Math 6345 Advanced ODEs
Homework 3

1. For the following system differential equations

$$
\begin{aligned}
(i) \quad \dot{x} &= x + y, \\
\dot{y} &= \left(\frac{\sin t + \cos t}{2 + \sin t - \cos t} \right) y, \\
(ii) \quad \dot{x} &= -\sin 2t x + (\cos 2t - 1) y, \\
\dot{y} &= (\cos 2t + 1) x + \sin 2t y.
\end{aligned}
$$

(i) Find a matrix $A(t)$ and period T such that $A(t+T) = A(t)$

(ii) Find the fundamental matrix $\Phi(t)$

(iii) Find a matrix C such that $\Phi(t+T) = \Phi(t)C$

(iv) Find matrices $P(t)$ and B such that

$$\Phi(t) = P(t)e^{Bt}$$

(v) Show that $C = e^{BT}$

(vi) Show that under $x = P(t)y$, the system simplifies to $\dot{y} = By$