Class-XI

Mathematics

Introduction to Three Dimensional Geometry

1. A point is on the x-axis. What are its y-coordinates and z-coordinates?

Ans. If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

2. A point is in the XZ-plane. What can you say about its y-coordinate?

Ans. If a point is in the XZ-plane, then its y-coordinate is zero.

3. Name the octants in which the following points lie:

i. (1,2,3)
ii. (4,-2,-5)
iii. (-4,2,5)

Ans.

i. The x-coordinate, y-coordinate and z-coordinate of point (1, 2, 3) are all positive.

Therefore, this point lies in octant **I**.

ii. The x-coordinate, y-coordinate and z-coordinate of point (4, -2, -5) are positive, negative and negative respectively.

Therefore, this point lies in octant **VIII**.

iii. The x-coordinate, y-coordinate and z-coordinate of point (-4, 2, 5) are negative, positive and positive respectively.

Therefore, this point lies in octant **II**.

4. Fill in the blanks:

Ans.

i. The x-axis and y-axis taken together determine a plane known as **XY-plane.**
ii. The coordinates of point in the XY-plane are of the form **(x,y,0).**
iii. Coordinate planes divide the space into **eight** octants.

5. Find the distance between the pair of points (-1,3,-4) and (1,-3,4).

Ans. The distance between points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) is given by

PQ=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

∴The distance between points (-1,3,-4) and (1,-3,4)

=$\sqrt{(1+1)^2(-3-3)^2(4+4)^2}$

=$\sqrt{(2)^2+(-6)^2+(8)^2}$

=$\sqrt{4+36+64}$

=$\sqrt{104}$

=2$\sqrt{26}$

6. Show that the points (-2,3,5), (1,2,3) and (7,0,-1) are collinear.

Ans. let points (-2,3,5),(1,2,3) and (7,0,-1) be denoted by P,Q and R respectively.

Points P,Q and R are collinear if they lie on a line.

PQ=$\sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$

=$\sqrt{(3)^2+(-1)^2+(-2)^2}$

=$\sqrt{9+1+4}$

=$\sqrt{14}$

QR=$\sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$

=$\sqrt{(6)^2+(-2)^2+(-4)^2}$

=$\sqrt{36+4+16}$

=$\sqrt{56}$

=2$\sqrt{14}$

PR=$\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$

=$\sqrt{(9)^2+(-3)^2+(-6)^2}$

=$\sqrt{81+9+36}$

=$\sqrt{126}$

=3$\sqrt{14}$

Here, PQ+QR=$\sqrt{14}+2\sqrt{14}$=3$\sqrt{14}$=PR

Hence, points P(-2,3,5),Q(1,2,3) and R(7,0,-1) are collinear.

7. Verify the following:

i. (0,7,-10), (1,6,-6) and (4,9,-6) are the vertices of an isosceles triangle.

Ans. Let points (0,7,-10), (1,6,-6) and (4,9,-6) be denoted by A,B and C respectively.

AB=$\sqrt{(1-0)^2+(6-7)^2+(-6+10)^2}$

=$\sqrt{(1)^2+(-1)^2+(4)^2}$

=$\sqrt{1+1+16}$

$=\sqrt{18}$

$=3\sqrt{2}$

BC=$\sqrt{(4-1)^2+(9-6)^2+(-6+6)^2}$

$=\sqrt{(3)^2+(3)^2}$

$=\sqrt{9+9}$

$=\sqrt{18}$

$=3\sqrt{2}$

CA=$\sqrt{(0-4)^2+(7-9)^2+(-10+6)^2}$

$=\sqrt{(-4)^2+(-2)^2+(-4)^2}$

$=\sqrt{16+4+16}$

$=\sqrt{36}$

=6

Here, AB=BC≠CA

Thus, the given points are the vertices of an isosceles triangle.

8. Find the equation of the set of points which are equidistant from the points (1,2,3) and (3,2,-1).

Ans. Let P(x,y,z) be the point that is equidistant from points A(1,2,3) and B(3,2,-1).

Accordingly, PA=PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z+1)^2$

$\Rightarrow x^2-2x+1+y^2-4y+4+z^2-6z+9=x^2-6x+9+y^2-4y+4+z^2+2z+1$

$\Rightarrow -2x-4y-6z+14=-6x-4y+2z+14$

$\Rightarrow -2x-6z+6x-2z=0$

$\Rightarrow 4x-8z=0$

$\Rightarrow x-2z=0$

Thus, the required equation is x-2z=0.

9. Find the coordinates of the point which divides the line segment joining the points (-2,3,5) and (1,-4,6) in the ratio
   i. 2:3 internally
   ii. 2:3 externally

Ans.

i) Let P(x, y, z) be any point which divides the line segment joining the points A(-2, 3, 5) and B(1, -4, 6) in the ratio 2 : 3 internally.

$$\text{Then } x = \frac{2\times1+3\times(-2)}{2+3} = \frac{2-6}{5} = \frac{-4}{5}$$

$$y = \frac{2\times(-4)+3\times3}{2+3} = \frac{-8+9}{5} = \frac{1}{5}$$

$$z = \frac{2\times6+3\times5}{2+3} = \frac{12+15}{5} = \frac{27}{5}$$

$$\therefore \quad \text{Coordinates of } P \text{ are } \left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$$

(ii) Let P(x, y, z) be any point which divides the line segment joining the points 71 (-2, 3, 5) and B(1, -4, 6) in the ratio 2 : 3 externally. Then

$$x = \frac{2\times1-3\times(-2)}{2-3} = \frac{2+6}{-1} = -8$$

$$y = \frac{2\times(-4)-3\times3}{2-3} = \frac{-8-9}{-1} = 17$$

$$z = \frac{2\times6-3\times5}{2-3} = \frac{12-15}{-1} = 3$$

$$\therefore \quad \text{Coordinates of } P \text{ are } (-8, 17, 3)$$

10. Given that P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) are collinear. Find the ratio in which Q divides PR.

    Ans: Let Q(5, 4, -6) divides the line segment joining the points P(3, 2, -4) and R(9, 8, -10) in the ratio k:1 internally.

    $$\therefore \quad \text{Then coordinates of } Q \text{ are}$$

    $$\left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1}\right)$$

    But it is given that coordinates of Q are $(5, 4, -6)$

    $$\therefore \quad \frac{9k+3}{k+1} = 5 \Rightarrow 9k+3 = 5k+5$$

    $$\Rightarrow \quad 4k = 2 \quad \Rightarrow \quad k = \frac{1}{2}$$

    Thus Q divides the line segment joining the points P and R in the ratio $\frac{1}{2}:1$ *i.e.,* $1:2$.

11. Using section formula, show that the points A(2, -3, 4), B(-1, 2, 1) and C(0,13,2) are collinear.
    Ans: Let the points A(2, -3, 4), B(-l, 2,1) and C(0,13,2) be the given points. Let the point P divides AB in the ratio k : 1. Then coordinates of P are (−k+2k+1,2k−3k+1,k+4k+1)
    Let us examine whether for some value of k, the point P coincides with point C.

On putting $\frac{-k+2}{k+1}=0$, we get $k=2$

When $k=2$, then $\frac{2k-3}{k+1}=\frac{4-3}{3}=\frac{1}{3}$ and

$$\frac{k+4}{k+1}=\frac{2+4}{2+1}=2$$

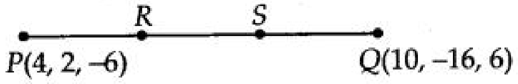
Therefore, $C\left(0, \frac{1}{3}, 2\right)$ is a point which divides

AB internally in the ratio 2:1.

Hence A, B, C are collinear.

12. Find the coordinates of the points which trisect the line segment joining the points P(4, 2, -6) and Q(10, -16, 6).

Ans: Let R and S be two points which trisect the line segment PQ.



$\because$ $PR = RS = SQ$

$\therefore$ Point R divides the join of PQ in the ratio $1:2$

$\therefore$ Coordinates of R are

$$\left(\frac{1\times10+2\times4}{1+2}, \frac{1\times(-16)+2\times2}{1+2}, \frac{1\times6+2\times(-6)}{1+2}\right)$$

$$=(6, -4, -2)$$

Also, point S divides the join of PQ in the ratio $2:1$

$\therefore$ Coordinates of S are

$$\left(\frac{2\times10+1\times4}{2+1}, \frac{2\times(-16)+1\times2}{2+1}, \frac{2\times6+1\times(-6)}{2+1}\right)$$

$$=(8, -10, 2)$$

HOME WORK: NCERT:EX-12.1 - 3.

EX-12.2- 1(i,ii,iv), 3(ii,iii), 5.