A novel approach for two person zero-sum matrix game with intuitionistic fuzzy 2-tuple linguistic information

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Abstract In this paper, an attempt has been made to establish the solution procedure for two person zero-sum game payoffs involving intuitionistic 2-tuple linguistic information (I2TLI). In real life, there are many decision-making problems which present the qualitative aspects that are complex to judge by means of numerical quantities. The I2TLI overcomes this issue and provides more freedom to the decision maker. The method proposed in this paper is demonstrated with an illustrative example.

Keywords I-fuzzy set · Two person Zero-Sum Game · 2-tuple linguistic variable

1 Introduction and Motivation

In many decision making problems, because there is fuzziness and uncertainty, the evaluation values of attributes are easier represented by linguistic terms (LTs) rather than crisp numbers, especially for qualitative attributes. The concept of linguistic variables (LVs) was first introduced by Zadeh [27], to depict the qualitative information for decision making problems. In order to avoid the drawback of the information distortion in the operational process, Herrera and Martinez [13] proposed a 2-tuple linguistic (2TL) model with respect to computing with words.Herrera and Martinez [12] proposed an approach to set up the conversion relation between LTs and numbers based on the 2TL model in decision-making. Martinez and Herrera [19] also gave a comprehensive overview of the research on the 2TL models in decision making.

Intuitionistic fuzzy sets (IFSs) proposed by Atanassov [4] can more easily express fuzzy information than fuzzy sets by the membership functions and the non-membership functions, whereas fuzzy sets [26] only have the membership functions. In intuitionistic fuzzy numbers (IFNs), the membership degree and non-membership degree are represented by crisp values. However, sometimes the membership degree and non-membership degree are difficult to described by numerical values because of fuzziness of decision making problems, especially for qualitative information, which can be easily described by LTs. Chen et al. [10] used LTs to represent the membership degrees and the non-membership degrees of IFNs, which are called linguistic intuitionistic fuzzy numbers (LIFNs). Zhang et al. [28] proposed the concept of 2-tuple intuitionistic fuzzy linguistic preference relations. Beg and Rashid [8] proposed the I2TLI model and proposed some correlated averaging operators to aggregate the I2TLI. They have easily described fuzzy information via I2TLI model and utilized it to deal with multiple attribute group decision making. Recently, Liu and chen [18] developed a new approach for dealing with group decision making problems with the I2TLI. They extended the t-norm and t-conorm aggregating operators for I2TLI, to deal with muti-attribute group decision making problems. Thus motivating us to aaply the same on two person zero-sum mateix game having I2TLI.

In recent years, attempts have been made to extend the results of the crisp game theory to the fuzzy games. The motivating force behind these extensions is the advancement in the duality theory for fuzzy linear

programming. The earliest study of a two person zero-sum matrix game with fuzzy pay-offs is due to [9].Also, [5,6] interpreted model of [9] in context of the fuzzy linear programming duality and showed that solving a two person zero-sum matrix game with fuzzy goals and, or fuzzy pay-offs are equivalent to solving an appropriate pair of primal-dual fuzzy linear programming problems. On the lines of [5–7], [2] studied duality for I-fuzzy linear programming problems and discussed its application in I-fuzzy matrix games. A vast literature is present on solving matrix games under fuzzy environment, [14–17,20,21,24,25] to name a few.
In recent years, Singh et al. [23] presented a non-cooperative two player constant sum matrix game problem in the 2-tuple linguistic framework. Also, Singh and Gupta [22] designed a methodology to solve the two players constant sum game, where the two-players have the knowledge of their payoffs in terms of interval-valued 2-tuple linguistic variables.
In this work an attempt is made to establish the solution procedure for two person zero sum game with I2TLI, based on the results of [18]. Thus, I2TL two person zero-sum game is denoted by I2TL-TPZSG. The intuitionistic 2-tuple linguistic representation model in [18] is utilized to avoid the loss information in the process of linguistic information. Thus, defining the solution of the zero-sum game with I2TLI model.
This paper unfolds as follows. Section 1.1 includes the basic definitions related to I-fuzzy sets and linguistic 2-tuples. Its subsequent subsections reviews the Intuitionistic 2-tuple linguistic variables and classical two person zero-sum game. Section 2 presents the proposed approach, for two person zero sum game with payoffs involving intuitionistic fuzzy 2-tuple linguistic terms. Section 3 presents an illustrative example in the support of theory. Section 4 is an concluding section. Lastly, Section 5 explains the future scope.

1.1 Preliminaries

In this section, some of the standard definitions of intuitionistic fuzzy sets and linguistic sets and some important associated concepts from [12] and [8,18], respectively, are presented. The motivation is that these well defined terms are extensively used in the rest of the work.

1.1.1 Intuitionistic Fuzzy Set

Definition 1 Intuitonistic Fuzzy Set [29,11,5]
In an underlying set X of objects, an intuitonistic fuzzy set (IFS) $\tilde{A}$ is a set of ordered fuzzy triples,

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) | x \in X\}$$

where $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are functions from X into $[0, 1]$. For each $x \in X$, $\mu_{\tilde{A}}(x)$ represents the degree of membership of the element x to the subset A of X, and $\nu_{\tilde{A}}(x)$ gives the degree of non-membership. For the functions $\mu_{\tilde{A}}(x),\ \nu_{\tilde{A}}(x) : X \to [0, 1]$, the conditions $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$ holds.

For each IFS A in X, if $h_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)),\ \forall\ x \in X$. Then $h_{\tilde{A}}(x)$ is called the degree of indeterminacy of x to A. If $h_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)) = 0,\ \forall\ x \in X$ then the IFS A reduces to fuzzy set.

1.1.2 Linguistic Term Sets and Linguistic 2-Tuples

Definition 2 Linguistic Term Set (LTS) [12]

A linguistic term set $S = \{s_0, s_1, \ldots, s_t\}$ must satisfy the following conditions:

1. the LTs $s_0, s_1, \ldots, s_t$ should be ranked in an ascending order, i.e. from the worst to the best in semantics.
2. the number of LTs in the LTS, S should be an odd number, i.e. t is an even number.

For convenience, we use $S_{[0,t]}$ to express the LTS $S = \{s_0, s_1, \ldots, s_t\}$. Because the LTS $S_{[0,t]}$ is a discrete set, in order to relieve the drawback of information loss in an operational process, Herrera and Martinez [12] presented a 2TL model to process the linguistic information, defined as follows.

Definition 3 Linguistic 2-tuples (2TL) [12,19]

Let $S_{[0,t]}$ be LTS and $\beta \in [0, t]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\triangle : [0, t] \longrightarrow S \times [-0.5, 0.5)$$
$$\triangle(\beta) = (s_i, \alpha) \text{ where } i = round(\beta) \text{ and } \alpha = \beta - i \text{ such that } \alpha \in [-0.5, 0.5)$$

where round(.) is the usual rounding operation. Then α is called symbolic translation.

Definition 4 [12,19]

Let $S_{[0,t]}$ be LTS, (s_i, α) be 2-tuple, and $\beta \in [0, t]$ (t is positive integer). There is a function $\triangle^{-1}$ which can convert a 2-tuple into a real number β with the equivalent information, where $\beta \in [0, t]$,

$$\triangle^{-1} : S \times [-0.5, 0.5) \longrightarrow [0, t],$$
$$\triangle^{-1}(s_i, \alpha) = i + \alpha = \beta.$$

Remark 1 From Definitions 3 and 4, it is seen that the conversion of a linguistic 2-tuple consist of adding a value zero symbolic translation. If $s_i \in S_{[0,t]}$ can be expressed as 2-tuple $(s_i, 0)$.

1.1.3 Intuitionistic 2-Tuple Linguistic Variables

Intuitionistic fuzzy sets proposed by Atanassov [4] can be used to describe fuzzy information. However, in some real applications, it is difficult to describe the membership degree and the non-membership degree of an intuitionistic fuzzy set by numerical values between zero and one. Therefore, Chen et al. [10] proposed the linguistic intuitionistic fuzzy numbers in which both the membership degree and the non-membership degree are expressed by LTs, defined as follows.

Definition 5 Intuitionistic linguistic term (ILT) [18]

Let $s_\alpha, s_\beta \in S_{[0,t]}$ and $\gamma = (s_\alpha, s_\beta)$, If $\alpha + \beta \leq t$, then γ is intuitionistic linguistic term defined on $S_{[0,t]}$.

Definition 6 Intuitionistic 2-tuple linguistic term (I2TLT) [10]

Let (s_α, s_β) be an element of ILT set, an intuitionistic 2-tuple linguistic model is $((s_i, \alpha), (s_j, \beta))$, where s_i, s_j are LTs and $\alpha, \beta \in [-0.5, 0.5)$, are the numeric values representing the symbolic translation, respectively. The set of I2TLTs is denoted by $\Gamma^*[0, t]$.

Definition 7 [10] Let $((s_i, \alpha), (s_j, \beta))$ be an I2TLT set, and (ξ, η) be an ordered numerical pair. There is a function ∇^{-1} which can convert the I2LT $((s_i, \alpha), (s_j, \beta))$ to (ξ, η) with the equivalent information, as follows:

$$\nabla^{-1} : (S \times [-0.5, 0.5)) \times (S \times [-0.5, 0.5)) \longrightarrow [0, t] \times [0, t],$$

such that

$$\nabla^{-1}((s_i, \alpha), (s_j, \beta)) = (\Delta^{-1}(s_i, \alpha), \Delta^{-1}(s_j, \beta)) = (i + \alpha, j + \beta) = (\xi, \eta)$$

where $\xi, \eta \in [0, t]$ with the condition $0 \leq \xi + \eta \leq t$, where $t + 1$ being the cardinality of $S_{[0,t]}$.

Definition 8 [10] Let $S_{[0,t]}$ be a LT set and (ξ, η) be an intuitionistic order pair of two numbers representing the aggregation results of linguistic symbolic. The function ∇ from intuitionistic 2-tuple $((s_i, \alpha), (s_j, \beta))$ to an ordered pair numerical values $(\xi, \eta \in [0, t] \times [0, t])$ defined as follows:

$$\nabla : [0, t] \times [0, t] \longrightarrow (S \times [-0.5, 0.5)) \times (S \times [-0.5, 0.5)),$$

such that

$$\nabla(\xi, \eta) = (\Delta(\xi), \Delta(\eta)) = ((s_i, \alpha), (s_j, \beta))$$

where $i = round(\xi)$, $j = round(\eta)$, and $\alpha = \xi - i$, $\beta = \eta - j$, s_i and s_j has the closest index label to ξ, η and α, β are the value of the symbolic translations of s_i and s_j, respectively.

Remark 2 The negation operator for I2TLT

$$\tilde{\kappa} = ((s_i, \alpha), (s_j, \beta)) \text{ is taken by } neg(\tilde{\kappa}) = neg\left(((s_i, \alpha), (s_j, \beta))\right) = \left(\Delta\left(t - \Delta^{-1}(s_i, \alpha)\right), \Delta\left(t - \Delta^{-1}(s_j, \beta)\right)\right).$$

Remark 3 Using the linearity property and the monotonic increasing property of the ∇ operator, it is observed that for I2TLTs $\tilde{\kappa}_2 = ((s_i, \alpha_i), (s_j, \beta_j))$ and $\tilde{\kappa}_2 = ((s_p, \alpha_p), (s_q, \beta_q))$, the following hold:

1. $neg(neg(\tilde{\kappa}_1)) = \tilde{\kappa}_1$;
2. $\min\{neg(\tilde{\kappa}_1), neg(\tilde{\kappa}_2)\} = neg(\max\{\tilde{\kappa}_1, \tilde{\kappa}_2\})$;
3. $\max\{neg(\tilde{\kappa}_1), neg(\tilde{\kappa}_2)\} = neg(\min\{\tilde{\kappa}_1, \tilde{\kappa}_2\})$; where the min and max of two I2TLTs are taken in the spirit of Definition 8.

Remark 4 Composition of ∇ and ∇^{-1} is an identity mapping, i.e. $\nabla(\nabla^{-1}((s_i, \alpha), (s_j, \beta)) = ((s_i, \alpha), (s_j, \beta))$.

Remark 5 [8] If $(s_i, \alpha_i), (s_j, \beta_j)) \leq ((s_p, \alpha_p), (s_q, \beta_q))$. Then $\nabla^{-1}((s_i, \alpha_i), (s_j, \beta_j)) \leq \nabla^{-1}((s_p, \alpha_p), (s_q, \beta_q))$.

We review the method of comparing I2TLTs based on score function and accuracy function in [8,10], shown as follows:

Definition 9 Let $\tilde{\kappa}_1 = ((s_i, \alpha_i), (s_j, \beta_j))$ and $\tilde{\kappa}_2 = ((s_p, \alpha_p), (s_q, \beta_q))$ be three I2TLs. Let Ls and Lh be the score function and the accuracy function of I2TLI $\tilde{\kappa}_1$, respectively, where $Ls(\tilde{\kappa}_1) = i+\alpha_i-(j+\beta_j)$ and $Lh(\tilde{\kappa}_2) = i+\alpha_i+j+\beta_j$. Then,

1. if $Ls(\tilde{\kappa}_1) < Ls(\tilde{\kappa}_2)$, then $\tilde{\kappa}_1$ is smaller than $\tilde{\kappa}_2$, denoted by $\tilde{\kappa}_1 < \tilde{\kappa}_2$.
2. if $Ls(\tilde{\kappa}_1) = Ls(\tilde{\kappa}_2)$, then
   (a) if $Lh(\tilde{\kappa}_1) < Lh(\tilde{\kappa}_2)$, then $\tilde{\kappa}_1$ is smaller than $\tilde{\kappa}_2$, denoted by $\tilde{\kappa}_1 < \tilde{\kappa}_2$.

(b) if $Lh(\tilde{\kappa}_1) = Lh(\tilde{\kappa}_2)$, then $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ represent the same information, denoted by $\tilde{\kappa}_1 = \tilde{\kappa}_2$.

[18] It is obvious that $(s_0, s_t) \leq ((s_i, \alpha_i), (s_j, \beta_j)) \leq (s_t, s_0)$ for any $((s_i, \alpha_i), (s_j, \beta_j)) \in \Gamma^*[0, t]$.

Definition 10 [8] Let (ξ_i, η_i) and (ξ_j, η_j) be two intuitionistic order pairs and $0 \leq k_1, k_2 \leq 1$ with $0 \leq k_1 + k_2 \leq 1$, then

1. $k_1(\xi_i, \eta_i) \cdot k_2(\xi_j, \eta_j) = (k_1\xi_i \cdot k_2\xi_j,\ k_1\eta_i \cdot k_2\eta_j)$
2. $(\xi_i, \eta_i)^\lambda = (\xi_i^\lambda, \eta_i^\lambda)$ for all $0 \leq \lambda \leq 1$
3. $k_1(\xi_i, \eta_i) + k_2(\xi_j, \eta_j) = (k_1\xi_i + k_2\xi_j, k_1\eta_i + k_2\eta_j)$

Definition 11 Linguistic Weighted Average Operator for I2TLI [8]

Let $\{((s_{i_r}, \alpha_{i_r}), (s_{j_r}, \beta_{j_r}))$, where $i_r, j_r \in \{0, 1, \ldots t\}$ and $r = 1, 2, \ldots, g\}$ be a set of I2TLTs and $\omega = (\omega_1, \ldots, \omega_g)^T$ be the weight vector satisfying $0 \leq \omega_r \leq 1, r = 1, 2, \ldots, g,\ \sum_{r=1}^{g} \omega_r = 1$. Then, the weighted average operator is defined as

$$\begin{aligned} LWA[((s_{i_r}, \alpha_{i_r}), (s_{j_r}, \beta_{j_r})) : r = 1, 2, \ldots, g] =& ((s_{i_1}, \alpha_{i_1}), (s_{j_1}, \beta_{j_1}))\omega_1 \oplus \\ & ((s_{i_2}, \alpha_{i_2}), (s_{j_2}, \beta_{j_2}))\omega_2 \oplus \ldots \\ & \ldots \oplus ((s_{i_g}, \alpha_{i_g}), (s_{j_g}, \beta_{j_g}))\omega_g \\ =& \nabla\left(\sum_{r=1}^{g} \omega_r \overset{-1}{\nabla}((s_{i_r}, \alpha_{i_r}), (s_{j_r}, \beta_{j_r}))\right) \end{aligned}$$

Consequently, $\nabla^{-1}\left(\bigoplus_{r=1}^{g}((s_{i_r}, \alpha_{i_r}), (s_{j_r}, \beta_{j_r}))\omega_r\right) = \sum_{r=1}^{g} \omega_g \nabla^{-1}\left((s_{i_r}, \alpha_{i_r}), (s_{j_r}, \beta_{j_r})\right)$.

1.1.4 Classical two person zero-sum game (TPZSG)

Let $A \in \mathbb{R}^{m \times n}$ be $m \times n$ matrix and $e^T = (1, ..., 1)$ be a vector of *ones* whose dimension is specified as per the specific context. A crisp two person non-cooperative zero-sum matrix game G is denoted by the triplet $G = (S^m, S^n, A)$, where $S^m = \{x \in \mathbb{R}_+^m | e^T x = 1\}$ and $S^n = \{y \in \mathbb{R}_+^n | e^T y = 1\}$ denote the strategy sets fo player I (P-I) and player II (P-II), respectively, and A is the payoff matrix. Therefore, for $x \in S^m$, $y \in S^n$, the scalar $x^T A y$ is the payoff to player I, and since the game is zero sum, the payoff to player II is $-x^T A y$.

A pure strategy occurs when P-II selects a single move j and P-I selects a single move i, so P-II has to pay a_{ij} units to P-I ($a_{ij} < 0$ means that P-I has to pay $|a_{ij}|$ units to P-II). So, there is a single value that maximizes the game. Whereas, mixed strategy in a zero sum game implies that the loss of P-I is the gain of P-II, so P-I wants to obtain a combination of moves i that maximizes the minimum payoffs given all possible moves j of P-II, which is called max-min decision making principle. A mathematical programming formulation of P-I problem is as follows:

$$\max_{i \in S^m}\{\min\left(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, ..., \sum_{i=1}^{m} a_{in}x_i\right) : \sum_{i=1}^{m} x_i = 1, x_i \in \mathbb{R}_+^m\}.$$

Analogus problem can be formulated for P-II. The equivalent LP formulation for P-I is:

$$\max_{i \in S^m}\{z = v : v - \sum_{i=1}^{m} a_{ij}x_i \leq 0\ \forall j \in S^n, \sum_{i=1}^{m} x_i = 1, x_i \in \mathbb{R}_+^m, v \in \mathbb{R}\}.$$

where v is the auxiliary variable which operates as

$$v = \min\{\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, ..., \sum_{i=1}^{m} a_{in}x_i\}.$$

Similarly, the LP formulation for P-II (equivalent to the dual for P-I) is :

$$\min_{j\in S^n}\{z = w : w - \sum_{j=1}^{n} a_{ij}y_j \geq 0 \ \forall i \in S^m, \sum_{j=1}^{n} y_j = 1, y_j \in \mathbb{R}_+^n, w \in \mathbb{R}\},$$

where w is an auxiliary variable which operates as

$$w = \max\{\sum_{j=1}^{n} a_{1j}y_j, \sum_{j=1}^{n} a_{2j}y_j, ..., \sum_{j=1}^{n} a_{mj}y_j\}.$$

The two person zero-sum games have been extensively studied both in classical sense and fuzzy sense, too. It is extremely difficult to restate each and every contribution in the literature. [1,3,5–7,**?**] are few who incorporated the study with state of art. Although fuzzy set and its generalized variants have been used to overcome the difficulties in restricting pay-offs to precision, they also require knowledge of membership or non-membership functions and their shapes. The linguistic variables provides an improved and more flexible framework to the players to express the granularity of the information and opinions of the payoffs by using the linguistic term set LT. The study of linguistic matrix games, to the best of our knowledge has been done by [23]

2 Proposed Approach

The approach found in [23] suggested the application of linguistic variable framework to constant-sum game theory. The fact that linguistic variables framework expresses the granularity of information, thus [23] utilized the linguistic framework to define the opinions of players in constant-sum game. We propose a new approach for solving, two person zero-sum game with intuitionistic fuzzy 2-tuple linguistic information (I2TL-TPZSG).

Definition 12 An intuitionistic 2-tuple linguistic two person zero-sum matrix game (I2TL-TPZSG) is defined by $\tilde{G} = \left(S^m, S^n, S_{[0,t]}, \tilde{A}\right)$, where S^m and S^n are the strategy sets, $S_{[0,t]}$ is the LT set for both the players and $\tilde{A}$ is the linguistic payoff matrix of player I and player II.

The expected payoff of the crisp two person zero-sum game is defined as the statistical expectation of information in the payoff matrix. In a linguistic matrix game with uncertain information of the payoff matrix, the accuracy of the expected value is explainable. Thus, the expected payoff for the linguistic matrix game is defined as follows:

Definition 13 Let $x = (x_1, x_2, \ldots, x_m) \in S^m$ and $y = (y_1, y_2, \ldots, y_n) \in S^n$ be a pair of mixed strategies for player I and player II respectively. The linguistic expected payoff of player I is defined as

$$\tilde{E}_{\tilde{A}}(x, y) = \bigoplus_{i=1}^{m} \left(x_i \left(\bigoplus_{j=1}^{n} \tilde{a}_{ij} y_j \right) \right)$$

The linguistic expected payoff of player II is $\mathrm{neg}(\tilde{E}_{\tilde{A}}(x, y))$

The payoff matrix $\tilde{A}$ is defined as:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{bmatrix}$$

where each $\tilde{a}_{ij} \in \Gamma^*[0,t]$, is a I2TLT. Here, $\tilde{a}_{ij} \in \Gamma^*[0,t]$ denotes payoff of player I chooses ith strategy while player II chooses jth strategy. Consequently, $\tilde{a}_{ij} = ((s_{r_{ij}}, k_{ij}), (s_{q_{ij}}, l_{ij}))$ for each $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$. These, $k_{ij}, q_{ij} \in [-0.5, 0.5)$ represents the values of the symbolic translations of $s_{r_{ij}}$ and $s_{q_{ij}}$, respectively. The mixed strategies of player I and player II are $x = (x_1, x_2, \dots, x_m) \in S^m$ and $y = (y_1, y_2, \dots, y_n) \in S^n$, respectively. Let $\tilde{v} \in \Gamma^*[0,t]$ be the I2TL value of I2TL game for player I and $\tilde{\omega} \in \Gamma^*[0,t]$ be the value for player II. Therefore, $\tilde{v} = ((v_\alpha, \rho), (v_\beta, \nu))$ and $\tilde{\omega} = ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu'))$, where $\rho, \nu \in [-0.5, 0.5)$ are the symbolic translations of v_α and v_β, respectively. And $\rho', \nu' \in [-0.5, 0.5)$ are symbolic translations of $\omega_{\alpha'}$ and $\omega_{\beta'}$, respectively. Thus, the problem for player I will becomes :

$$\begin{aligned} \text{(I2TLP-I)} \quad & \max\ \tilde{v} \\ & \text{subject to,} \\ & \tilde{a}_{11}x_1 \oplus \tilde{a}_{21}x_2 \oplus \dots \oplus \tilde{a}_{m1}x_m \geq \tilde{v}, \\ & \tilde{a}_{12}x_1 \oplus \tilde{a}_{22}x_2 \oplus \dots \oplus \tilde{a}_{m2}x_m \geq \tilde{v}, \\ & \vdots \\ & \tilde{a}_{1n}x_1 \oplus \tilde{a}_{2n}x_2 \oplus \dots \oplus \tilde{a}_{mn}x_m \geq \tilde{v}, \\ & x_1 \oplus x_2 \oplus \dots \oplus x_m = 1, \\ & x_1, x_2, \dots x_m \geq 0. \end{aligned}$$

Similarly, the problem for player II is:

$$\begin{aligned} \text{(I2TLP-II)} \quad & \min\ \tilde{\omega} \\ & \text{subject to,} \\ & \tilde{a}_{11}y_1 \oplus \tilde{a}_{12}y_2 \oplus \dots \oplus \tilde{a}_{1n}y_n \leq \tilde{\omega}, \\ & \tilde{a}_{21}y_1 \oplus \tilde{a}_{22}y_2 \oplus \dots \oplus \tilde{a}_{2n}y_n \leq \tilde{\omega}, \\ & \vdots \\ & \tilde{a}_{m1}y_1 \oplus \tilde{a}_{m2}y_2 \oplus \dots \oplus \tilde{a}_{mn}y_n \leq \tilde{\omega}, \\ & y_1 \oplus y_2 \oplus \dots \oplus y_n = 1, \\ & y_1, y_2, \dots y_n \geq 0. \end{aligned}$$

Since ∇^{-1} is monotonically increasing [10], by applying it to the constraints of model (I2TLP-I) and (I2TLP-II) and using Definition 11, the models become:

$$
\begin{aligned}
\text{(I2TLP-I)} \quad & \max \ \nabla^{-1}(\tilde{v}) \\
& \text{subject to,} \\
& \nabla^{-1}(\tilde{a}_{11})x_1 + \nabla^{-1}(\tilde{a}_{21})x_2 + \ldots + \nabla^{-1}(\tilde{a}_{m1})x_m \geq \nabla^{-1}(\tilde{v}), \\
& \nabla^{-1}(\tilde{a}_{12})x_1 + \nabla^{-1}(\tilde{a}_{22})x_2 + \ldots + \nabla^{-1}(\tilde{a}_{m2})x_m \geq \nabla^{-1}(\tilde{v}), \\
& \vdots \\
& \nabla^{-1}(\tilde{a}_{1n})x_1 + \nabla^{-1}(\tilde{a}_{2n})x_2 + \ldots + \nabla^{-1}(\tilde{a}_{mn})x_m \geq \nabla^{-1}(\tilde{v}), \\
& x_1 + x_2 + \ldots + x_m = 1, \\
& x_1, x_2, \ldots x_m \geq 0.
\end{aligned}
$$

and

$$
\begin{aligned}
\text{(I2TLP-II)} \quad & \min \ \nabla^{-1}(\tilde{\omega}) \\
& \text{subject to,} \\
& \nabla^{-1}(\tilde{a}_{11})y_1 + \nabla^{-1}(\tilde{a}_{12})y_2 + \ldots + \nabla^{-1}(\tilde{a}_{1n})y_n \leq \nabla^{-1}(\tilde{\omega}), \\
& \nabla^{-1}(\tilde{a}_{21})y_1 + \nabla^{-1}(\tilde{a}_{22})y_2 + \ldots + \nabla^{-1}(\tilde{a}_{2n})y_n \leq \nabla^{-1}(\tilde{\omega}), \\
& \vdots \\
& \nabla^{-1}(\tilde{a}_{m1})y_1 + \nabla^{-1}(\tilde{a}_{m2})y_2 + \ldots + \nabla^{-1}(\tilde{a}_{mn})y_n \leq \nabla^{-1}(\tilde{\omega}), \\
& y_1 + y_2 + \ldots + y_n = 1, \\
& y_1, y_2, \ldots y_n \geq 0.
\end{aligned}
$$

Now, using Definition 7, the $\nabla^{-1}(\tilde{v}) = (\alpha + \rho, \beta + \nu)$ and $\nabla^{-1}(\tilde{\omega}) = (\alpha' + \rho', \beta' + \nu')$. Similarly, if for each $i = 1, 2, ..., m$, $j = 1, 2, ..., n$, $\tilde{a}_{ij} = ((s_{r_{ij}}, k_{ij}), (s_{q_{ij}}, l_{ij}))$. Then, $\nabla^{-1}(\tilde{a}_{ij}) = (r_{ij} + k_{ij}, q_{ij} + l_{ij})$. Moreover, $\alpha + \rho,\ \beta + \nu \in [0, t]$ with the condition $0 \leq \alpha + \rho + \beta + \nu \leq t$, where $t + 1$ being the cardinality of $S_{[0,t]}$. Likewise, same is true for $r_{ij} + k_{ij} + q_{ij} + l_{ij}$.

Thus, the equivalent I2TL-TPZSG problems becomes:

$$
\begin{aligned}
\text{(EI2TLP-I)} \quad & \max \ (\alpha + \rho, \beta + \nu) \\
& \text{subject to,} \\
& (r_{11} + k_{11}, q_{11} + l_{11})x_1 + (r_{21} + k_{21}, q_{21} + l_{21})x_2 + ... \\
& \qquad \ldots + (r_{m1} + k_{m1}, q_{m1} + l_{m1})x_m \geq (\alpha + \rho, \beta + \nu), \\
& (r_{12} + k_{12}, q_{12} + l_{12})x_1 + (r_{22} + k_{22}, q_{22} + l_{22})x_2 + ... \\
& \qquad \ldots + (r_{m2} + k_{m2}, q_{m2} + l_{m2})x_m \geq (\alpha + \rho, \beta + \nu), \\
& \vdots \\
& (r_{1n} + k_{1n}, q_{1n} + l_{1n})x_1 + (r_{2n} + k_{2n}, q_{2n} + l_{2n})x_2 + ... \\
& \qquad \ldots + (r_{mn} + k_{mn}, q_{mn} + l_{mn})x_m \geq (\alpha + \rho, \beta + \nu), \\
& x_1 + x_2 + \ldots + x_m = 1, \\
& 0 \leq \alpha + \rho + \beta + \nu \leq t, \\
& -0.5 \leq \rho,\ \nu \leq 0.5, \\
& x_1, x_2, \ldots x_m \geq 0.
\end{aligned}
$$

and for player II the Equivalent I2TL problem is:

$$
\begin{aligned}
\text{(EI2TLP-II)} \quad & \min\ (\alpha' + \rho', \beta' + \nu') \\
& \text{subject to,} \\
& (r_{11} + k_{11}, q_{11} + l_{11})y_1 + (r_{12} + k_{12}, q_{12} + l_{12})y_2 + \ldots \\
& \qquad \ldots + (r_{1n} + k_{1n}, q_{1n} + l_{1n})y_n \leq (\alpha' + \rho', \beta' + \nu'), \\
& (r_{21} + k_{21}, q_{21} + l_{21})y_1 + (r_{22} + k_{22}, q_{22} + l_{22})y_2 + \ldots \\
& \qquad \ldots + (r_{2n} + k_{2n}, q_{2n} + l_{2n})y_n \leq (\alpha' + \rho', \beta' + \nu'), \\
& \qquad \vdots \\
& (r_{m1} + k_{m1}, q_{m1} + l_{m1})y_1 + (r_{m2} + k_{m2}, q_{m2} + l_{m2})y_2 + \ldots \\
& \qquad \ldots + (r_{mn} + k_{mn}, q_{mn} + l_{mn})y_n \leq (\alpha' + \rho', \beta' + \nu'), \\
& y_1 + y_2 + \ldots + y_n = 1, \\
& 0 \leq \alpha' + \rho' + \beta' + \nu' \leq t, \\
& -0.5 \leq \rho',\ \nu' \leq 0.5, \\
& y_1, y_2, \ldots y_n \geq 0.
\end{aligned}
$$

The comparison of I2TLT's is done on the basis of score function [10, 18] which is the most reliable of all the methods. Using Definition 9, the problems (EI2TLP-I) and (EI2TLP-II) are further reduced to their equivalent crisp linear programming problems. For player I

$$
\begin{aligned}
\text{(CEI2TLP-I)} \quad & \max\ [\alpha + \rho - (\beta + \nu)] \\
& \text{subject to,} \\
& [r_{11} + k_{11} - (q_{11} + l_{11})]x_1 + [r_{21} + k_{21} - (q_{21} + l_{21})]x_2 + \ldots \\
& \qquad \ldots + [r_{m1} + k_{m1} - (q_{m1} + l_{m1})]x_m \geq [\alpha + \rho - (\beta + \nu)], \\
& [r_{12} + k_{12} - (q_{12} + l_{12})]x_1 + [r_{22} + k_{22} - (q_{22} + l_{22})]x_2 + \ldots \\
& \qquad \ldots + [r_{m2} + k_{m2} - (q_{m2} + l_{m2})]x_m \geq [\alpha + \rho - (\beta + \nu)], \\
& \qquad \vdots \\
& [r_{1n} + k_{1n} - (q_{1n} + l_{1n})]x_1 + [r_{2n} + k_{2n} - (q_{2n} + l_{2n})]x_2 + \ldots \\
& \qquad \ldots + [r_{mn} + k_{mn} - (q_{mn} + l_{mn})]x_m \geq [\alpha + \rho - (\beta + \nu)], \\
& x_1 + x_2 + \ldots + x_m = 1, \\
& 0 \leq \alpha + \rho + \beta + \nu \leq t, \\
& -0.5 \leq \rho,\ \nu \leq 0.5, \\
& x_1, x_2, \ldots x_m \geq 0.
\end{aligned}
$$

and for player II

$$
\begin{aligned}
\text{(CEI2TLP-II)} \quad & \min\ [\alpha' + \rho' - (\beta' + \nu')] \\
& \text{subject to,} \\
& [r_{11} + k_{11} - (q_{11} + l_{11})]y_1 + [r_{12} + k_{12} - (q_{12} + l_{12})]y_2 + \ldots \\
& \qquad \ldots + [r_{1n} + k_{1n} - (q_{1n} + l_{1n})]y_n \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& [r_{21} + k_{21} - (q_{21} + l_{21})]y_1 + [r_{22} + k_{22} - (q_{22} + l_{22})]y_2 + \ldots \\
& \qquad \ldots + [r_{2n} + k_{2n} - (q_{2n} + l_{2n})]y_n \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& \qquad \vdots \\
& [r_{m1} + k_{m1} - (q_{m1} + l_{m1})]y_1 + [r_{m2} + k_{m2} - (q_{m2} + l_{m2})]y_2 + \ldots \\
& \qquad \ldots + [r_{mn} + k_{mn} - (q_{mn} + l_{mn})]y_n \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& y_1 + y_2 + \ldots + y_n = 1, \\
& 0 \leq \alpha' + \rho' + \beta' + \nu' \leq t, \\
& -0.5 \leq \rho',\ \nu' \leq 0.5, \\
& y_1, y_2, \ldots y_n \geq 0.
\end{aligned}
$$

The crisps linear programming problems (CEI2TLP-I) and (CEI2TLP-II) can be easily solved on LINGO software to obtain the optimal values of the game with the optimal strategies $(x_1^*, x_2^*, ..., x_m^*)$ and $(y_1^*, y_2^*, ..., y_n^*)$, respectively for player I and player II. The optimal value for player I obtained is $\tilde{v}^* =$, where as the optimal value for player II obtained is $\tilde{\omega}^*$.

3 Numerical Illustration

Consider the I2TL-TPZSG with payoffs from the LT set
$S_{[0,6]} = \{s_0 =$ Extremely Poor (EP), $s_1 =$ Very Poor (VP), $s_2 =$ Poor (P), $s_3 =$ Medium (M), $s_4 =$ Good (G), $s_5 =$ Very Good (VG), $s_6 =$ Extremely Good (EG)$\}$ and the payoff matrix

$$
\tilde{A} = \begin{pmatrix}
((M,-0.2),(P,-0.1)) & ((VG,-0.2),(EP,0.4)) & ((P,-0.2),(M,0)) & ((P,0.2),(M,-0.5)) \\
((P,-0.2),(M,-0.1)) & ((P,0.2),(P,0)) & ((G,0.1),(VP,-0.3)) & ((G,0.2),(VP,-0.2)) \\
((M,-0.1),(VP,0.4)) & ((M,0.1),(P,0)) & ((G,0.2),(EP,0)) & ((VP,0.2),(M,0.3)) \\
((VG,0.2),(VP,0.5)) & ((M,0.1),(VP,0.1)) & ((P,0),(M,0.2)) & ((G,-0.5),(EP,0.3)) \\
((G,0.1),(VP,-0.2)) & ((M,-0.5),(M,0.3)) & ((P,0.1),(M,-0.1)) & ((VG,0.1),(EP,0.4))
\end{pmatrix}
$$

Consider the mixed strategies $x = (x_1, x_2, x_3, x_4, x_5),\ x_i \geq 0,\ i = 1, 2, ..., 5, \sum_{i=1}^{5} x_i = 1$, for player I and $y = (y_1, y_2, y_3, y_4),\ y_j \geq 0,\ j = 1, 2, ..., 4, \sum_{j=1}^{4} y_j = 1$, for player II. The $\tilde{v} = ((v_\alpha, \rho), (v_\beta, \nu))$ and $\tilde{\omega} = ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu'))$ are observed as the value of the game for player I and player II, respectively. Constructing the I2TL models of the game, for player I (I2TLP-I) is as follows:

$$
\begin{aligned}
\text{(I2TLP-I)} \quad & \max\ \tilde{v} = ((v_\alpha, \rho), (v_\beta, \nu)) \\
& \text{subject to,} \\
& ((s_3, -0.2), (s_2, -0.1))x_1 \oplus ((s_2, -0.2), (s_3, -0.1))x_2 \oplus \\
& \quad ((s_3, -0.1), (s_1, 0.4))x_3 \oplus ((s_5, 0.2), (s_1, -0.5))x_4 \oplus \\
& \qquad ((s_4, 0.1), (s_1, -0.2))x_5 \geq ((v_\alpha, \rho), (v_\beta, \nu)), \\
& ((s_5, -0.2), (s_0, 0.4))x_1 \oplus ((s_2, 0.2), (s_2, 0))x_2 \oplus \\
& \quad ((s_3, 0.1), (s_2, 0))x_3 \oplus ((s_3, 0.1), (s_1, 0.1))x_4 \oplus \\
& \qquad ((s_3, -0.5), (s_3, -0.3))x_5 \geq ((v_\alpha, \rho), (v_\beta, \nu)), \\
& ((s_2, -0.2), (s_3, 0))x_1 \oplus ((s_4, 0.1), (s_1, -0.3))x_2 \oplus \\
& \quad ((s_4, 0.2), (s_0, 0))x_3 \oplus ((s_2, 0), (s_3, 0.2))x_4 \oplus \\
& \qquad ((s_2, 0.1), (s_3, -0.1))x_5 \geq ((v_\alpha, \rho), (v_\beta, \nu)), \\
& ((s_2, 0.2), (s_3, -0.5))x_1 \oplus ((s_4, 0.2), (s_1, -0.2))x_2 \oplus \\
& \quad ((s_1, 0.2), (s_3, 0.3))x_3 \oplus ((s_4, -0.5), (s_0, 0.3))x_4 \oplus \\
& \qquad ((s_5, 0.1), (s_0, 0.4))x_5 \geq ((v_\alpha, \rho), (v_\beta, \nu)), \\
& x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5 = 1, \\
& x_1, x_2, x_3, x_4, x_5 \geq 0.
\end{aligned}
$$

The I2TL model of the game, for player II (I2TLP-II) is:

$$
\begin{aligned}
\text{(I2TLP-II)} \quad & \min\ \tilde{\omega} = ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu')) \\
& \text{subject to,} \\
& ((s_3, -0.2), (s_2, -0.1))y_1 \oplus ((s_5, -0.2), (s_0, 0.4))y_2 \oplus \\
& \quad ((s_2, -0.2), (s_3, 0))y_3 \oplus ((s_2, 0.2), (s_3, -0.5))y_4 \leq (((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu')), \\
& ((s_2, -0.2), (s_3, -0.1))y_1 \oplus ((s_2, 0.2), (s_2, 0))y_2 \oplus \\
& \quad ((s_4, 0.1), (s_1, -0.3))y_3 \oplus ((s_4, 0.2), (s_1, -0.2))y_4 \leq ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu')), \\
& ((s_3, -0.1), (s_1, 0.4))y_1 \oplus ((s_3, 0.1), (s_2, 0))y_2 \oplus \\
& \quad ((s_4, 0.2), (s_0, 0))y_3 \oplus ((s_1, 0.2), (s_3, 0.3))y_4 \leq ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu')), \\
& ((s_5, 0.2), (s_1, -0.5))y_1 \oplus ((s_3, 0.1), (s_1, 0.1))y_2 \oplus \\
& \quad ((s_2, 0), (s_3, 0.2))y_3 \oplus ((s_4, -0.5), (s_0, 0.3))y_4 \leq ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu')), \\
& ((s_4, 0.1), (s_1, -0.2))y_1 \oplus ((s_3, -0.5), (s_3, -0.3))y_2 \oplus \\
& \quad ((s_2, 0.1), (s_3, -0.1))y_3 \oplus ((s_5, 0.1), (s_0, 0.4))y_4 \leq ((\omega_{\alpha'}, \rho'), (\omega_{\beta'}, \nu')), \\
& y_1 \oplus y_2 \oplus y_3 \oplus y_4 = 1, \\
& y_1, y_2, y_3, y_4 \geq 0.
\end{aligned}
$$

On applying ∇^{-1} operator on (I2TLP-I) and (I2TLP-II), the two problems are reduced to equivalent problems (EI2TLP-I) and (EI2TLP-II), respectively.

For player I:

$$
\begin{aligned}
\text{(EI2TLP-I)}\quad & \max\ (\alpha+\rho,\beta+\nu)\\
& \text{subject to,}\\
& (2.8,1.9)x_1+(1.8,2.9)x_2+(2.9,1.4)x_3+(5.2,0.5)x_4+\\
& \qquad (4.1,0.8)x_5 \geq (\alpha+\rho,\beta+\nu),\\
& (4.8,0.4)x_1+(2.2,2.0)x_2+(3.1,2.0)x_3+(3.1,1.1)x_4+\\
& \qquad (2.5,2.7)x_5 \geq (\alpha+\rho,\beta+\nu),\\
& (1.8,3.0)x_1+(4.1,0.7)x_2+(4.2,0)x_3+(2,3.2)x_4+\\
& \qquad (2.1,2.9)x_5 \geq (\alpha+\rho,\beta+\nu),\\
& (2.2,2.5)x_1+(4.2,1.8)x_2+(1.2,3.3)x_3+(3.5,0.3)x_4+\\
& \qquad (5.1,0.4)x_5 \geq (\alpha+\rho,\beta+\nu),\\
& x_1+x_2+x_3+x_4+x_5=1,\\
& 0 \leq \alpha+\rho+\beta+\nu \leq 6,\\
& -0.5 \leq \rho,\ \nu \leq 0.5,\\
& x_1,x_2,x_3,x_4,x_5 \geq 0.
\end{aligned}
$$

Similarly, for player II:

$$
\begin{aligned}
\text{(EI2TLP-II)}\quad & \min\ (\alpha'+\rho',\beta'+\nu')\\
& \text{subject to,}\\
& (2.8,1.9)y_1+(4.8,0.4)y_2+(1.8,3.0)y_3+(2.2,2.5)y_4 \leq (\alpha'+\rho',\beta'+\nu'),\\
& (1.8,2.9)y_1+(2.2,2.0)y_2+(4.1,0.7)y_3+(4.2,0.8)y_4 \leq (\alpha'+\rho',\beta'+\nu'),\\
& (2.9,1.4)y_1+(3.1,2.0)y_2+(4.2,0)y_3+(1.2,3.3)y_4 \leq (\alpha'+\rho',\beta'+\nu'),\\
& (5.2,0.5)y_1+(3.1,1.1)y_2+(2,3.2)y_3+(3.5,0.3)y_4 \leq (\alpha'+\rho',\beta'+\nu'),\\
& (4.1,0.8)y_1+(2.5,2.7)y_2+(2.1,2.9)y_3+(5.1,0.4)y_4 \leq (\alpha'+\rho',\beta'+\nu'),\\
& y_1+y_2+y_3+y_4=1,\\
& 0 \leq \alpha'+\rho'+\beta'+\nu' \leq 6,\\
& -0.5 \leq \rho',\ \nu' \leq 0.5,\\
& y_1,y_2,y_3,y_4 \geq 0.
\end{aligned}
$$

Thus, utilizing score function comparison method of intuitionistic 2-tuple linguistic variables as employed by [10,18], the problems reduce to equivalent crisp programmming problems. For player I, the crisp equivalent problem is:

$$
\begin{aligned}
\text{(CEI2TLP-I)} \quad & \max\ [\alpha + \rho - (\beta + \nu)] \\
& \text{subject to,} \\
& 0.9x_1 - 1.1x_2 + 1.5x_3 + 4.7x_4 + 3.3x_5 \geq [\alpha + \rho - (\beta + \nu)], \\
& 4.4x_1 + 0.2x_2 + 1.1x_3 + 2.0x_4 - 0.2x_5 \geq [\alpha + \rho - (\beta + \nu)], \\
& -1.2x_1 + 3.4x_2 + 4.2x_3 - 1.2x_4 - 0.8x_5 \geq [\alpha + \rho - (\beta + \nu)], \\
& -0.3x_1 + 3.4x_2 - 2.1x_3 + 3.2x_4 + 4.7x_5 \geq [\alpha + \rho - (\beta + \nu)], \\
& x_1 + x_2 + x_3 + x_4 + x_5 = 1, \\
& 0 \leq \alpha + \rho + \beta + \nu \leq 6, \\
& -0.5 \leq \rho,\ \nu \leq 0.5, \\
& x_1, x_2, x_3, x_4, x_5 \geq 0.
\end{aligned}
$$

and for player II

$$
\begin{aligned}
\text{(CEI2TLP-II)} \quad & \min\ [\alpha' + \rho' - (\beta' + \nu')] \\
& \text{subject to,} \\
& 0.9y_1 + 4.4y_2 - 1.2y_3 - 0.3y_4 \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& -1.1y_1 + 0.2y_2 + 3.4y_3 + 3.4y_4 \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& 1.5y_1 + 1.1y_2 + 4.2y_3 - 2.1y_4 \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& 4.7y_1 + 2y_2 - 1.2y_3 + 3.2y_4 \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& 3.3y_1 - 0.2y_2 - 0.8y_3 + 4.7y_4 \leq [\alpha' + \rho' - (\beta' + \nu')], \\
& y_1 + y_2 + y_3 + y_4 = 1, \\
& 0 \leq \alpha' + \rho' + \beta' + \nu' \leq 6, \\
& -0.5 \leq \rho',\ \nu' \leq 0.5, \\
& y_1, y_2, y_3, y_4 \geq 0.
\end{aligned}
$$

The optimal solution of problem (CEI2TLP-I) for player I is obtained as ($x_1^* = 0.1402519$, $x_2^* = 0.3241958$, $x_3^* = 0.2320240$, $x_4^* = 0.3035283$, $x_5^* = 0.0000$) and $\alpha = 3.772115$, $\rho = 0.000$, $\beta = 2.227885$, $\nu = 0.000$. That implies, $\tilde{v}^*$ being the optimal value of the I2TL game for player I is obtained as $\tilde{v}^* = ((v_\alpha, \rho), (v_\beta, \nu))$. Thus, $\tilde{v}^* = ((s_{3.772115}, 0), (s_{2.227885}, 0))$.

Similarly, for player II, the optimal solution of problem (CEI2TLP-II) is obtained as ($y_1^* = 0.1054664$, $y_2^* = 0.4254982$, $y_3^* = 0.2917715$, $y_4^* = 0.1772639$), whereas $\alpha' = 3.781903$, $\rho' = 0.000$, $\beta' = 1.718097$, $\nu' = 0.479958$. Hence, the optimal value of the I2TL game for player II is $\tilde{\omega}^* = ((s_{3.781903}, 0), (s_{1.718097}, 0.479958))$.

4 Conclusion

In this work, a novel approach is proposed to resolve the solution of I2TL-TPZSG. Our proposed method is different from all other previous techniques for two person zero-sum game due to the fact that the proposed method use intuitionistic 2-tuple fuzzy linguistic information, thus causing no loss of information in the process. Hence, making it feasible and efficient for real-world decision making applications. Moreover, in the proposed technique comparision of I2TLVs is based on score function, which is the most common approach used in literature for comparing fuzzy values.

5 Future Scope

In addition, the method for group decision-making based on multi-granularity intuitionistic fuzzy linguistic information are also worthy of consideration for future research. Also, the technique can be employed on bimatrix games with intuitionistic 2-tuple linguistic information.

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