A Mathematical Model to Calculate Variation in Volume Flux for the Problem of Blood Flow in Artery

Amit Gupta[#1], Gajendra Saraswat[*2], Ravendra Singh[$3]
[#1,*2] *Department of Mathematics, Mangalayatan University, (UP), India*
[$3]*K R Mangalam University, Sohna l, (Haryana), India*

Abstract-This chapter deals with a mathematical modal described to calculate blood flow in a normal artery from pressures measured at two separated points. A assumption in considered for the frictional term which cannot be expressed in the form of these non-liner partial differential equations. The importance of the result to change in frictional term is validated. A method is given on the basis of the skin friction on a linear approximation. The differential equation are solved analytically as well as numerically using finite difference technique and observed the variation for one period of oscillation in a canine artery in volume flux.

Key words: *Blood flow, pulsatile flow, skin wall friction, finite difference techniques*

I.INTRODUCTION:

The behavior of an elastic tube containing a viscous fluid submitted to pulsatile flow has been the object of many studies concerning the analysis of blood flow. Quite a few models has been developed treating the movement of the arterial wall as a function of its rheological properties and of the motion of the fluid contained in the vessel (wherlow and rouleau1965 and womersley1957). The numerical technique can lodge nonlinearity and has the benifit that solution can be foundas distance along the vessel and of function of time.

Strreter et al. (1963), Olsen and Shapiro (1967) Hunt (1969), Anliken et al.(1971), and wimple and Mockros (1972) used same basic one-dimensional equation to calculate blood flow.. The techniques employed by these entire researchers were the method of characteristic. The method of finite differences in the various approaches used by Ling and Atabek (1972), Rains et al. (1974) and the present authors is in the manner in which account is taken of the motion of the wall and skin friction the skin friction must be independently determined in integrated out and thus the resulting equations have only the crosssectional mean velocity,

The use of method of characteristics results in a limitation on the form of the skin as taken by anliker al. (1971), it is possible to circumvent this limitation of the method of characteristics by engaging an approximation and iterative of the effects of wall visco elasticity.

The formulation of ling and atabek (1972) does not callof pressure gradient as well as the pressures at two distinct points. Taylor and Gerrard (1977) have taken the pressure radius relationship which draw account of the wall motion.

Imacda et al. (1980) worked on the analysis of non linear pulsatile blood flow in arteries .chaturani and upadhya (1981) worked on a two-fluid model for blood flow though small diameter tubes with non-zero coutne stress boundary condition at the interface, devid(1997) discussed on the blood flow in arteries .

Pontrelli(2000) gave an idea on blood flow through a circular pipe with an impulsive pressure gradient. Formaggia et, Al .(2003) discussed the one dimensional model for blood flow in arteries young et. Al. (2006) observed the effects of non Newtonian behavior of blood on pulsatile flows in stenotic arteries. Mandalet.al . (2007) demonstrated the effect of body acceleration on unsteady pulsatile flow of non-newtonian fluid through a stenosedartery. Sankar and hemalatha(2007) gave an idea non-linear mathematical model for blood flow through tapered tubes.

II. FORMULATION OF THE PROBLEM :

To simplify the analysis, we additionally make the following suppositions :

The flow is laminar and there rotational symmetry of flow consider blood density as constant quantity and the flow is Newtonian which is justified in the larger arteries. The fluid properties are homogeneous and isotropic. Taylor and relationship and according which the excess pressure P is related to the radius R by :

$$\frac{Eh_0}{3R_0}\left(1-\frac{R_0^4}{R^4}\right)\theta_0 = P \qquad \text{……………(1)}$$

Taking E as young modulusof the wall material, h_0=Thickness of wallunder zero excess pressure, R_0=internal radius at zero excess pressure andthickness factor=

$$\theta_0 = \left(1+\frac{h_0}{2R_0}\right)\left(1+\frac{h_0}{R_0}\right)^2$$

The wave velocity for a tube filled with in viscous is given by:

$$a^2 = \frac{R}{2\rho}\frac{\partial P}{\partial R} \qquad \text{………..(2)}$$

According to Taylor (1972) the expression for the skin friction is:

$$\tau = \frac{4\mu\nu}{R_0} \qquad \text{..........(3)}$$

Where μ is the coefficient of viscosity

A solution of the basic equation of motion of the fluid is derived bystreeter et al. (1963), The equation of continuity is

$$\frac{\partial(\nu A)}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{............(4)}$$

And the equation of momentum :

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \nu}{\partial r}\right) - \frac{2\tau}{R} = \frac{\partial \nu}{\partial t} \qquad \text{............(5)}$$

Where Internal area of cross-section of the vessel is taken as A, cross sectional mean velocity= ν ,density of fluid= ρ pressure=P, Internal radius of the vessel=Rand skin friction between the fluid and well= τ .

When $\tau = 0$ and the amplitude is very small then these equations treated as the wave equation with wave speed squared

$$\frac{R}{2\rho}\frac{\partial P}{\partial R} = a^2$$

Now using the value of a from equation (2) in equation (4) then the equation of continuity becomes:

$$\nu\frac{\partial P}{\partial x} + \frac{\partial P}{\partial t} + Pa^2\frac{\partial \nu}{\partial x} + \left(\frac{2\rho\nu a^2}{R_0}\right)\frac{\partial R_0}{\partial x} = 0$$

.........(6)

In equation last term arises from the tapes of the vessel .The initial and boundary conditions of the problem are:

$$\nu = 0 \;\; on \; r = R$$

$$\frac{\partial \nu}{\partial r} = 0 \; on \, r = 0$$

2.1 Method of solution

2.1.1. Analytical solution

Let the solution of ν and P are set in the following forms:

$$\nu(r,t) = \bar{\nu}(r)e^{i\omega t} \qquad \text{............(7)}$$

$$-\frac{\partial P}{\partial z} = Pe^{i\omega t} \qquad \text{..............(8)}$$

And substitute this set of equation (7) and (8) in equation (6)we get :

$$\frac{\partial^2 \bar{\nu}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\nu}}{\partial r} + \frac{P}{\mu} - \frac{2\tau}{R\mu} = \frac{i\omega\nu P}{\mu}$$

..............(9)

While the boundary conditions becomes :

$$\bar{\nu} = 0 \;\; on \; r = R$$

.............(10)

$$\frac{\partial \bar{\nu}}{\partial r} = 0 \; on \, r = 0$$

............(11)

After applying boundary condition solution of equation (9) can be given as :

$$\bar{\nu} = -\left(P - \frac{2\tau}{R}\right)\frac{i}{\omega\rho}\left[1 - \frac{j_0\left(i^{3/2}\alpha s\right)}{j_0\left(i^{3/2}\alpha\right)}\right]$$

$$where\, \alpha^2 = \frac{\omega\rho R^2}{\mu} = \frac{\omega R^2}{\nu}\, and\; s = \frac{r}{R}$$

Where j_0 is expressed as the Bessel function of zero order with complex argument .Now for axial velocitythe resulting expression is given by:

$$\nu(r,t) = -\left(P - \frac{2\tau}{R}\right)\frac{i}{\omega\rho}\left[1 - \frac{j_0\left(i^{3/2}\alpha s\right)}{j_0\left(i^{3/2}\alpha\right)}\right]e^{i\omega t}$$

And Q i.e. volumetric flow rate can be calculated by the relation:

$$Q = \int_0^R 2\pi r\nu dr$$

2.1.2. Approximate solution :

Now applying finite differences methods for obtaining the solution of equation of motion (4) and (5) and assuming $\frac{\partial P}{\partial z} = 0$. After substitution the value of a^2 ,P and τ from equation (2) ,(1)and (3) respectively . forms of Central finite gradients of quantities were applied and this resulting equations as:

$$-\frac{1}{2\Delta x}\left[\frac{\left(P_{i+1}^k - P_{i-1}^k\right)}{\rho} + \nu_i^k\left(\nu_{i+1}^k - \nu_{i-1}^k\right)\right] - \frac{2\tau_i^k}{\rho R_i^k} = \frac{\nu_i^{k+1} - \nu_i^{k-1}}{2\Delta t}$$

..................(12)

and

$$\frac{\left(P_i^{k+1}-P_i^{k-1}\right)}{2\Delta t}=-\frac{\left[v_i^k\left(P_{i+1}^k-P_{i-1}^k\right)+\left(\rho a_0{}^2-2P_i^k\right)\left(v_{i+1}^k-v_{i-1}^k\right)+\frac{2v_i^k a^{2k}}{R_0\left(\frac{\partial R_0}{\partial x}\right)}\right]}{2\Delta x}$$

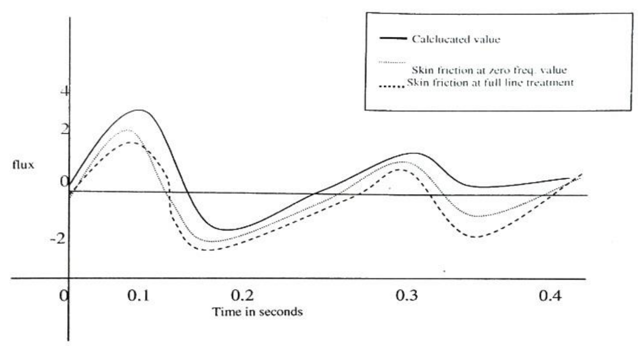
………(13)

Where the number of the distance step along the vessel is refer by suffixes i length and the number of the time steps is refers by superscripts k. At the ends of the vessel pressure is given as the function of time. Distance (Δx) and time(Δt) steps small as compared with the tube length and the period respectively . Also $\frac{\Delta x}{\Delta t}$ greater than the speed of pulse to ensure stability of theproduce an accurate result as well as computation .

III. RESULT AND DISCUSSION:

The calculation of Flux was depending on fixed diameter at the ends. Since calculation of flux as with a cuff type of electromagnetic flow which remains in contact with the arterial wall. It was considered that the quoted diameters corresponding to the measuring stations with minimum pressure..

The value of a corresponding to the minimum pressurewave from will be quoted for solutions with skin friction equated to τ_0 its value for zero frequency a =444cm/sec is taken and the speed for corresponding wave for the three harmonics are 418, 782 and 1256 cm/sec. For the calculation of skin friction the present full line treatment a= 482 cm/sec for these the corresponding wave speeds are 404 ,754 and 1434 cm/sec.



Volume flux for one period of oscillation in artery

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