Graph-Theoretic Approach for Increasing Participation in Social Sensing

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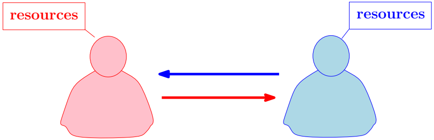
Motivation



- A key to **sustainable cooperation** is that agents share their *resources/capabilities/measurements*.

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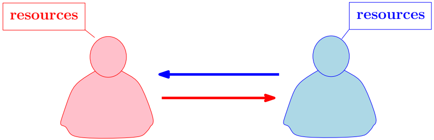
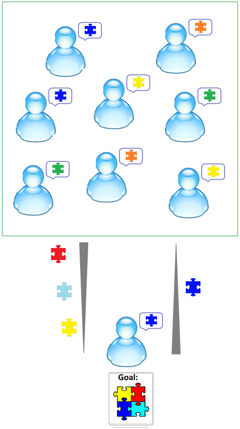




- A key to **sustainable cooperation** is that agents share their *resources/capabilities/measurements*.
- **Heterogeneous** resources/data.
- Users **participate** if they find sufficient value in participation.

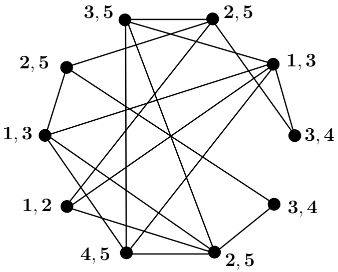
How can we modify or design networks to maximize participation of users with heterogeneous resources?

Contributions

- A graph-theoretic concept to model participation of users with different attributes in networks.
  - **(r, s)-core** in networks,
- To maximize users' participation, we propose two strategies.
  - based on **anchors** - users incentivized to never leave the network.
  - based on **relabeling** - users incentivized to change their attributes.
- Preliminary results
  - **problem complexity**
  - **heuristics** to select anchors and relabel networks to maximize users' participation.
  - **numerical evaluation**.

Network Model

- **Network graph** $G(V, E)$
  - set of users: V
  - interaction between users: E



Network Model

- **Network graph** $G(V, E)$
  - set of users: V
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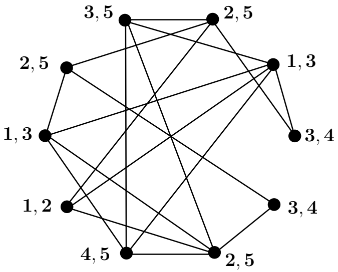
- **User's attributes**
  - set of labels: $R = \{1, 2, \cdots, r\}$
  - each user has s labels
  - labels assignment function:

$$\ell : V \longrightarrow [R]_s$$



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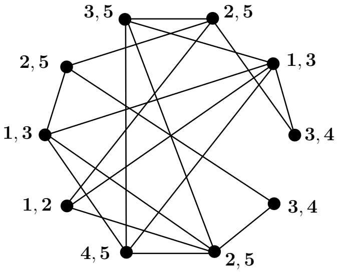
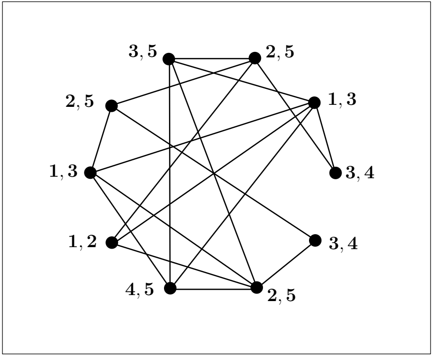


- **User participation rule**
  - A user x participates in the network as long as its neighbors $N(x)$ provide the labels that are missing from the x's own label set $\ell(x)$, i.e.,

$$\bigcup_{y \in (\{x\} \cup N(x))} \ell(y) = R.$$

Cascading Effect

- A node leaving the network can have a cascading effect, as it may also cause its neighbors to leave the network.
- Here, $R = \{1, 2, \cdots, 5\}$, and $s = 2$.



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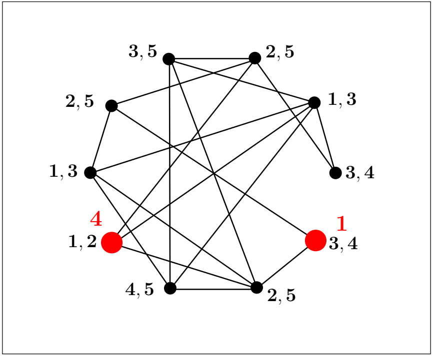

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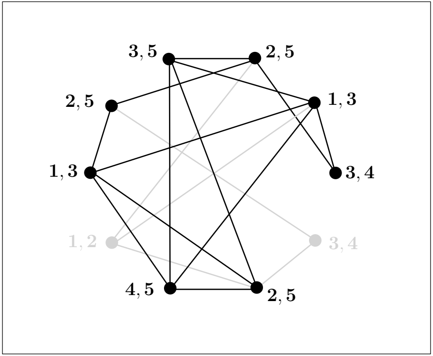

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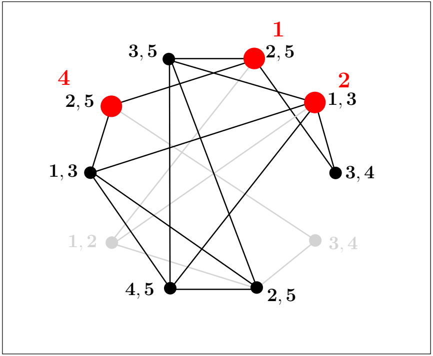

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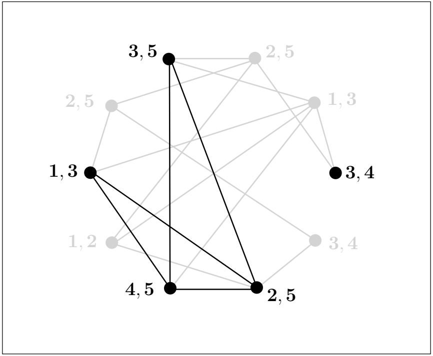

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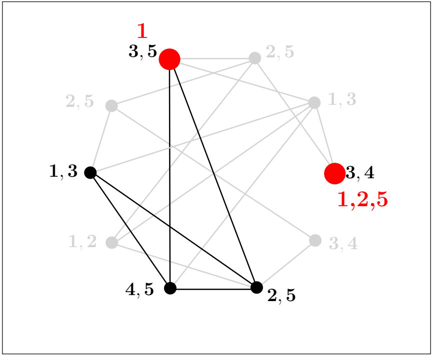

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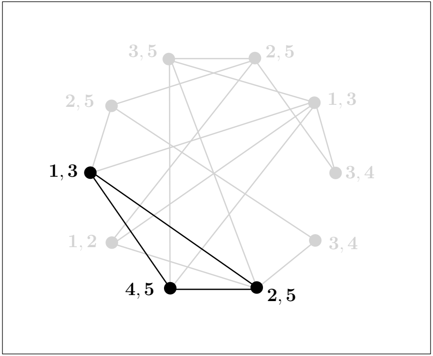

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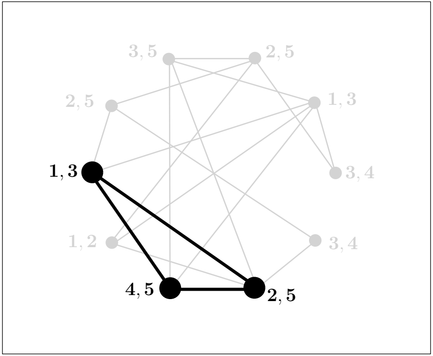

(r, s)-Core of the Network

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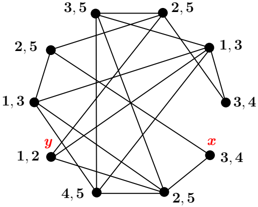
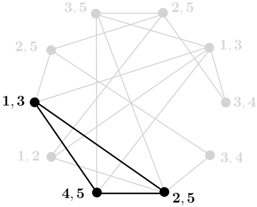




(5,2)-core

(r, s)-Core

The **(r, s)-core of G**, denoted by $\tilde{G}(\tilde{V}, \tilde{E})$, is the maximal subgraph in which every node has all r labels in its neighborhood in $\tilde{G}$, i.e.,

$$\bigcup_{y \in (\{x\} \cup N(x)) \cap \tilde{V}} \ell(y) = R, \quad \forall x \in \tilde{V}.$$

Main Objective

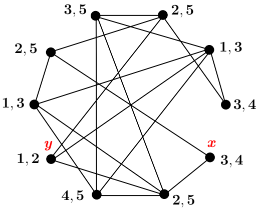
Maximizing (r, s)-core

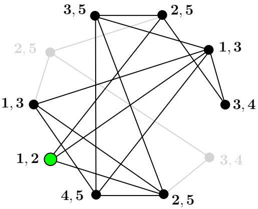
For a given network G, label set R, and a positive integer s, how can we modify or design our network so as to maximize the size of its (r, s)-core?

We propose two ways for achieving the above objective:

- by incentivizing a small subset of nodes, called **anchors**.
- by assigning different labels to nodes, that is **relabeling**.

- **Anchors**
  - Users that do not leave the network even if the participation rule is not satisfied.
- **Basic idea**
  - Prevent the cascading effect by incentivizing few users.

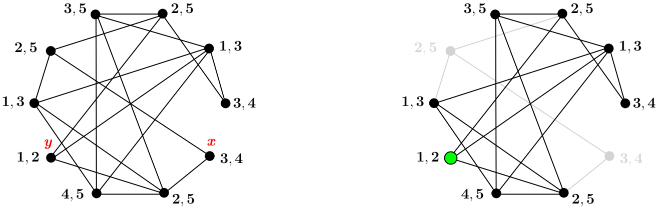




(r, s)-Core with Anchors

The **(r, s)-core with anchors A**, denoted by $\tilde{G}_A(\tilde{V}_A, \tilde{E}_A)$, is the maximal subgraph of G consisting of

- all the anchor nodes, and
- non-anchor nodes having all r labels in their respective neighborhoods.



Problem Statement

Problem (Anchors' selection)

Given a graph

- $G(V, E)$,
- a label set R,
- an integer s,
- a labeling function ℓ, and
- number of anchor nodes α;

find a set of anchor nodes $\boldsymbol{A} \subset \boldsymbol{V}$ such that $|A| \leq \alpha$ and the resulting (r, s)-core with anchors is of **maximum** size.

Problem Complexity

NP-Hardness

Determining if there exists a set A of at most α anchor nodes that results in an anchored (r, s)-core whose cardinality is at least K is an **NP-hard problem**.

We provide two heuristics to select a given number of anchors,

1. **Greedy** heuristic,
2. **Noisy best response** based strategy.

Heuristics to Select Anchors

Greedy

Basic idea

- In each iteration,
  - out of all the non-anchor nodes, select the one as an anchor that maximizes the (r, s)-core.
- Repeat the above step until α anchors are selected.

Heuristics to Select Anchors

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Noisy-best response

Basic idea

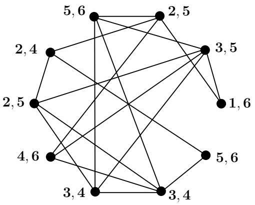
- Randomly select α anchors, $\boldsymbol{A}$.
- In each iteration,
  - randomly pick $x \in A$, and $y \in (V \setminus A)$.
  - replace x with y with 'high probability' if (r, s)-core with y is larger as compared to the one with x.
- Repeat the above step.

Increasing Users' Participation through Relabeling

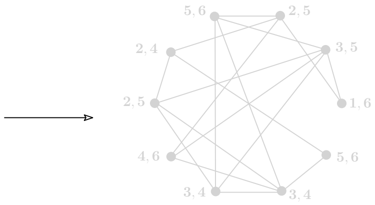
How can we maximize the size of (r, s)-core by relabeling nodes?

Increasing Users' Participation through Relabeling

Initial graph ($s = 2, r = 6$)



(6, 2)-core



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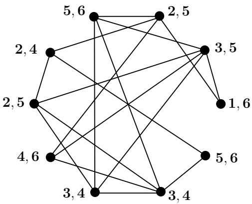

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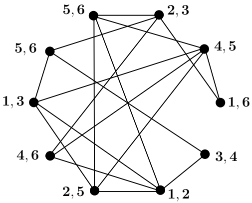
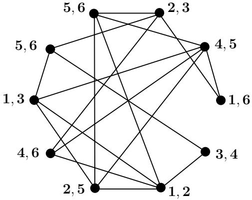


Relabeled graph



(6, 2)-core

- The size of the (r, s)-core depends on both the
  - **structure** of the network,
  - and the assignment of **labels** ℓ to nodes.
- Consequently, by **relabeling** a subset of nodes, the size of (r, s)-core can be maximized.

Optimal labeling

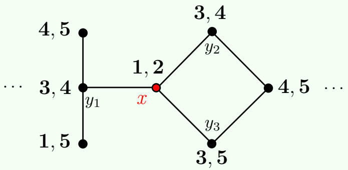
Given a graph G, a positive integer s, and a set of r labels; determining if there exists a labeling ℓ that gives an (r, s)-core consisting of G is **NP-hard**.

Heuristic to Relabel Nodes

- Again, we use a **noisy-best response** based strategy.
- First, we define the **utility** of labels assigned to a node x, i.e. $\ell(x)$.

Utility of labels

Consider an example.



- Here, $\ell(x) = \{1, 2\}$.
- Utility of $\ell(x) = 1 + 2 + 2 = 5$.

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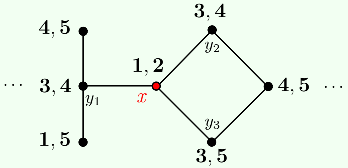

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Noisy-best response

Basic idea

- In each iteration,
  - randomly select a node x.
  - randomly select a subset of s labels from labeling set R.
  - replace $\ell(x)$ with new labels with 'high probability' if the utility of new labels is better than the utility of $\ell(x)$.
- Repeat the above step.

We evaluate our results on two types of networks:

- **Social network**
  - a social network of 4,039 Facebook users[1], and

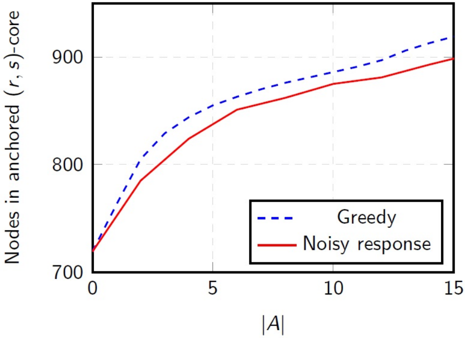

Figure: Size of anchored $(15, 1)$-core as a function of $|A|$.

[1] Leskovec and Mcauley, "Learning to discover social circles in ego networks," *NIPS 2012*

Numerical Results

- **Preferential attachment network**
  - randomly-generated preferential attachment networks with 500 nodes.

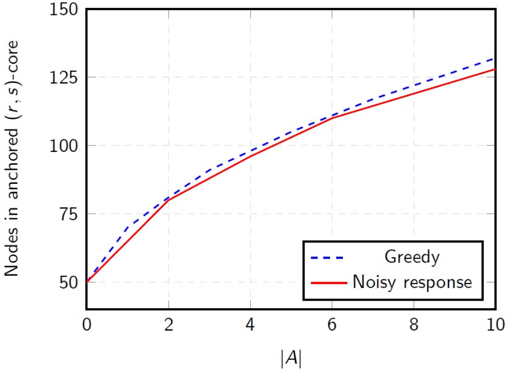

Figure: Size of anchored $(6, 2)$-core as a function of the number of anchors.

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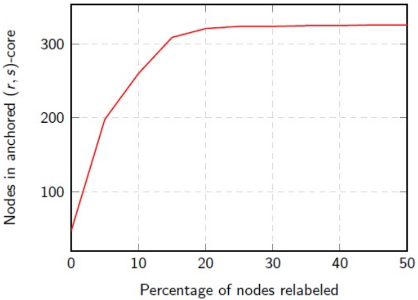

Figure: Size of anchored (6, 2)-core as a function of the percentage of relabeled nodes.

Conclusions

- **(r,s)-core** models the phenomenon of participation among heterogeneous nodes within a network.
- The number of users that actively participate can be increased by **incentivizing** a small subset of users to
  - remain active in all conditions (**anchors**),
  - change their attributes (**relabel**).

Future work

- A **generalized** solution by combining two approaches re-assigning labels and selecting anchors to maximize the number of users participating.
- **characterize** networks for which (r, s)-cores of larger sizes are possible.
- studying (r, s)-cores in **adversarial** setting.
- **distributed** algorithms to compute (r, s)-cores and anchor nodes.

Acknowledgments

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Thank You