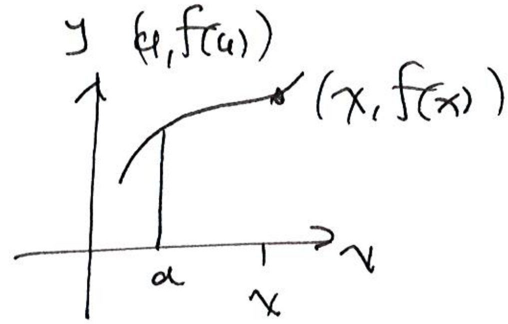
Math 1496 Calc I

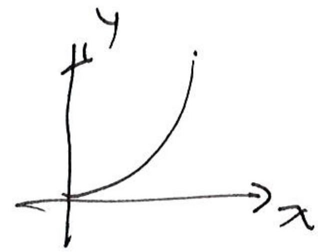
Derivatives



Derivative at a pt.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

ex Find $f'(2)$ when $f(x) = x^2$



$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = \lim_{x \to 2} x+2 = 4$$

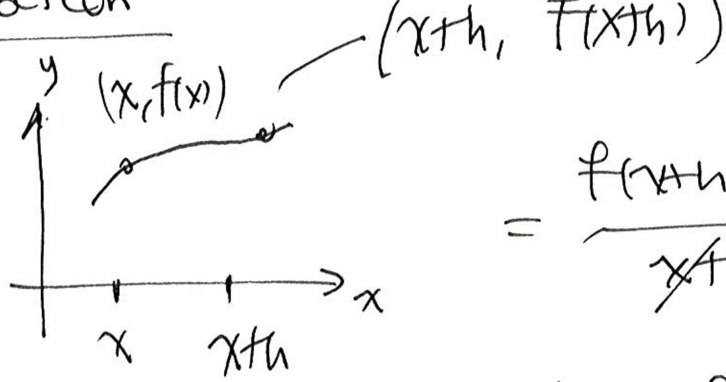
maybe $f'(3)$

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$$

etc.

so can I do this for all x

Variation



$$= \frac{f(x+h)-f(x)}{\cancel{x}+h-\cancel{x}}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

Ex $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0} 2x+h = 2x$$

so $f'(x) = 2x$

Note: $f'(1) = 2$, $f'(2) = 4$ $f'(3) = 6$ like before

Ex $f(x) = 3x^2 - 2x + 1$

$$\lim_{h\to 0} \frac{3(x+h)^2 - 2(x+h) + 1 - [3x^2 - 2x + 1]}{h}$$

$$\lim_{h\to 0} \frac{3\cancel{x^2}+6xh+3h^2-\cancel{2x}-2h+\cancel{1}-\cancel{3x^2}+\cancel{2x}-\cancel{1}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(6x+3h-2)}{\cancel{h}} = 6x-2$$

Rules

(i) $f(x) = c$ const. $f'(x) = 0$

Proof $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c-c}{h} = 0$ ✓

(i) $f(x) = x^n$ $f'(x) = nx^{n-1}$

Proof $\lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$ $(x+h)^n$ Binomial expansion

$$\lim_{h \to 0} \frac{\cancel{x^n} + nx^{n-1}h + \cdots h^n - \cancel{x^n}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h \cdots h^{n-1})}{\cancel{h}}$$

$$= nx^{n-1}$$ ✓

(3) const. mult.

$$F(x) = c f(x) \qquad F'(x) = c f'(x)$$

Proof $$F'(x) = \lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h}$$

$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c f'(x)$$

(4) Sum/Difference

Sum $(f+g)' = f' + g'$

$$\lim_{h \to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f' + g'$$

Similar for difference

so $\frac{d}{dx}$ $f = 3x^2 - 2x + 1$

$$f' = 3 \frac{d}{dx} x^2 - 2 \frac{d}{dx}(x) + \frac{d}{dx}(1) = 3 \cdot 2x - 2(1) + 0$$
$$= 6x - 2$$