration for one-through-five-point payments.

II. Significance of Project

One of the few major options for mortgage financing, point payment enables a borrower to bargain for a lower interest rate over the life of the mortgage, *ceteris paribus*. Federal government's Internal Revenue Code, meanwhile, permits mortgage point payment to be tax-deductible in the fiscal year when the closing takes place. Unfortunately, neither the lender nor the borrower, barring a few finance gurus, has any idea what is the minimum duration of ownership in order for the borrower to breakeven her point payment. The analysis of this project provides an answer to that question by varying the three most salient variables, to wit the points paid, the borrower's opportunity cost of capital, and the marginal tax rate of the borrower. We hope the plain exposition of the analysis will enable more borrowers to understand the points option before utilizing it, and more lenders to offer such option that is mutually beneficial to both parties.

III. Relation of Proposed Work

Knight and Knight (1995) document that points are deductible as interest if taxpayers pay them out of their own funds to a bank or financial institution for the use of money - usually to secure a lower interest rate. Points are not deductible if they are withheld from the loan proceeds or if the taxpayer pays them with borrowed funds. Lieber (2006) documents that only 1.4% of all homebuyers hold the mortgage long enough to breakeven.

General plan

a. **Project design**

Since the intended “readers” and/or “users” of this analysis are the homebuyers and those who advise them, it becomes imperative that simplicity or plainness of expression cannot be compromised. Ergo, we design our analysis along that plain and simple line for the readership of the general public.

b. **Methods and procedures**

We begin with a numerical example that illustrates the use of 2-point payment by assuming a homebuyer’s cost of capital, marginal tax rate, and $1.0 million in principal amount that she needs 8 years 7 months and 24 days to breakeven. The choice of $1 million principal is arbitrary at this point. Later, we will prove analytically the breakeven duration is independent of the principal amount.

Case I: no point paid with 6% p.a. mortgage rate

$1.0m principal, 30-year traditional fixed rate, 6% per annum without any point traditional mortgage. Translating from annual frequency mode to monthly frequency mode, we use 360 months at 0.5% per month for the $1.0m principal. Applying the regular present value of annuity formula that says the present of an annuity is the product of the annuity, A, and the present value interest factor annuity, PVIFA, we have:

Present value of annuity = Annuity * Present value interest factor annuity

$$PV = A * PVIFA_{r\%,,T} = A * \left[\frac{1 - \frac{1}{(1+r)^T}}{r} \right]$$

$$1{,}000{,}000 = A * PVIFA_{6\%/12,360} = A * \left[\frac{1 - \frac{1}{\left(1 + \frac{0.06}{12}\right)^{360}}}{\left(\frac{0.06}{12}\right)} \right]$$

Solving for A, we get A = 5,995.51 $/month

Case II: 2 points paid made at t=0 to reduce mortgage rate to 5.75%

$1.0m principal, 30-year fixed rate, 5.75% p.a. with 2 points traditional mortgage. Assuming the homebuyer has $20,000 cash up-front for the 2-point payment that earns 4% p.a. in a savings account, and she is in the 30% marginal tax bracket.

$$1{,}000{,}000 = A * PVIFA_{5.75\%/12,360} = A * \left[\frac{1 - \frac{1}{\left(1 + \frac{.0575}{12}\right)^{360}}}{\left(\frac{0.0575}{12}\right)} \right]$$

Solving for A, we get A = 5,835.73 $/month

So, monthly savings of the with-point mortgage payment over the without-point mortgage payment = 5995.51 – 5835.73 = 159.78 \$/month.

Assuming the homebuyer is in the 30% tax bracket, and the point payment of \$20,000 provides her with full tax shelter without any further restrictive qualifications.[1] This provides her with an after-tax savings of \$6,000, meaning she actually pays \$14,000 for the points. This \$14,000 is the amount that she has to "earn back" by monthly savings of the \$159.78.[2]

Diagrammatically, the cash flows, with downward arrow for outflow and upward arrow for inflow, manifest themselves in Diagram 1 as:

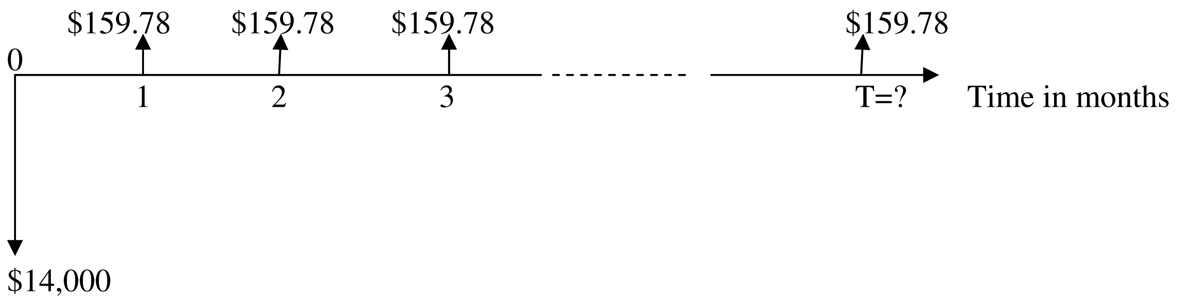

Diagram 1: After-tax cash flows of mortgage-point payment and the monthly savings incurred by the 2% points paid which reduce the 6% p.a. rate to 5.75% for a \$1 million principal 30-year fixed rate traditional mortgage

The question to ask after seeing Diagram 1 is: what is the interest rate to use to discount the monthly \$159.78 monthly savings? That rate corresponds to the *opportunity cost* of homebuyer. She either has the cash upfront or she can take another loan to finance the upfront points payment of \$14,000. If she takes the loan, the interest rate of the loan is the discount rate for the monthly savings. In this example, let's assume the homebuyer has sufficient savings in her bank that pays her 4% p.a., a rate that we will use to discount the \$159.78 monthly savings.

In the parlance of the present value algebra, we have:

$$159.78 * PVIFA_{(4/12)\%,T} = 14,000 = 20,000*(1 - 0.3)$$

$$159.78 * \left[\frac{1 - \dfrac{1}{\left(1 + \dfrac{0.04}{12}\right)^{T}}}{\left(\dfrac{0.04}{12}\right)} \right] = 14,000$$

To solve for T using the financial calculator, we enter:
I/Y = 4/12; PV = –14,000; PMT = 159.78; cpt N = 103.79749 months ≈ 8 yrs 7 months 24 days.

For those who still remember well high-school mathematics involving exponents, we can obtain the same result of 103.79748 months by solving for T using the natural logarithmic function of the expression $159.78 * \{[1 - 1/(1+0.04/12)^{T}]/(0.04/12)\} = 14,000$ using any

[1] Internal Revenue Code, section 461(g) 2 allows full deduction of mortgage-points payment as prepaid interest in the year of *purchase* if the property is the buyer's principal residence. For *refinancing*, Internal Revenue Code requires mortgage-points payment to be amortized. Our analysis in this paper assumes purchase, not refinancing.
[2] We ignore the time value of money created as a result of the \$20,000 payment upfront and the \$6,000 tax deduction obtained by the end of the fiscal year.

regular scientific calculator after making T the subject to one side of the equation. Using Microsoft Excel, the financial formula's syntax to find the same answer is:

=nper(4%/12,159.78,-14000,0,0) where "nper" stands for the number of periods.

This means the homebuyer needs to own her home for at least 8 years 7 months 24 days to breakeven for the 2 points paid at the inception of the mortgage. Ownership shorter than that duration is not financially advisable for her, while ownership longer than that duration works in her favor to pay for the 2 points upfront.

c. **Generalization**

Let's set up the general formula for solving the breakeven number of months as:

$$\left(\frac{M}{PVIFA_{(x/12)\%,360}}-\frac{M}{PVIFA_{(y/12)\%,360}}\right)*PVIFA_{(z/12)\%,T}=\left(\frac{p}{100}\right)*M*(1-t) \quad \ldots\ldots\ldots\ldots\ldots.. \quad (1)$$

where:
M = original principal amount of the mortgage in $
x = percent interest without points
y = percent interest with p points
z = opportunity cost the homebuyer has to forgo to pay for the point(s)
p = number of point(s)
t = marginal tax rate which the homebuyer faces
T = number of months for the homebuyer to breakeven the p-point payment upfront

It is obvious that the solution for T, the breakeven number of months is independent of the mortgage amount M since M can be factored out on both sides, and then be cancelled out. Moreover, if we assume one paid mortgage point, p=1, will incur an eighth of percent point or 12.5 basis points reduction, the array of outcomes we obtain further depends on the three variables, namely the level of opportunity costs z, the point payment p, and the marginal tax rate the homebuyer faces t.

For readers who are well-versed in capital-budgeting techniques, it should not be too hard to recognize that the solution for T is indeed the *discounted payback period*, dPB that so many corporate finance textbooks discuss *ad nauseam* although it manifests in this context in a masqueraded manner. The number of months needed to breakeven the point payment is simply the discounted payback period, expressed in months, the homebuyer needs to wait in recouping her lump sum point payment by discounting all future monthly savings at the opportunity cost she faces at z% per annum.

With the above general framework, we now proceed to estimate the arrays of triad combinations for practically meaningful domains on (points paid, opportunity cost of capital for points, marginal tax rate), or (p, z, t) using the notations listed in the above equation.

d. **Use of Excel spreadsheet**

To facilitate the computation of the numerous triad combinations of result, we use the Microsoft Excel's Data Table function. We rearrange Equation (1) above into Equation (2) below.

$$PVIFA_{(z/12)\%,T} = \frac{\left(\frac{p}{100}\right)*(1-t)}{\left[\left(\frac{1}{PVIFA_{(x/12)\%,360}}\right)-\left(\frac{1}{PVIFA_{(y/12)\%,360}}\right)\right]} \quad \text{.................................... (2)}$$

To generate the numerous triad combinations of results, we code Equation (2) above in Excel as follows:

	A	B	C	D	E	F	G	H
1	p	2			Point's opportunity cost, z			
2	x	0.06		=B11	0.01	0.02	0.03	0.04
3	y	=B2-0.125*B1/100	Marginal tax rate, t	0.1				
4	z	0.04		0.2				
5	t	0.3		0.3				
6	numerator	=B1*(1-B5)/100		0.4				
7	PVIFA_x%	=1/PMT(B2/12,360,-1,0,0)						
8	PVIFA_y%	=1/PMT(B3/12,360,-1,0,0)						
9	denominator	=(1/B7 - 1/B8)						
10	PVIFA_T	=B6/B9						
11	breakeven T	=NPER(B4/12,-1,B10,0,0)/12						

Table 1: Microsoft Excel codes used to generate the number of years needed to breakeven the points paid p, at the opportunity cost of borrower z and the marginal tax rate t for a 30-year fixed rate mortgage at 6% p.a. We assume 1 point paid will reduce the interest rate by 12.5 basis points as in cell B3.

To generate the table in cells D2 through H6, we enter "=B11" in Cell D2. We next enter practical range of opportunity costs for the points paid in cells E2 through H2, and marginal tax rates range in cells D3 through D6. Considering today's low interest rate environment for depositors, we use 1% through 4% as the borrower's opportunity costs for the points paid, and we use 10% through 40% marginal tax rates. Then, we select cells D2 through H6, choose *Data*, followed by *What-if Analysis*, followed by *Data Table*. A small interactive window will pop up for us to enter *Row input cell* as B4, and *Column input cell* as B5. Click the *OK* icon in the bottom of the interactive window will enable Excel to generate the results in Cells E3 through H6.

In short, given an interest rate for the mortgage, in this case it is 6% p.a. input in Cell B2, we can obtain the triad combination of results by entering values in Cells B1, B4 and B5. The breakeven year will show in Cell B11. The table in Cells D2 through H6 will then show sensitivity-analysis results when opportunity cost of points is in the 1%-to-4% domain and the marginal tax rate is in the 10%-to-40% domain. We repeat the above exercise for 2, 3, 4, and 5 points payment. Of course, we can change the numerical values in the z- and p-domain so that they are more indicative of the real rates the borrower faces. We present the results in Table 2.

Table 2: Number of years for a homebuyer to breakeven the point payment for a 6% per annum 30-year traditional fixed-rate mortgage. We assume each paid point will reduce the annual mortgage rate by 1/8 of a percent point or 12.5 basis points from the original 6% per annum rate without any paid point. We estimate the arrays of results for 1%, 2%, 3% and 4%

opportunity cost of paid points, and for marginal tax rates of 10%, 20%, 30%, and 40% for the homebuyer.

Marginal tax rates, t%	Paid point, p=1, y=5.875% p.a.			
	z=1%	z=2%	z=3%	z = 4%
10	9.83	10.37	11.00	11.75
20	8.69	9.11	9.58	10.13
30	7.56	7.87	8.22	8.62
40	6.45	6.67	6.92	7.19

Marginal tax rates, t%	Paid point, p=2, y=5.750% p.a.			
	z=1%	z=2%	z=3%	z = 4%
10	9.86	10.41	11.04	11.79
20	8.72	9.14	9.62	10.17
30	7.59	7.90	8.25	8.65
40	6.47	6.69	6.94	7.22

Marginal tax rates, t%	Paid point, p=3, y=5.625% p.a.			
	z=1%	z=2%	z=3%	z = 4%
10	9.89	10.44	11.08	11.84
20	8.75	9.17	9.65	10.21
30	7.61	7.93	8.28	8.68
40	6.49	6.71	6.96	7.24

Marginal tax rates, t%	Paid point, p=4, y=5.500% p.a.			
	z=1%	z=2%	z=3%	z = 4%
10	9.93	10.48	11.12	11.88
20	8.77	9.20	9.69	10.25
30	7.63	7.95	8.31	8.71
40	6.51	6.74	6.99	7.27

Marginal tax rates, t%	Paid point, p=5, y=5.375% p.a.			
	z=1%	z=2%	z=3%	z = 4%
10	9.96	10.51	11.16	11.93
20	8.80	9.23	9.72	10.29
30	7.66	7.98	8.34	8.75
40	6.53	6.76	7.01	7.29

In order the compare the effect of interest rate on the breakeven-year computations for 1 through 5 points payments, we repeat the above exercise using 3% p.a. Again, we assume 1 point will pare 12.5 basis points off the original 3% interest rate without any point. We use a different opportunity cost domain of .25% through 1% since the opportunity cost the homebuyer faces is typically lower than the mortgage rate of 3% p.a. assumed in this case. The 10%-to-40% marginal-tax domains for the homebuyer remain unchanged in this case. We present the results in Table 3. The repeat is prompted by the fact that we are in a low-interest-rate environment now, and the 3% p.a. rate is a more realistic rate than the 6% p.a. rate.

Table 3: Number of years for a homebuyer to breakeven the point payment for a 3% per annum 30-year traditional fixed-rate mortgage. We assume each paid point will reduce the annual mortgage rate by 1/8 of a percent point or 12.5 basis points from the original 6% per annum rate without any paid point. We estimate the arrays of results for 1%, 2%, 3% and 4% opportunity cost of paid points, and for marginal tax rates of 10%, 20%, 30%, and 40% for the homebuyer.

Marginal tax rates, t%	Paid point, p=1, y=2.875% p.a.			
	z=.25%	z=.50%	z=.75%	z = 1.00%
10	11.33	11.50	11.67	11.85
20	10.06	10.19	10.33	10.47
30	8.79	8.89	8.99	9.10
40	7.52	7.59	7.67	7.75

Marginal tax rates, t%	Paid point, p=2, y=2.750% p.a.			
	z=.25%	z=.50%	z=.75%	z = 1.00%
10	11.39	11.55	11.73	11.91
20	10.11	10.24	10.37	10.52
30	8.83	8.93	9.03	9.14
40	7.56	7.63	7.70	7.78

Marginal tax rates, t%	Paid point, p=3, y=2.625% p.a.			
	z=.25%	z=.50%	z=.75%	z = 1.00%
10	11.44	11.61	11.79	11.97
20	10.15	10.29	10.42	10.57
30	8.87	8.97	9.07	9.18
40	7.59	7.66	7.74	7.82

Marginal tax rates, t%	Paid point, p=4, y=2.500% p.a.			
	z=.25%	z=.50%	z=.75%	z = 1.00%
10	11.49	11.66	11.84	12.03
20	10.20	10.33	10.47	10.62
30	8.91	9.01	9.12	9.23
40	7.62	7.70	7.78	7.86

Marginal tax rates, t%	Paid point, p=5, y=2.375% p.a.			
	z=.25%	z=.50%	z=.75%	z = 1.00%
10	11.55	11.72	11.90	12.09
20	10.25	10.38	10.52	10.67
30	8.95	9.06	9.16	9.27
40	7.66	7.74	7.81	7.89

IV. Discussion

Breakeven duration increases with points paid and opportunity cost, but decreases with borrower's marginal tax rate. Across the five panels in Table 2, we observe that as the paid points increase from 1% to 5%, the breakeven duration increases, *ceteris paribus*. Within each panel, we observe that an increase in opportunity cost of paid points results in an increase in breakeven duration, and that an increase in marginal tax rate incurs a decrease in breakeven duration. These results imply that during low-interest-rate eras when the opportunity cost of paid points is low, a homebuyer should have paid as many points as her budget or credit limit permits, with those homebuyers in the higher marginal tax rates enjoy shorter breakeven duration than those in lower marginal tax rates do.

Breakeven duration increases with a decrease in mortgage interest rate. Table 3 exhibits the same trends as those discussed for Table 2 in the above paragraph. The purpose of generating Table 3, however, is to compare the breakeven number of years needed at a mortgage interest rate level that is only half of that used to generate the results in Table 2. At 3% p.a. of mortgage rate before point deduction, we see that the breakeven number of year in Table 3 increases when compared to the corresponding breakeven number of year at the 6% p.a. level in Table 2. For example, the (p,z,t) of (2%,4%,30%) breaks even at 8.68 years in Table 2 but the same (p,z,t) combination only breaks even at 10.82 years, nearly 25% increase, in Table

3. This means that points are expected to be more popular in use in a high-interest-rate environment than in a low-interest-rate one.

Another way to frame the same problem is through the net-present-value, NPV, method. The breakeven period or discounted payback period requires us to solve for the unknown T in Diagram 1. However, if the borrower is certain of living in the same house for the next 30 years, then Diagram 1 will become Diagram 2 as follows.

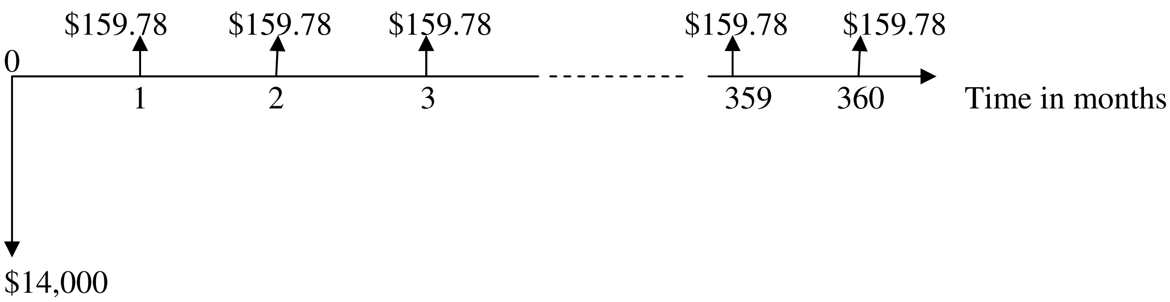

Diagram 2: After-tax cash flows of mortgage-point payment and the monthly savings incurred by the 2% points paid which reduce the 6% p.a. rate to 5.75% for a $1 million principal 30-year fixed rate traditional mortgage

Using 4% p.a. or (1/3)% per month as the after-tax opportunity cost of capital for this borrower, the NPV for the 2-point payment that reduces the 6% p.a. rate to 5.75% p.a. rate is simply

$$NPV = \sum_{1}^{360} \frac{159.78}{\left(1+\frac{.04}{12}\right)^{360}} - 14{,}000 = \$19{,}467.72.$$

Since the NPV is almost 140% of the initial cost of after-tax point payment at 4% p.a. cost of capital, the point payment is a value-adding option for the homebuyer.

The next natural question to ask is at what rate of the opportunity cost of capital will the NPV decrease to zero? That rate is the solution of r in the following equation.

$$NPV = \sum_{1}^{360} \frac{159.78}{\left(1+\frac{r}{12}\right)^{360}} - 14{,}000 = 0$$

The unknown “r” bears the moniker of *internal rate of return*, IRR, in capital budgeting practice. Its solution for this numerical example is 13.4475% p.a. or 1.1206% per month. This means the opportunity cost for the homebuyer can be as high as 13.4475% p.a. or 1.1206% per month for her to breakeven the $14,000 upfront after-tax point payment if she is certain of living that house for the next 30 years.

We can also use the Excel program in Table 1 to find the optimal (p,z,t) combination subject to a certain discounted payback period. For example, if a prospective homebuyer knows in advance that she will stay in a house for exactly 10 years, what is the (p,z,t) combination that will enable her to get there? To accomplish the objective, she will simply click *Data*, and then *Solver*. An interactive window will appear, and into it we will enter:

Set Target Cell: B11
Equal to: Value of 10
By changing cells: B1,B4, B5

Click the icon Solve in the top right hand corner.
The (p,z,t) results after clicking the Solve icon are p=2.0000; z=0.0636; t=0.2920.

These results mean that at a mortgage rate of 6% p.a., the homebuyer can pay 2 points to lower the 6% rate to 5.75% p.a., and if her opportunity cost is 6.36% and her marginal tax rate is 29.2%, then she will take exactly 10 years to breakeven the $14,000 after-tax point payment.

The discussions so far all hinge on points paid by the borrower to the mortgage firm, or positive points. In the mortgage industry, however, negative points are also available. In a negative-point arrangement, the mortgage firm pays the borrower a certain amount of cash, which the borrower uses for closing costs and other expenses, in exchange for a higher mortgage rate. The analyses of the negative points are almost symmetrical to those of the positive points. The asymmetric part is the lack of tax shelter, or simply t=0 in the framework established so far. In addition, the "–" sign in Cell B3 has to be changed to "+," the t rate must equal 0 in Cell B5, and the sign in B9 has to be reversed. Using the same set of sample values, we estimated that a 6% mortgage with 2 negative points which raise the rate to 6.25% p.a. will entail 13y3m23d to breakeven for a borrower with 4% opportunity cost of capital.

V. Conclusion

In planning for mortgage payment, the option to reduce the interest rate by paying for points upfront can be framed as an NPV exercise in which the breakeven duration of ownership corresponds to the discounted payback period. In estimating the breakeven duration, we found that the duration varies positively with the level of the point paid, and the opportunity cost of capital, and it varies inversely with the borrower's marginal tax rates.

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