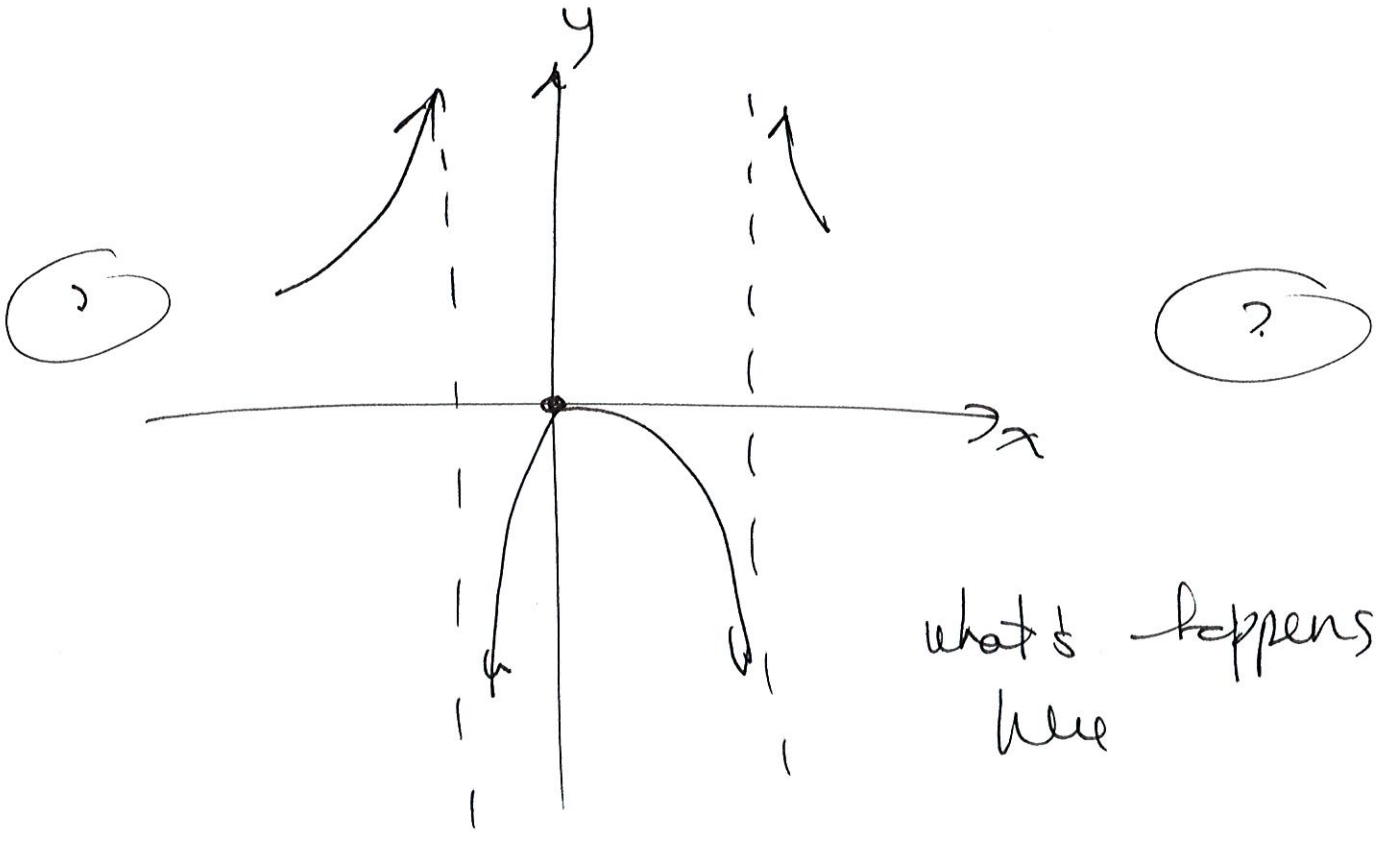
Math 1496 - Calc I

$$f(x) = \frac{2x^2}{x^2-x-2} = \frac{2x^2}{(x-2)(x+1)}$$

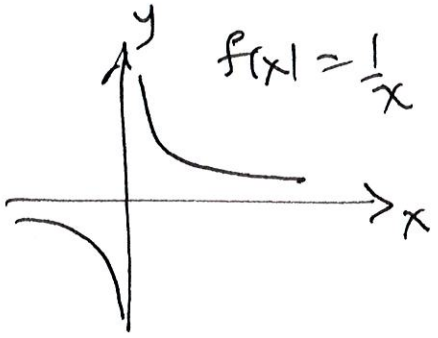
$f(-1)$ $f(2)$ DNE $f(0) = 0$

x		-1		0		2		
$2x^2$	+	+	+	0	+	+	+	+
$x-2$	−	−	−	−	−	0	+	+
$x+1$	−	0	+	+	+	+	+	+
$\frac{x^2}{(x-2)(x+1)}$	+	∞ VA	−	0	−	∞ VA	+	



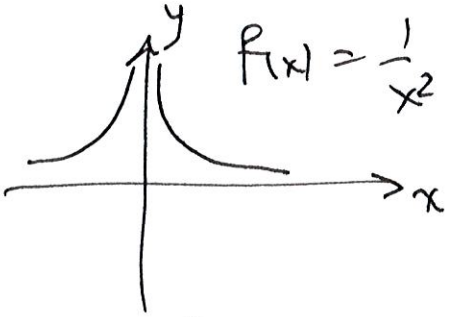
what happens here

Limits at Infinity

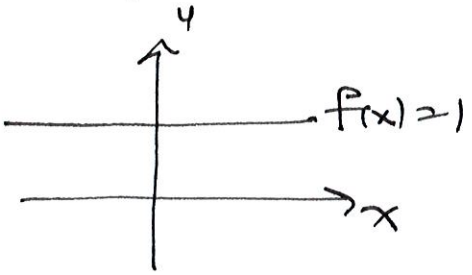


we see from the graph

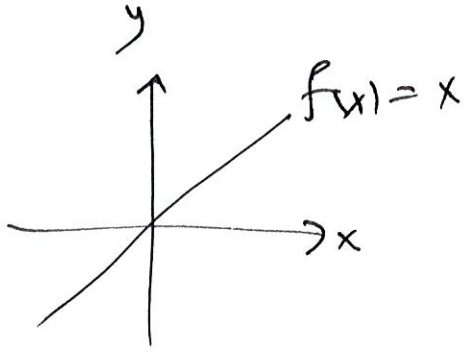
$$\lim_{x\to\infty} \frac{1}{x} = 0 \qquad \lim_{x\to-\infty} \frac{1}{x} = 0$$



$$\lim_{x\to\infty} \frac{1}{x^2} = 0 \qquad \lim_{x\to-\infty} \frac{1}{x^2} = 0$$



$$\lim_{x\to\infty} 1 = 1 \qquad \lim_{x\to-\infty} 1 = 1$$



$$\lim_{x\to\infty} x = \infty, \qquad \lim_{x\to-\infty} x = -\infty$$

so what about

$$\lim_{x\to\infty} \frac{2x+1}{3x+1} = \text{“}\frac{\infty}{\infty}\text{”}$$

we divide by the highest power in the denominator

so $$\lim_{x\to\infty} \frac{\frac{2x}{x} + \frac{1}{x}}{3\frac{x}{x} + \frac{1}{x}} = \lim_{x\to\infty} \frac{2 + \frac{1}{x}}{3 + \frac{1}{x}} = \frac{2+0}{3+0} = \frac{2}{3}$$

Ex $$\lim_{x\to-\infty} \frac{x^2 + x - 1}{7x^2 + 4x - 4} = \lim_{x\to-\infty} \frac{1 + \frac{1}{x} - \frac{1}{x^2}}{7 + \frac{4}{x} - \frac{4}{x^2}}$$

$$= \frac{1+0-1}{7+0-0} = 1/7$$

what about

$$\lim_{x\to\infty} \frac{2x+1}{7x^2+4x+1} = \lim_{x\to\infty} \frac{\frac{2}{x} + \frac{1}{x^2}}{7 + \frac{4}{x} + \frac{1}{x^2}} = \frac{0+0}{7+0+0} = 0$$

↑

and

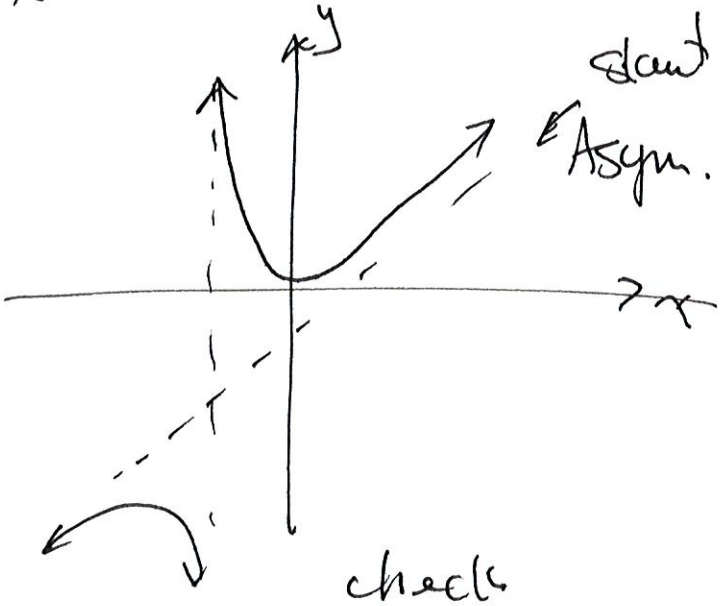
$$\lim_{x\to\infty} \frac{2x^2+1}{7x+1} = \lim_{x\to\infty} \frac{2x + \frac{1}{x}}{7 + \frac{1}{x}} = \frac{"\infty + 0"}{7+0} \to \infty$$

↑

Consider

$$f(x) = \frac{x^2}{x+1}$$

$$\lim_{x\to\infty} \frac{x^2}{x+1} = \lim_{x\to\infty} \frac{x}{1+\frac{1}{x}} \to \infty$$



Divide polynomial

$$\begin{array}{r} x-1 \\ x+1 \overline{)\, x^2} \\ x^2+x \\ \hline -x \\ -x-1 \\ \hline 1 \end{array}$$

check

$$\text{so } \frac{x^2}{x+1} = x-1+\frac{1}{x+1}$$

$$\left[= \frac{(x-1)(x+1)}{x+1} + \frac{1}{x+1} \right.$$

$$= \frac{x^2-1}{x+1} + \frac{1}{x+1}$$

$$= \frac{x^2-x+1}{x+1}$$

$$= \frac{x^2}{x+1} \quad \checkmark \quad]$$

$$\text{so } \lim_{x\to\infty} \frac{x^2}{x+1} = \lim_{x\to\infty} x-1 + \lim_{x\to\infty} \frac{1}{x+1}$$

$$\lim_{x\to\infty} \frac{1}{x+1} \to 0$$

$$\text{so } y \to x-1$$

$y = x-1$ is a slant Asymptote