Visualizations in Teaching Algebra

Ed Laughbaum

Also find the published version of the article posted at:
http://education.ti.com/educationportal/sites/US/nonProductMulti/communitycollege_feature_article.html

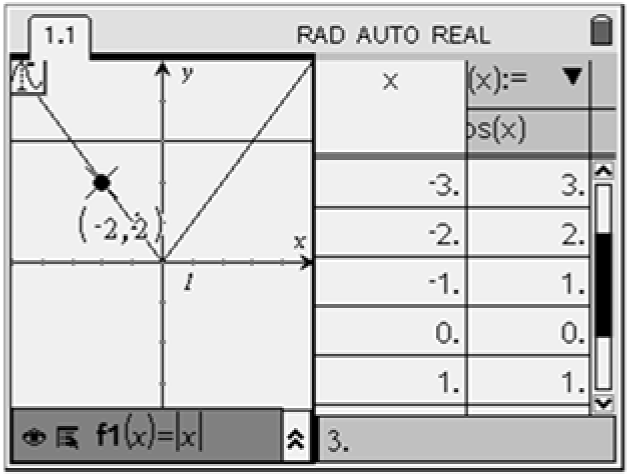
We have all heard it. We have all said it. We all believe it. But we may not use it appropriately in our teaching. In this paper we will explore the value of "it" in day-to-day teaching. What is it? “A picture is worth a thousand words.” As it turns out, a picture is worth much more than a thousand words. In addition, when we use appropriate dynamic visualizations in our teaching, we add an attention factor, and without attention, learning is not possible.

Odds are that readers know the occipital lobes process vision. You may not be aware that, “Neuroplasticity, … can reshape the brain so that a sensory region performs a sophisticated cognitive function” (Bagley, 99). As a result, Steven Pinker (359-360) argues that “Thanks to graphs, we primates grasp mathematics with our eyes and our mind's eye [occipital lobe]. In return, mathematical thinking offers new ways to understand the world. So, vision was co-opted for mathematical thinking, which helps us see [understand] the world.” Notice Pinker did not use “Thanks to symbols, we primates grasp mathematics …” Yet in this country, we often teach algebra as if it were all about symbols. The bottom line may be about symbols, but if you want to capitalize on basic brain function in teaching and learning of concepts and processes, then symbols must come last in the teaching process.

Let's consider the teaching of the Less-Than Theorem for Absolute Values. If $|x| \le a$ then for some positive number a, then $-a \le x \le a$.

The traditional option for teaching (and textbook presentation of) this theorem is to state the theorem, explain it with words, use it in several examples and then it is followed by extensive homework-based practice. All of this with no visualization of the absolute value function, no visualization of a constant function and no dynamic connections within the visualization. If we look at what neuroscience research tells us about visualizations, we find: “… after studying pictures along with the words, ... they [study subjects] easily reject items that do not contain the distinctive pictorial information they [brains] are seeking …” (Schacter, 103). So we are taking the chance that students will not learn, or even remember the theorem because we only using words. But there is more. “… [A]dvocates of dual coding theory argue that people retain information best when it is encoded in both visual and verbal codes” (Byrnes, 51-52). So we miss the opportunity to enhance memory of what we teach by not using visualizations. Ackerman argues “… because we have visual, novelty-loving brains, we're entranced by electronic media” (157). So, even the method of the delivery of visualizations becomes significant to the teaching process as we address student attention through the seamless integration of technology.

Capitalizing on a visual/function approach, we start with the graphical, numeric and symbolic representations of the function $y = |x|$ and the graph of a positive constant function like $y = 3$, and build a pattern leading students to generalize the theorem. Unlike the static visualization below, in the classroom every student sees an interactive event loaded with connections.



To see why connections are extremely important, please see http://www.math.ohio-state.edu/~elaughba/papers/EDITORIAL_Associations.pdf.

What do we discover about x as we trace or scroll back-and-forth on the $y = |x|$ function while it is less than, or equal to, 3? It is that x is on the interval [–3, 3]? Is this pattern true for $|x| \leq 2$ or $|x| \leq 1.7$ or $|x| \leq 19$? So it seems that if $|x| \leq a$ then $-a \leq x \leq a$.

Do we also learn something about x when $|x| \geq a$? When students have access to the TI-84 technology, for example, we can assign a pattern-building activity that is a part of homework the day **before** we teach the theorem in class. For example:

1. For the given positive constant 2, find all values of x that cause the function $|x|$ to be less than or equal to 2.
2. For the given positive constant 3, find all values of x that cause the function $|x|$ to be less than or equal to 3.
3. For the given positive constant 5.5, find all values of x that cause the function $|x|$ to be less than or equal to 5.5.
4. For a given positive constant a, make a conjecture on what values of x will cause the function $|x|$ to be less than or equal to a.

(Note: The average brain will generalize after 3 iterations, so more or fewer may be needed - based on class response.)

We use pattern-building activities because "… human brains operate fundamentally in terms of pattern recognition rather than logic [reasoning]. They are highly constructive in settling on given patterns and at the same time are constantly open to error. But after selection occurs … refinements can take place with increasing specificity. … [S]ubsequent application of observation, logic, and mathematics can yield laws or at least strong regularities" (Edelman, 83-84).

The idea is to replace the symbolic process of "explaining" with the pattern building activity as part of homework the day before teaching the theorem in class. This is followed by the dynamic visualization process described above. The visualization contains multiple representations and pattern building used for memory and understanding of the theorem. That is, "… more elaboration during encoding generally produces less transient memories [memory loss over time]" (Schacter, 25). Further, when students get to item four in the pattern-building activity (or in the teacher-directed activity in class), the brain must generalize which "…forms a persistent representation, or memory, for the sequence" (Hawkins, 128). Teachers often assume students will not, or cannot generalize. But this is often because teachers underestimate the innate ability of the brain to generalize patterns. But remember, and we repeat for emphasis: "…human brains operate fundamentally in terms of pattern recognition rather than logic" (Edelman, 83).

Looking at this process another way, "… in general, how well new information is stored in long-term memory depends very much on depth of processing … which produces substantially better memory for events than a structural or surface level of processing" (Thompson & Madigan, 33). Further, [relative to memory] "… many different circuits of neuronal groups could and do give a similar output. If one circuit fails to function, the other is likely to work" (Edelman, 33). So we always include visualizations in pattern-building activities that lead directly to a mathematical generalization. Connections add to the dept of processing. That is, in a function approach, students would have encountered the absolute value function on three previous occasions. They would have seen a connection to a variety of real-world contexts, and all algebra taught is connected through function representation and behaviors.

This is the power of connections embedded in the visualizations. That is, our students have more opportunities for recall because we evoked numerous neural networks. The graphing calculator is the tool used to process visualizations of mathematics. It is the tool that facilitates many of the activities that add to the novelty, the multiplicity of methods, the attention and the connections used in the teaching process.

Visualizations are a basic catalyst to understanding and memory, without them, our brain must work harder to understand concepts and skills, and will have a tendency to reject learning/understanding content presented without visualizations. Visual recognition of problem or situation is the primary, and most influential, connection to understanding, meaning, properties, uses, and skills related to the problem or situation. In addition, when visualization**s** are used before any symbolic development, it greatly increases the likelihood that the memory of the mathematical concept being taught will survive. So we see that there is more to the concept of visualizations than being worth only a thousand words.

References

Ackerman, D. (2004). *An alchemy of mind: The marvel and mystery of the brain*. Schibner. New York.

Diane Ackerman is the author of 10 books of literary nonfiction. http://www.dianeackerman.com/

Bagley, S. (2008). *Train your mind: Change your brain.* Ballantine Books. NY, NY.

Byrnes, J. P., (2001). *Minds, brains and learning: Understanding the psychological and educational relevance of neuroscientific research.* The Guilford Press. New York.

James P. Byrnes is a professor of Human Development in the College of Education at the University of Maryland, College Park, where he has been affiliated with the Neuroscientific and Cognitive Science program. http://www.education.umd.edu/EDHD/faculty2/Byrnes/

Edelman, G. M. (2006). *Second nature: Brain science and human knowledge.* Yale University Press. New Haven, CT.

Gerald Edelman is director of the Neurosciences Institute and president of Neurosciences Research Foundation. He is a Nobel Laureate in medicine. http://www.nsi.edu/public/overview.php

Hawkins, J. (2004). *On intelligence.* Times Books. New York.

Jeff Hawkins created the Redwood Neuroscience Institute http://www.rni.org/directors.html and is a member of the scientific board of Cold Spring Harbor Laboratory.

Pinker, S. (1997). *How the mind works*. W. W. Norton & Company. New York.

Steven Pinker was Professor of Psychology, Director of the Center for Cognitive Neuroscience at the Massachusetts Institute of Technology, and is now the Johnstone Family Professor in the Department of Psychology at Harvard University. http://pinker.wjh.harvard.edu/

Schacter, D. L. (2001). *The seven sins of memory: How the mind forgets and remembers.* Houghton Mifflin Company. Boston.

Daniel Schacter is chair of the Department of Psychology at Harvard University and the author of numerous books on memory and neuropsychology. http://www.wjh.harvard.edu/~dsweb/

Thompson, R. F. & Madigan, S. A. (2005). *Memory.* Joseph Henry Press. Washington, D.C.