CEC 2024 SPECIAL EDUCATION CONVENTION & EXPO SAN ANTONIO

What's Important in Mathematics at Tier 2 and 3

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#CEC2024

Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

Instructional Platform

What's Important in Mathematics at Tier 2 and 3?

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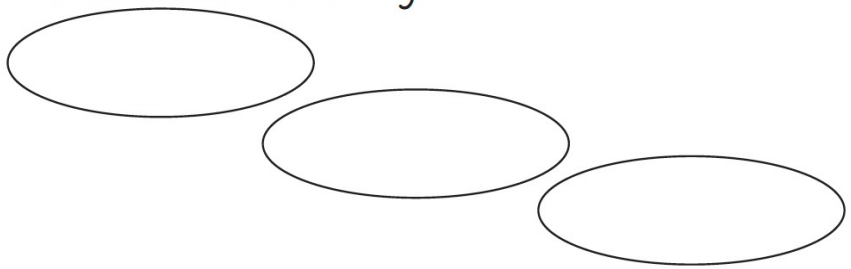
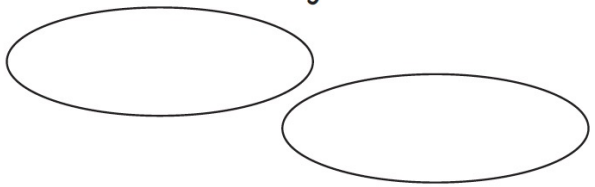
Instructional Platform

Instructional Delivery

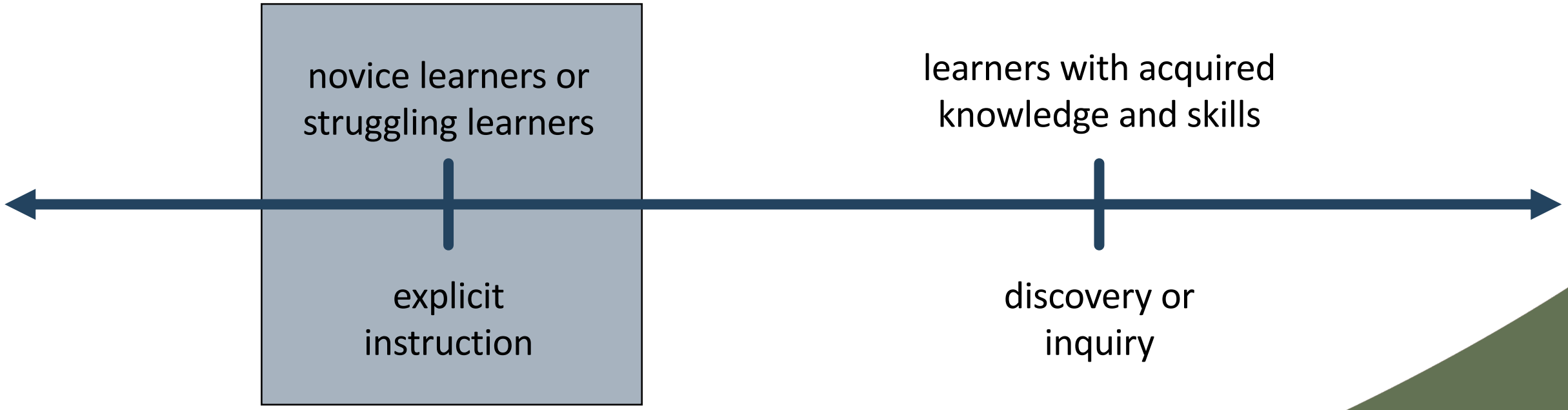


Instructional Strategies



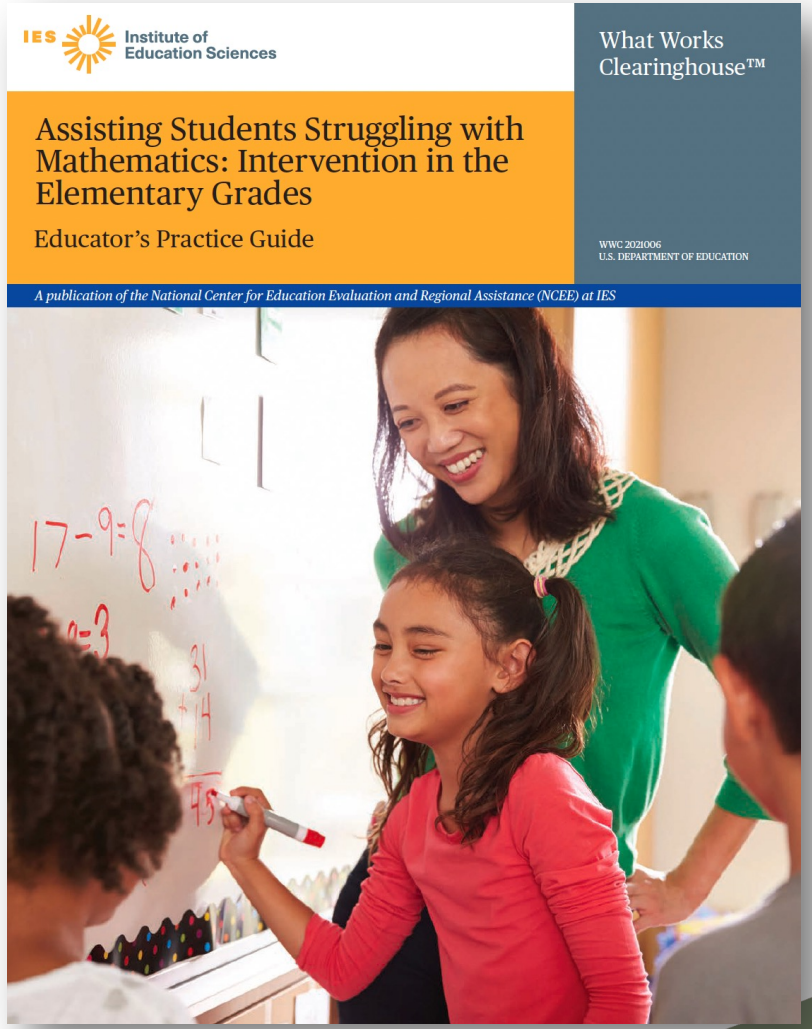
MATH

What's the continuum of mathematics support?



Anita Archer (2019)

Instructional Platform



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Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Vocabulary

VOCABULARY

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...

Instructional Platform

INSTRUCTIONAL DELIVERY

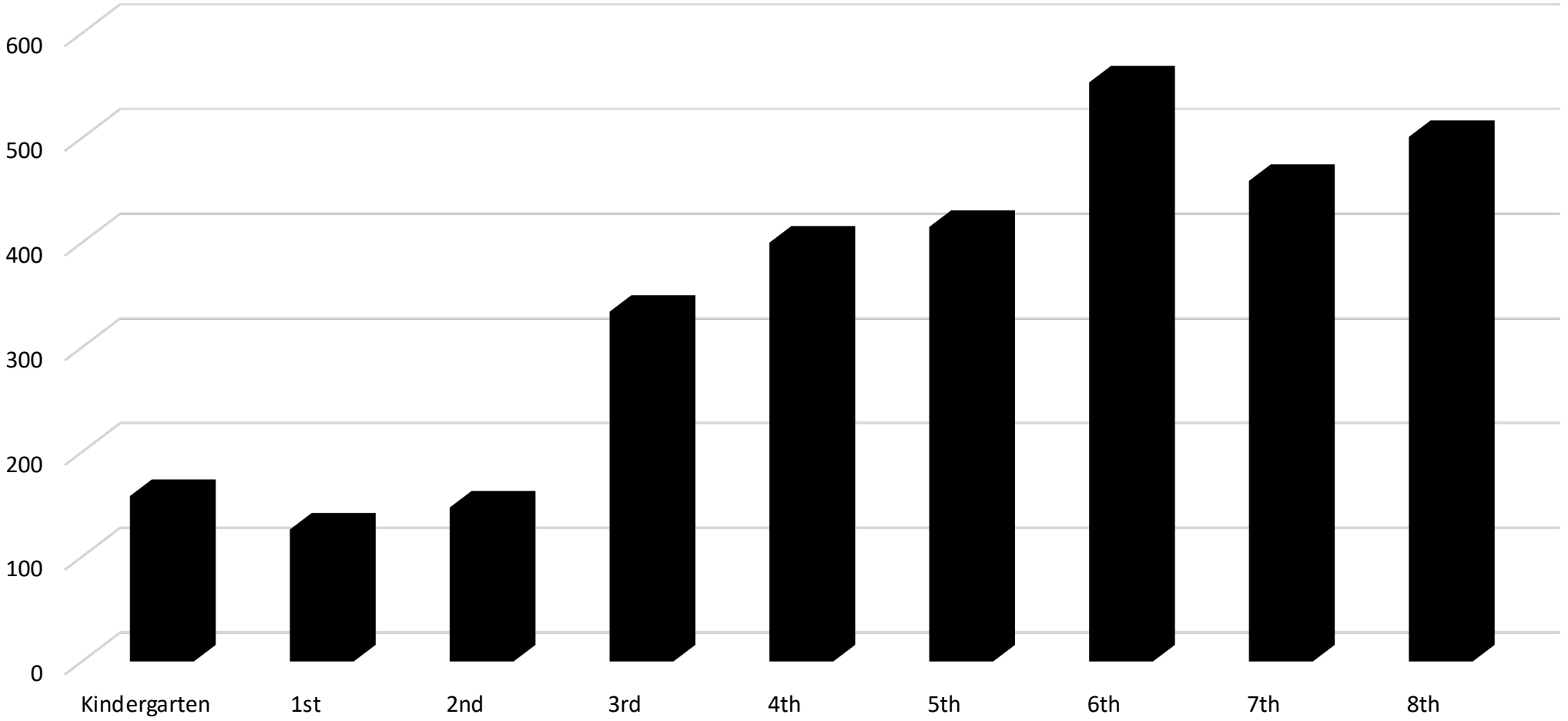
Vocabulary

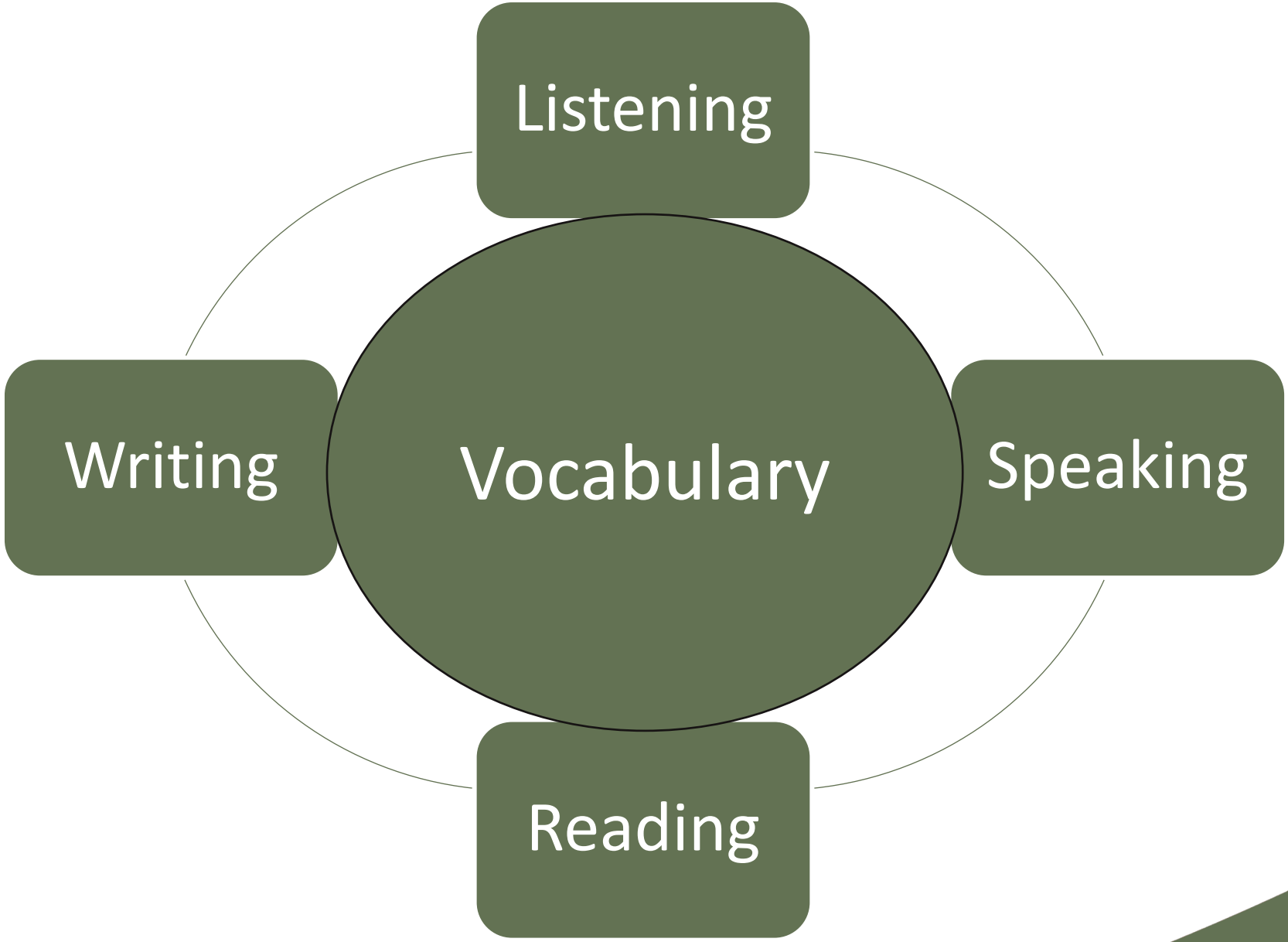
INSTRUCTIONAL STRATEGIES

Significant correlation (r = .49) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance (r = .58).
(Lin et al., 2021)

Early mathematics vocabulary related to mathematics and literacy.
(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.
(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)


600
500
400
300
200
100
0
Kindergarten
1st
2nd
3rd
4th
5th
6th
7th
8th


Listening
Writing
Vocabulary
Speaking
Reading

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

round

square

second

base

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

trapezoid

numerator

parallelogram

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

7. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

7. Some math concepts are verbalized in more than one way

8 Some informal terms may be used for formal math terms

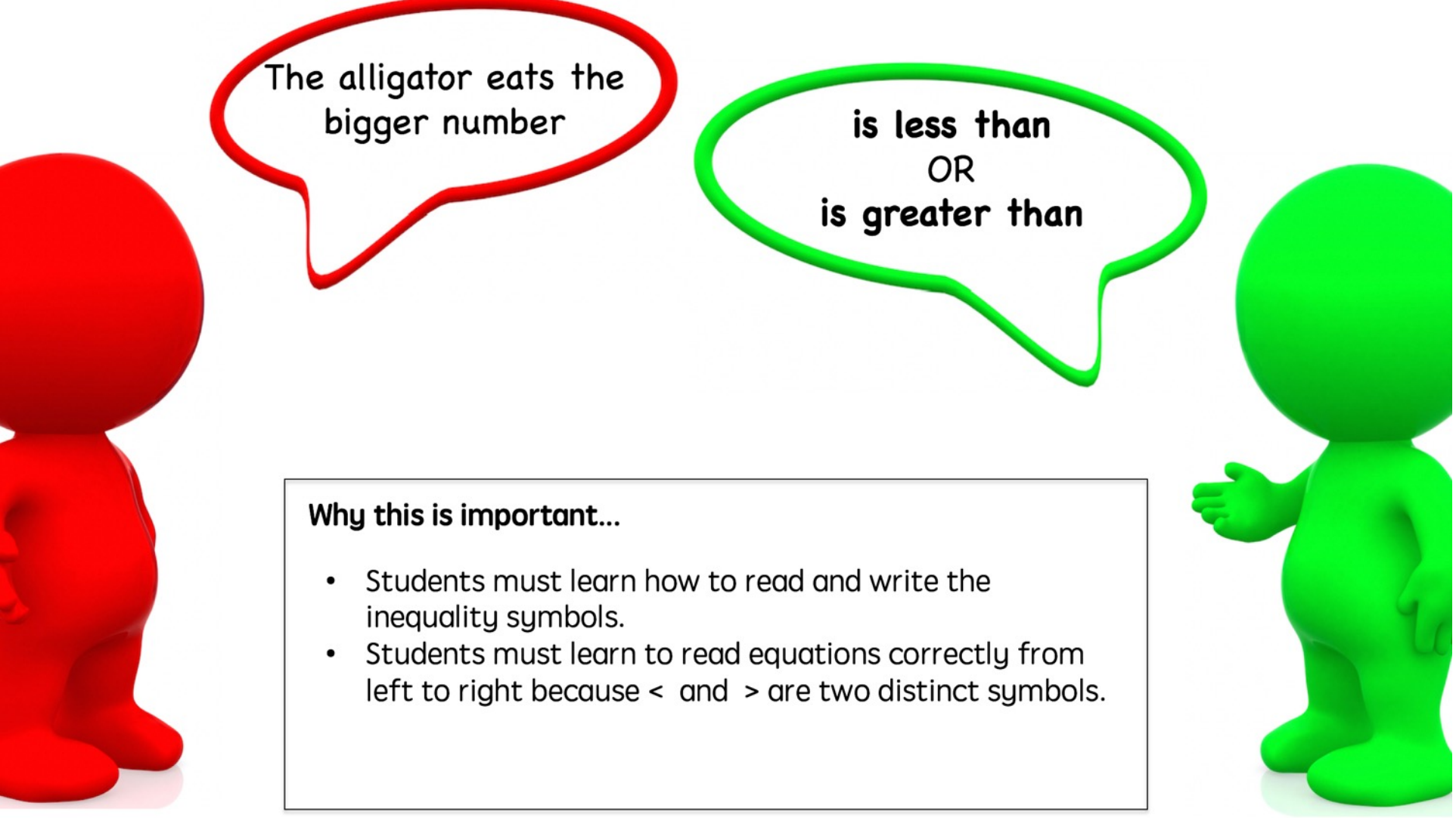
rhombus vs. diamond

vertex vs. corner

Why might your students have difficulty with math vocabulary?

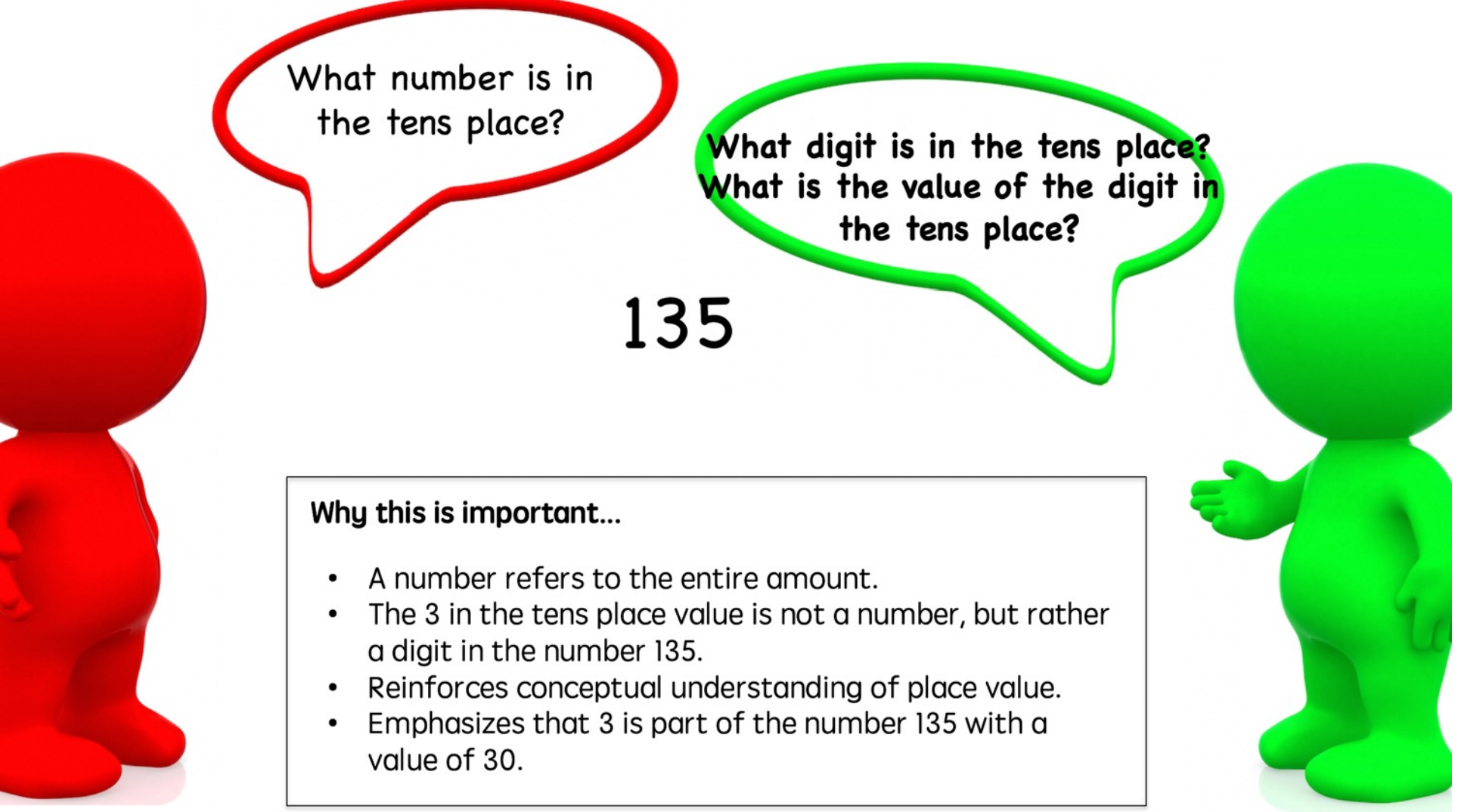
Use formal math language

Use terms precisely



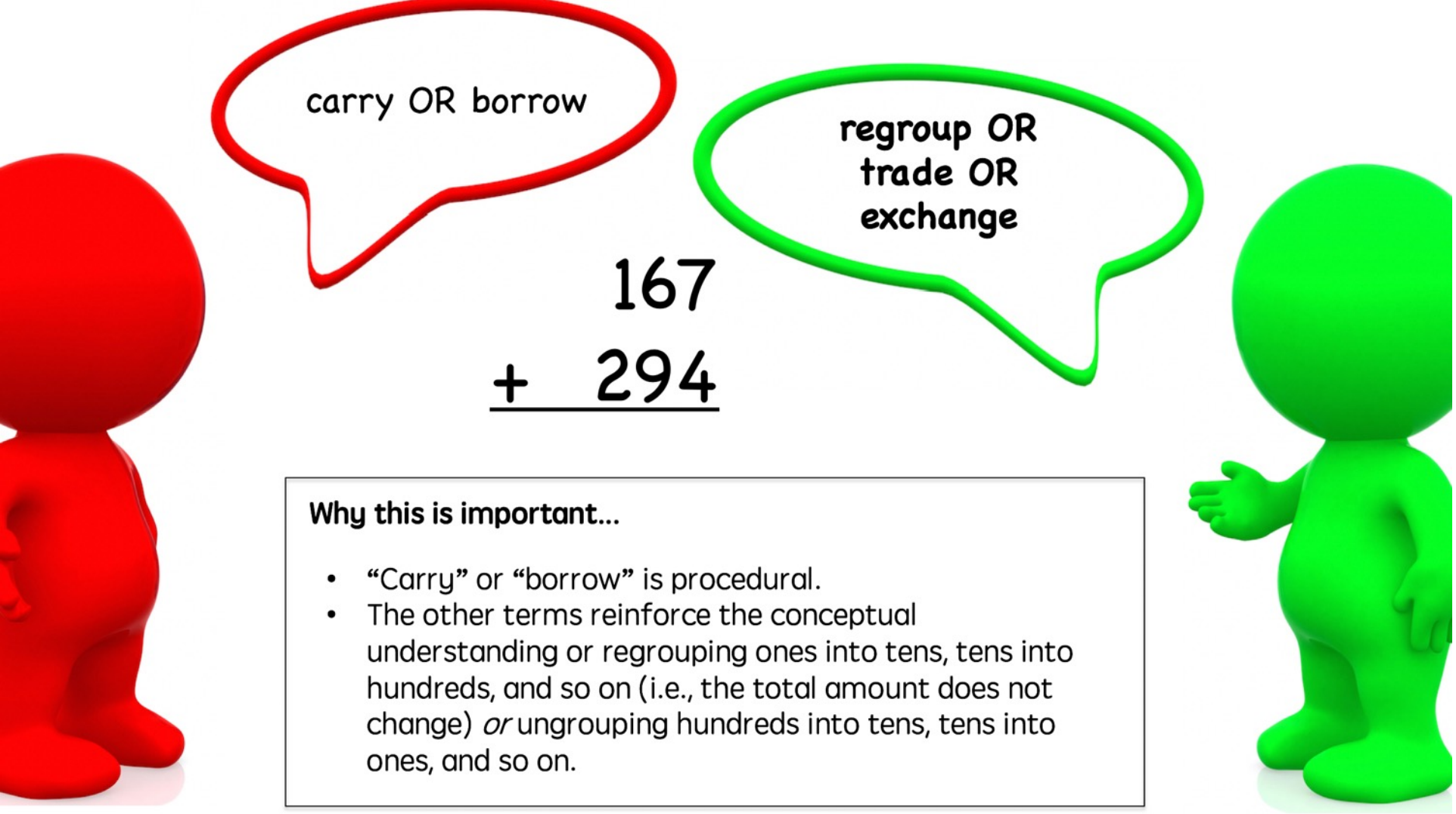
Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.



Why this is important...

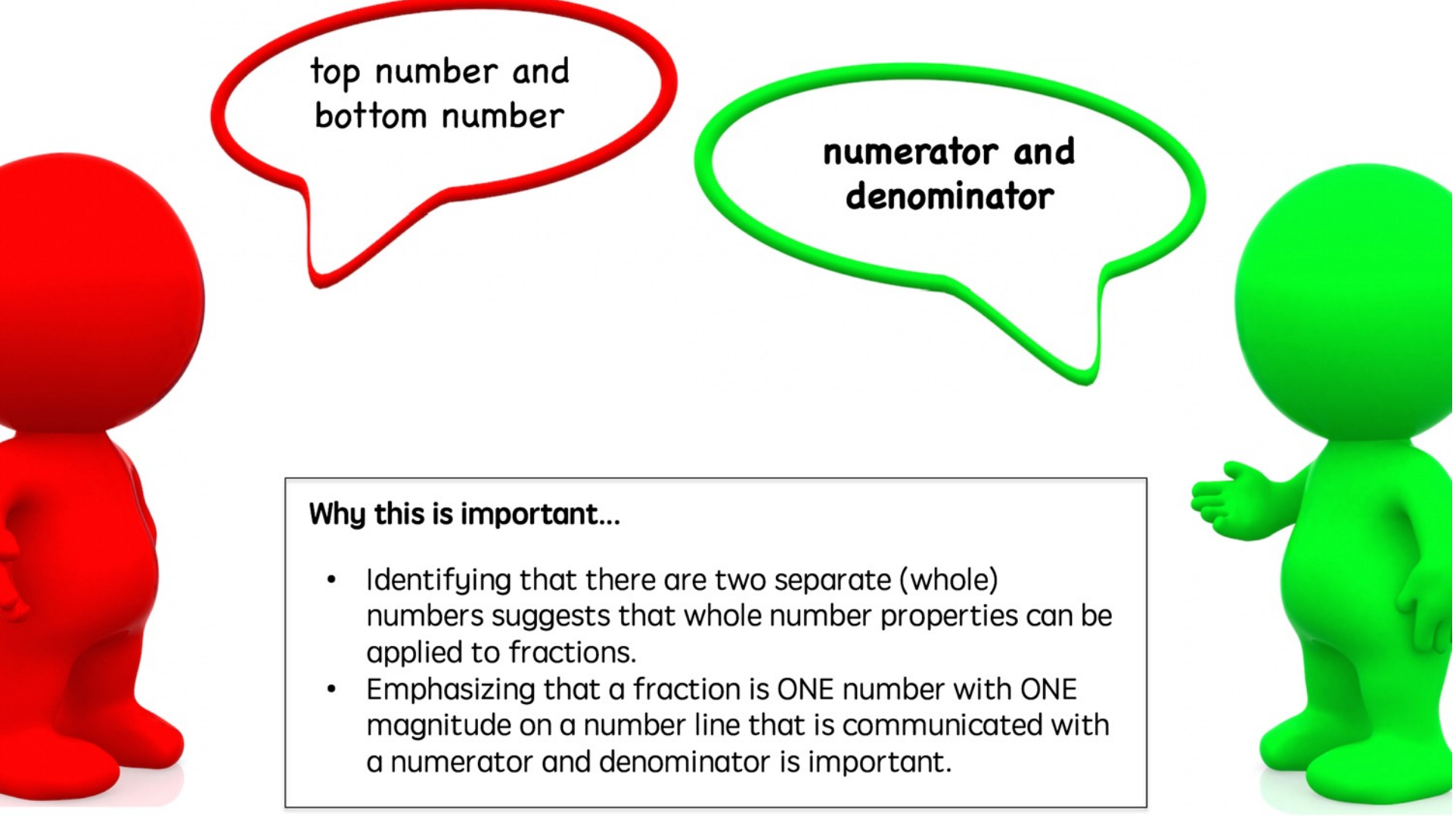
- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

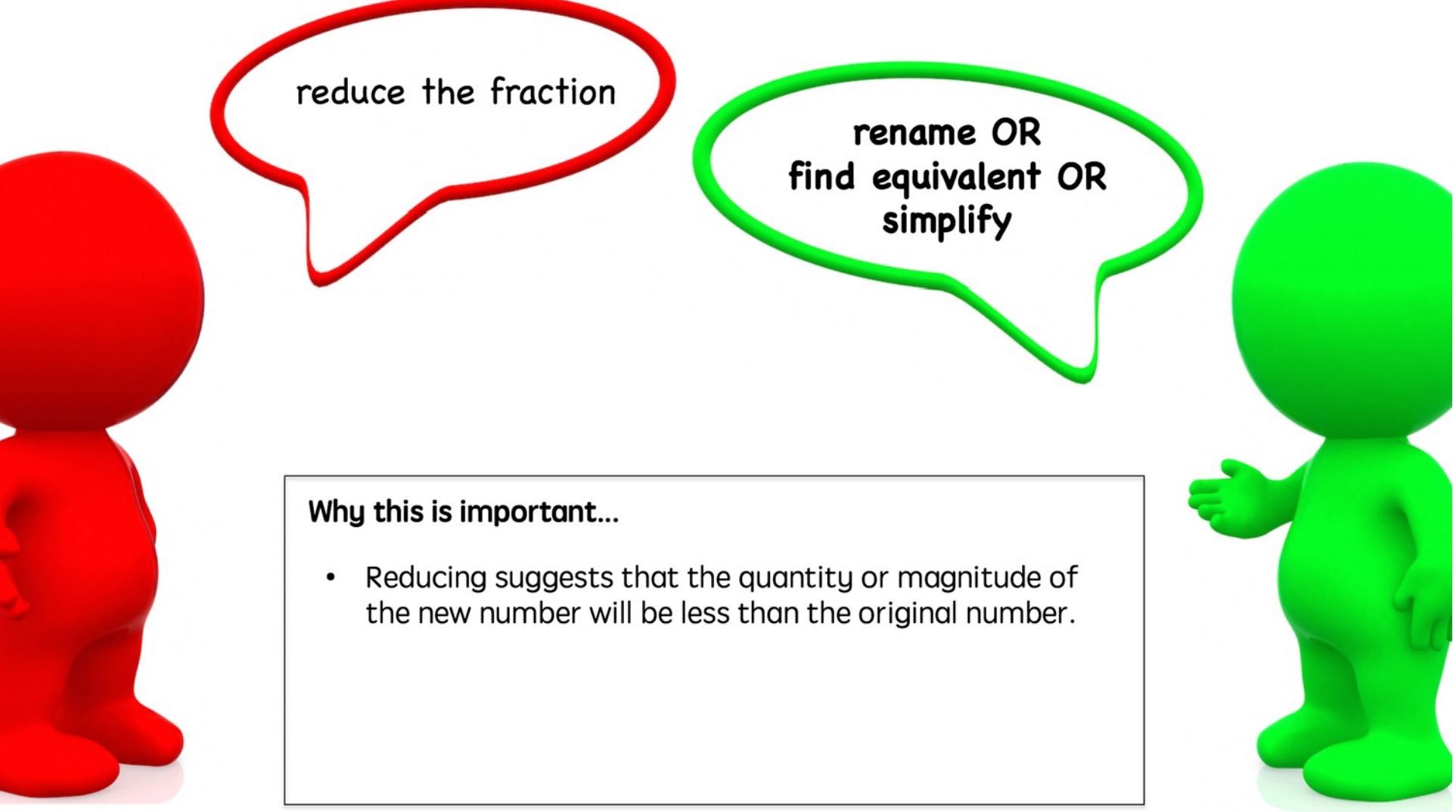
Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



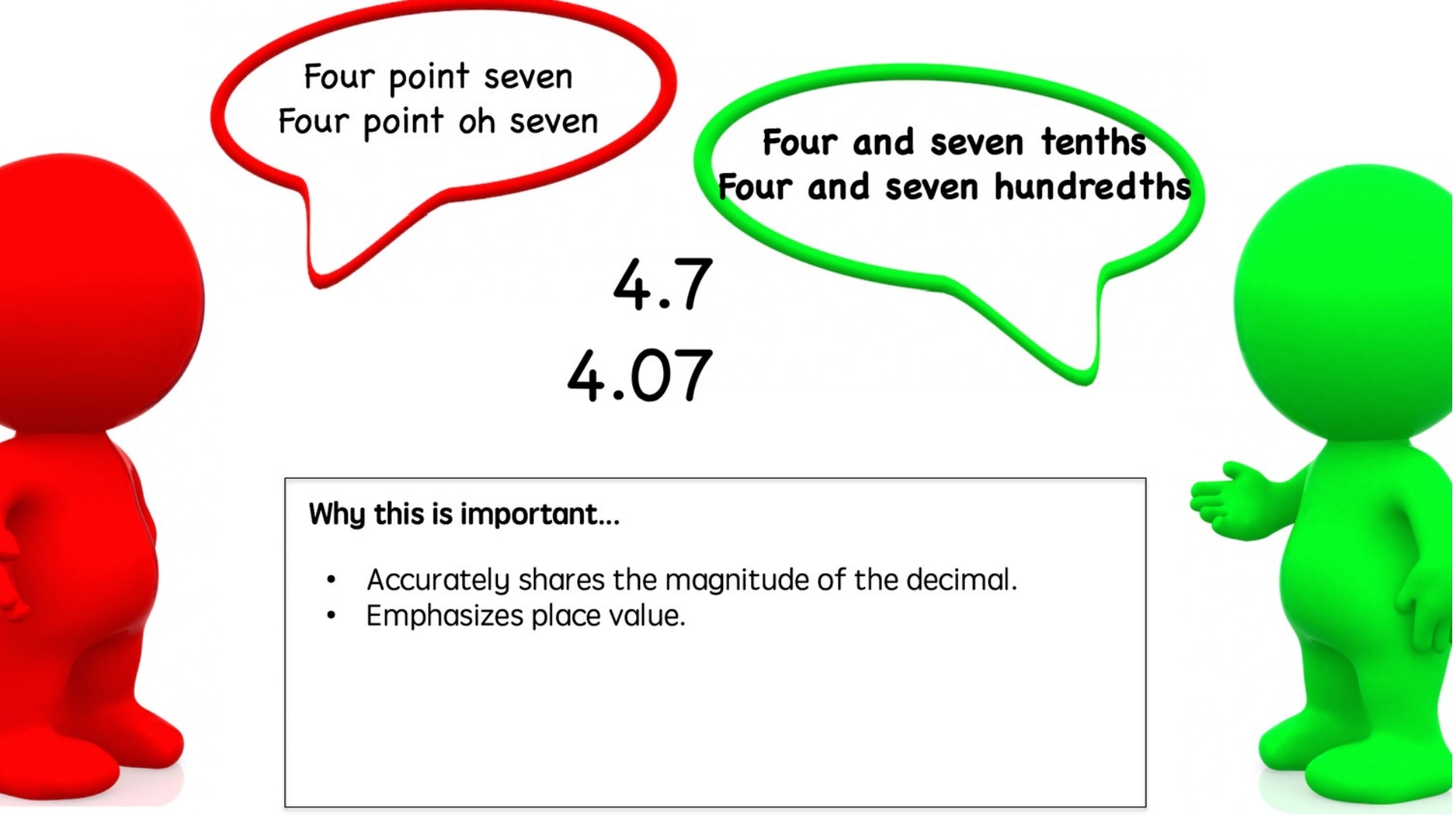
Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



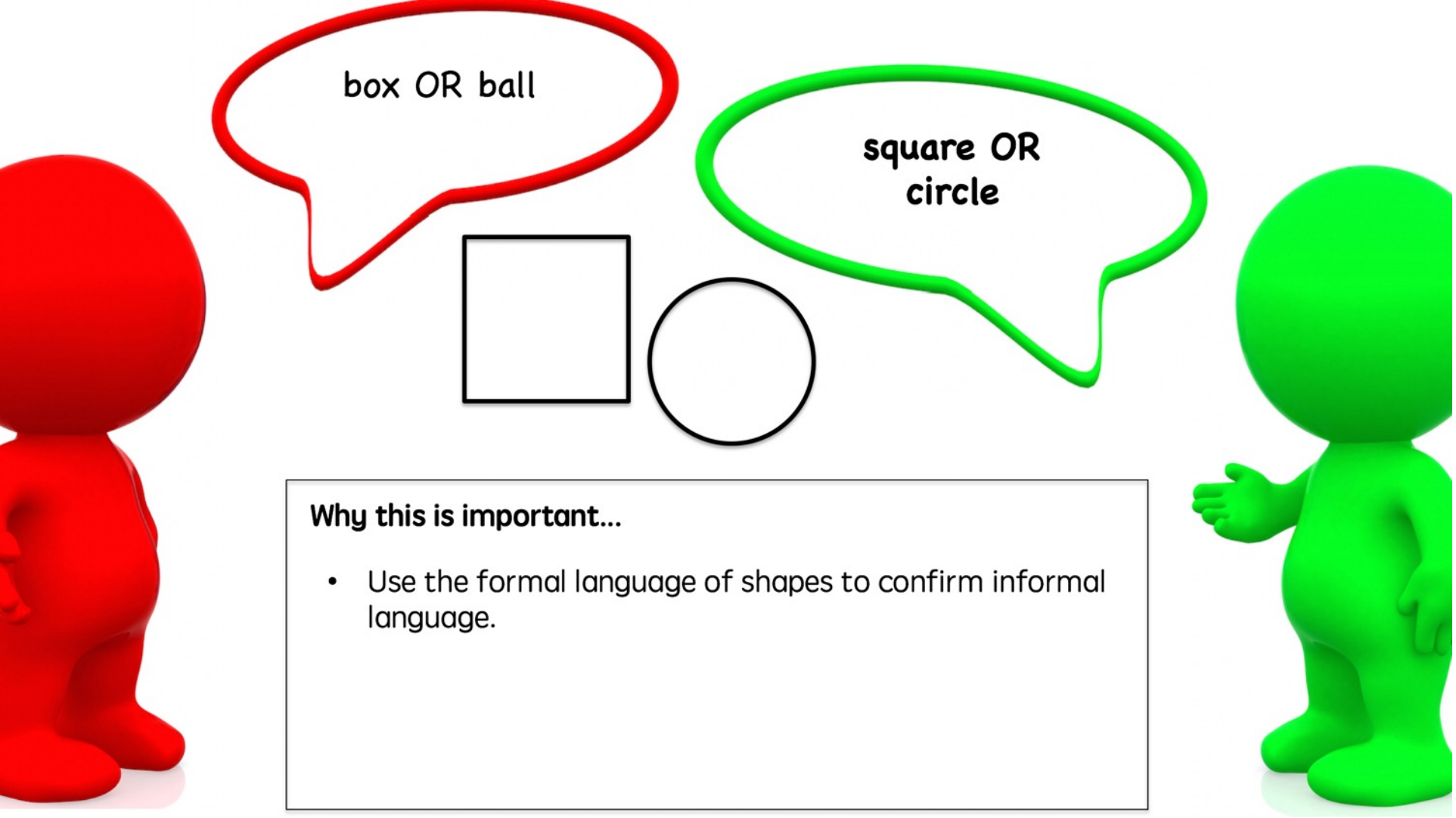
Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



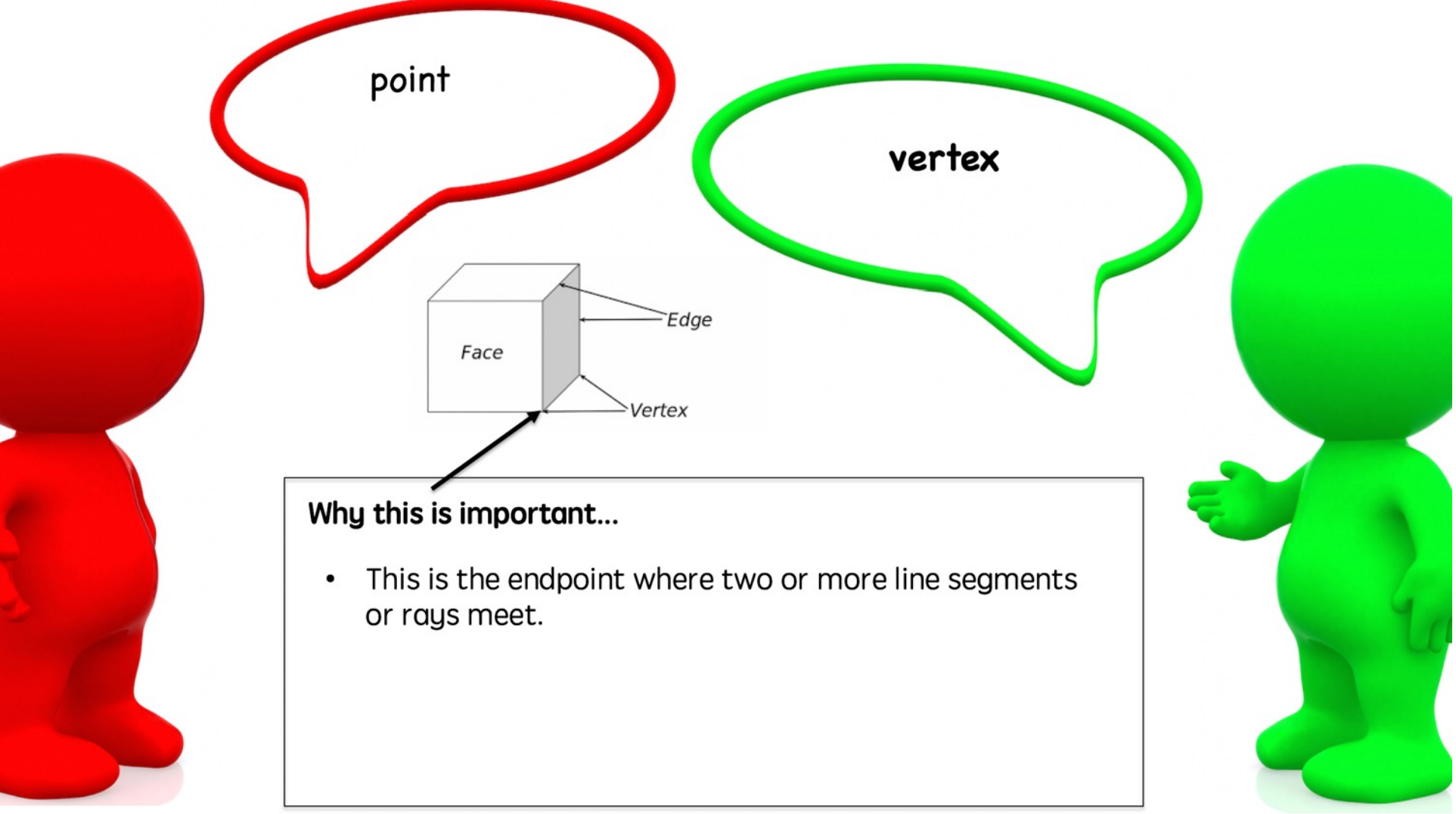
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



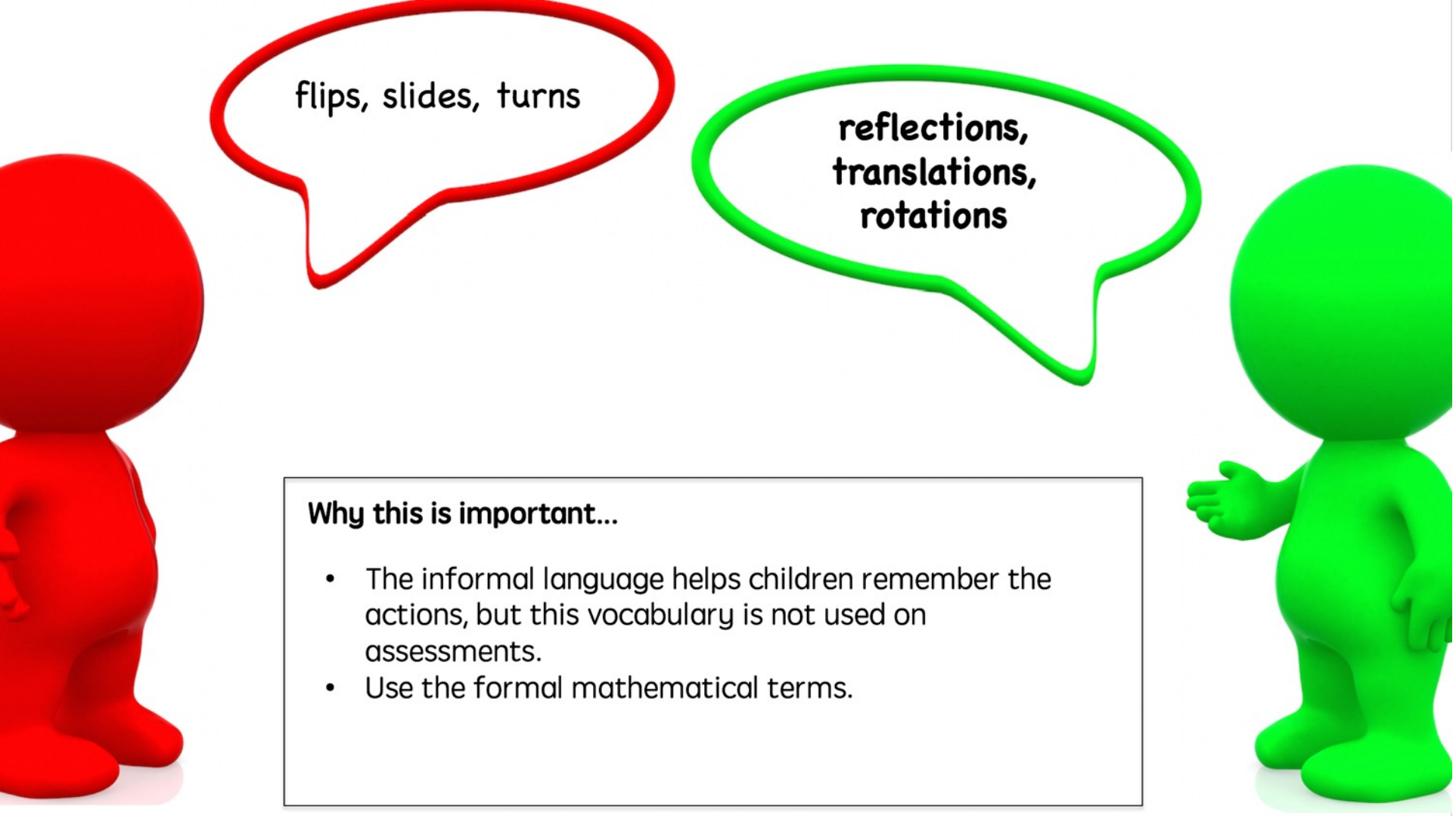
Why this is important...

- Use the formal language of shapes to confirm informal language.



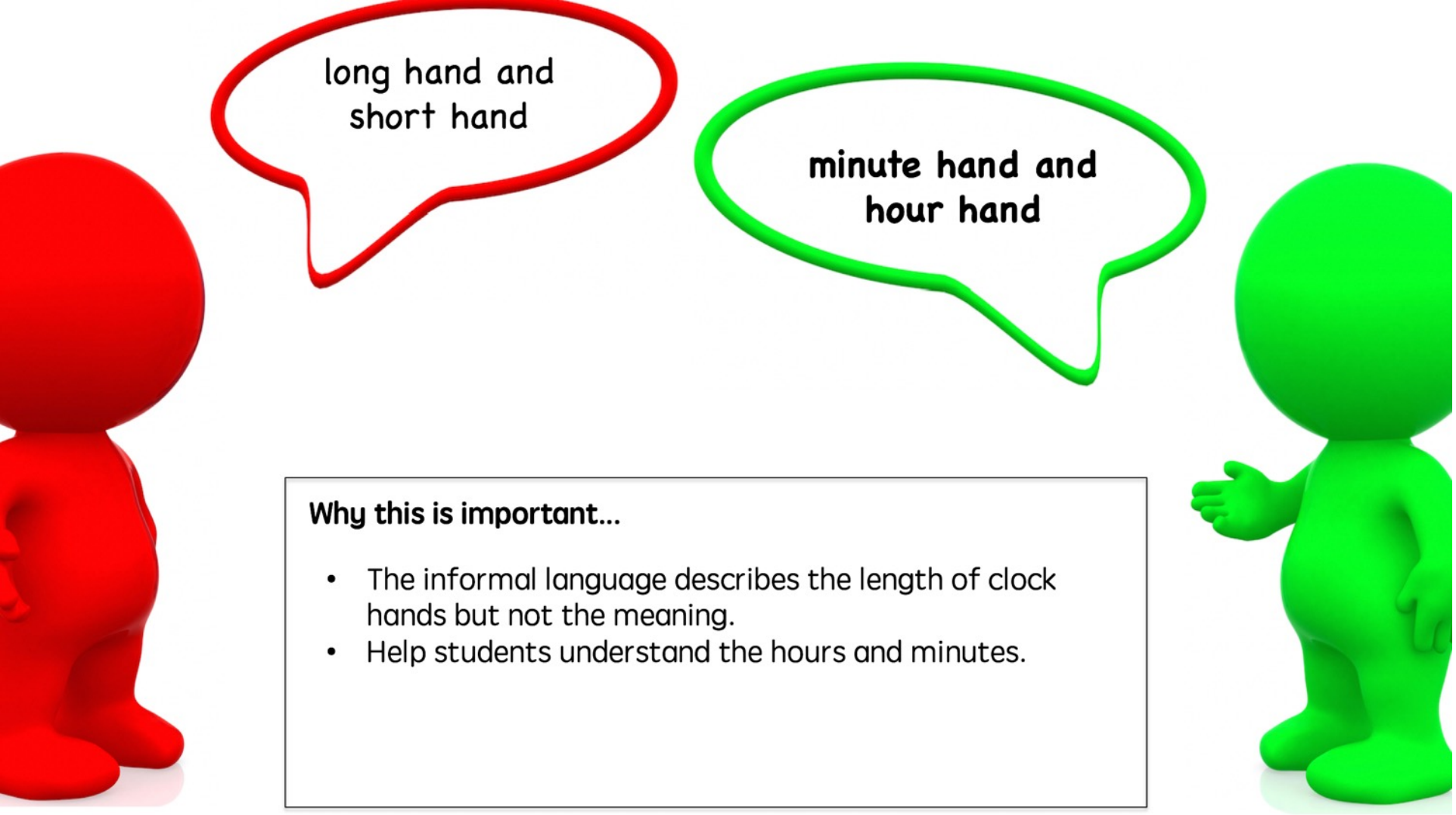
Why this is important…

- This is the endpoint where two or more line segments or rays meet.



Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

Identify examples of “Instead of ____, say ____.”

Use formal math language

Use terms precisely

VOCABULARY

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

MATH

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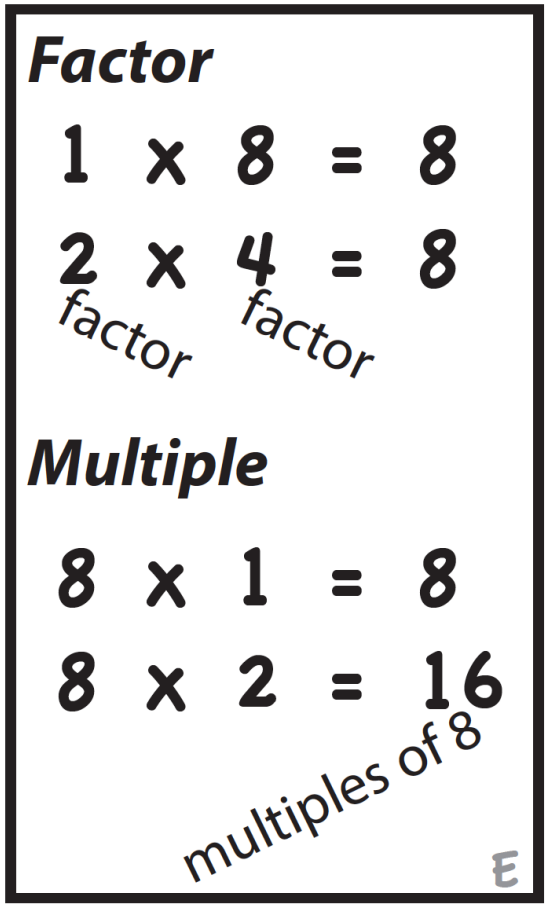

Factor
1 x 8 = 8
2 x 4 = 8
factor
factor
Multiple
8 x 1 = 8
8 x 2 = 16
multiples of 8

Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

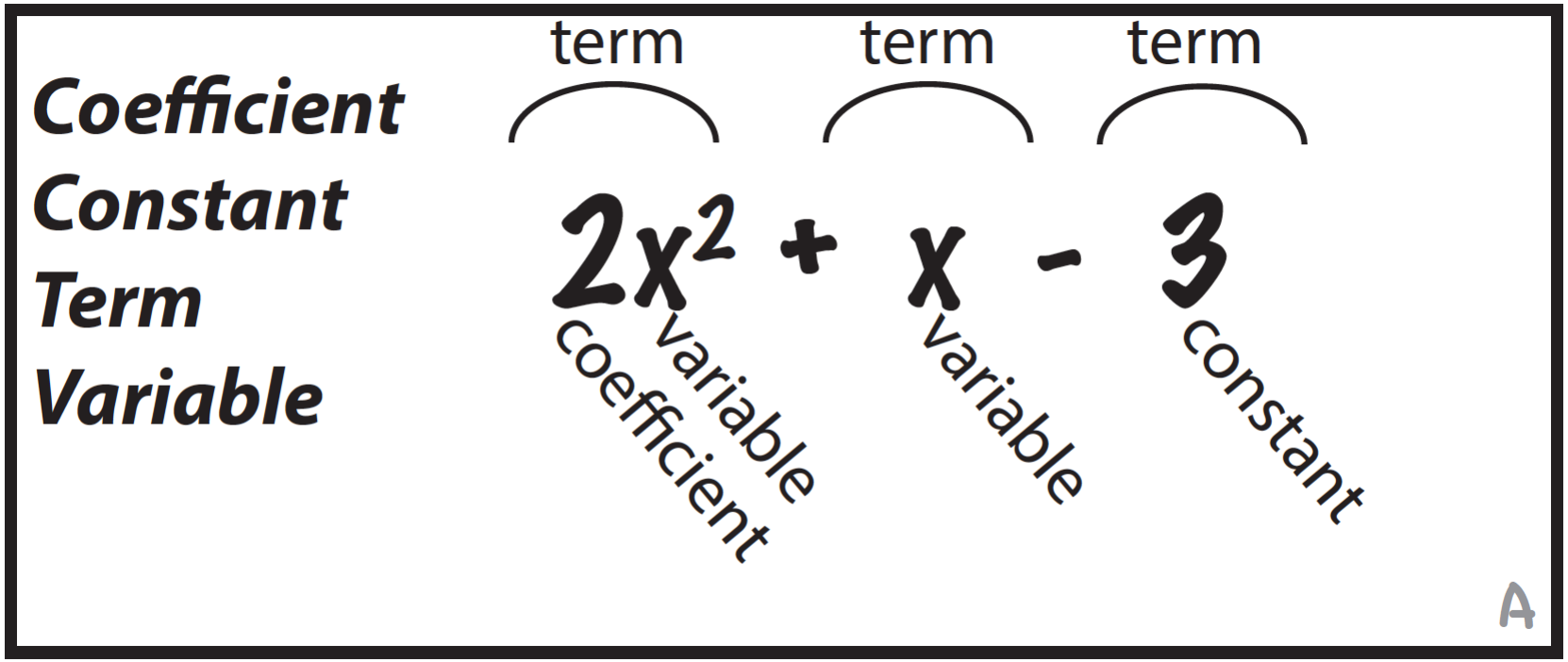
Proper fraction

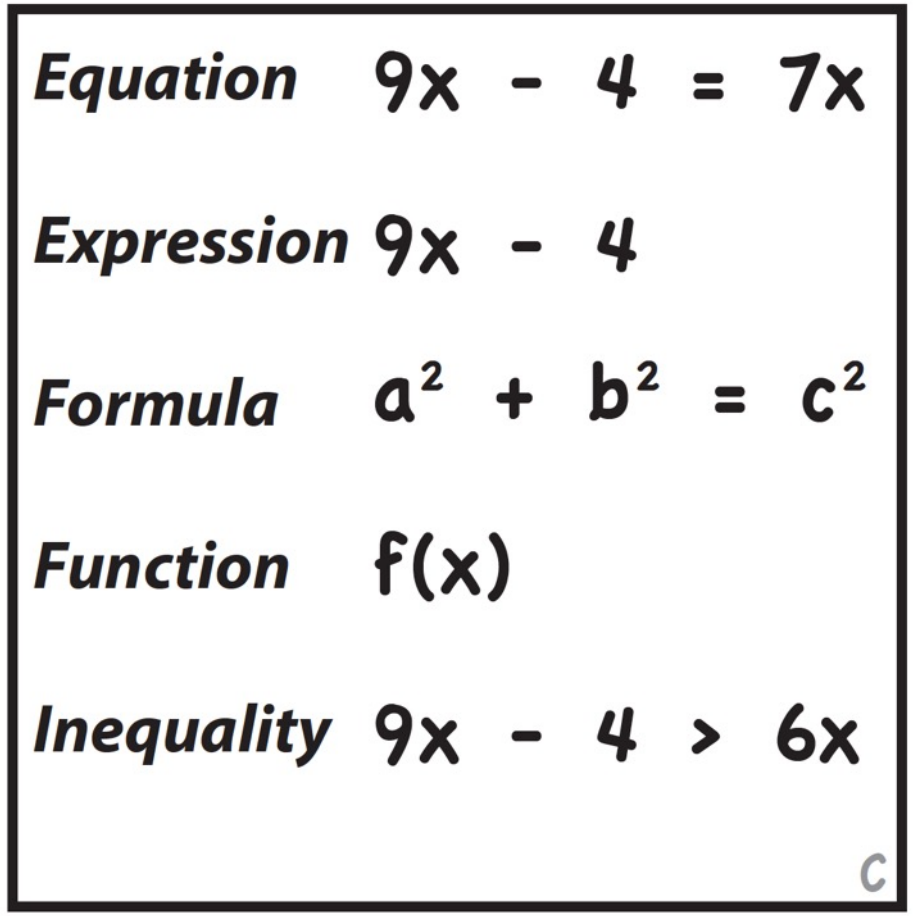
$$\frac{2}{9}$$

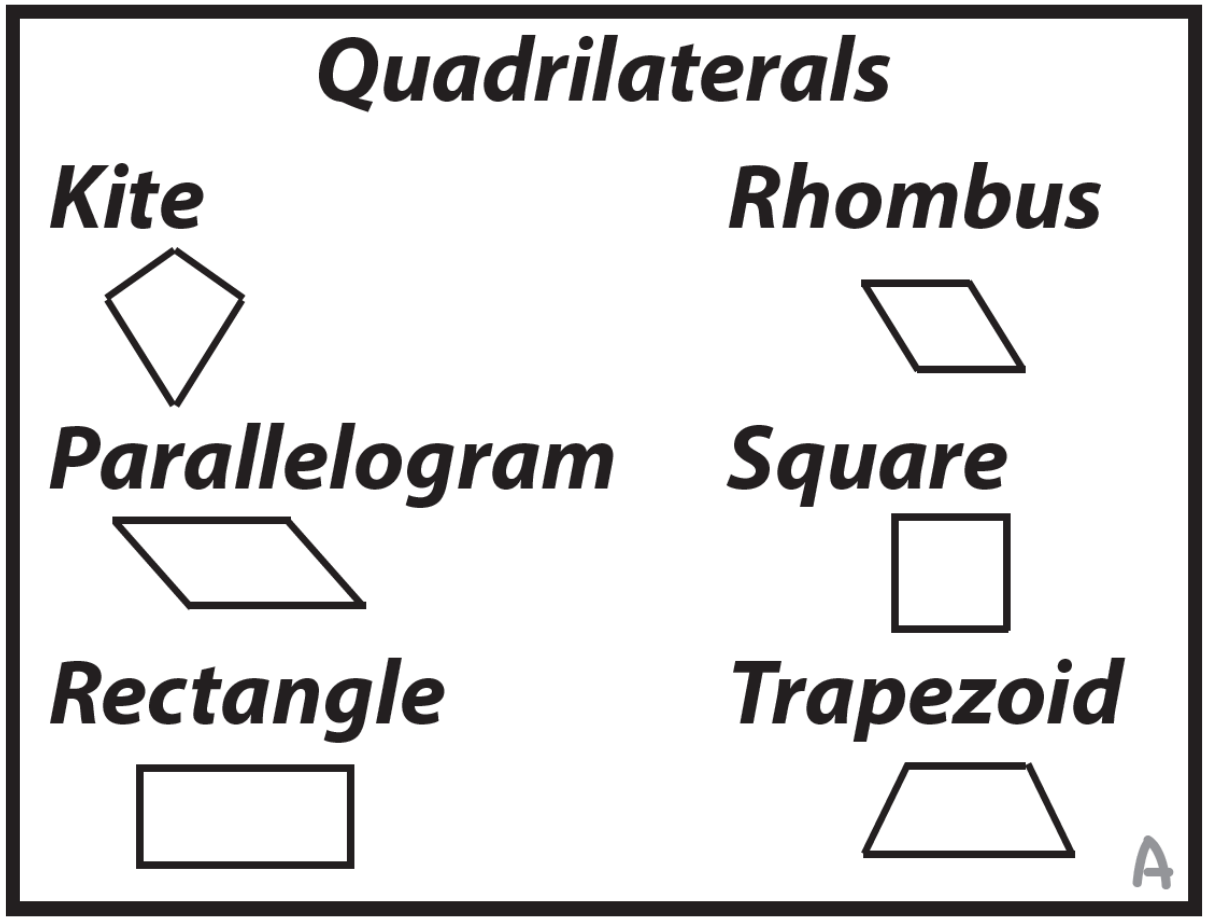
Unit fraction

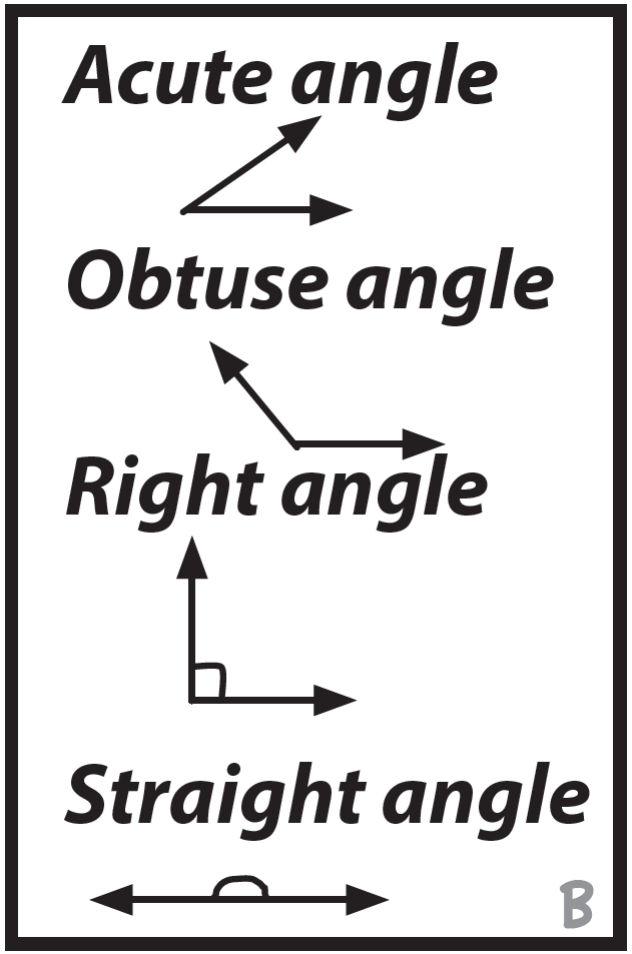
$$\frac{1}{6}$$

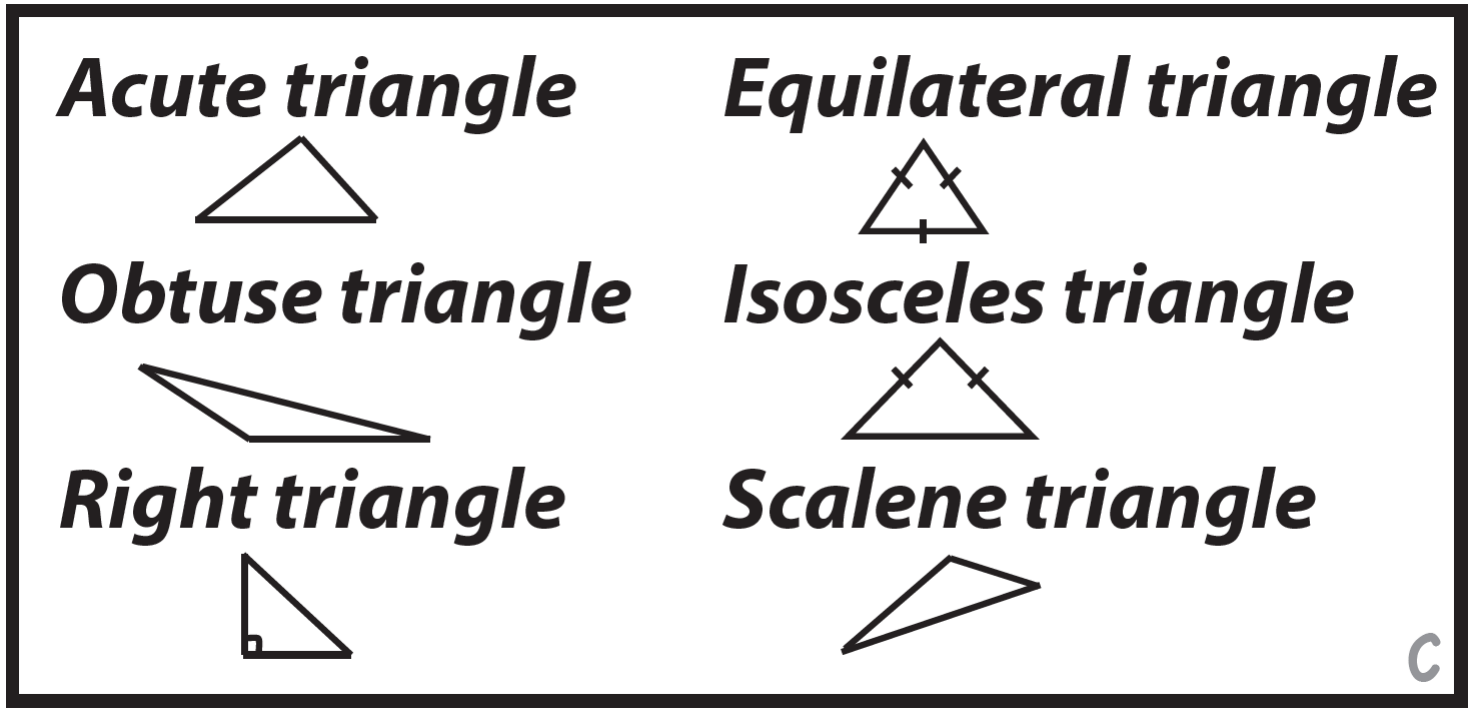
D


Coefficient
Constant
Term
Variable
term
term
term
2x²
+
x
-
3
variable
coefficient
variable
constant
A

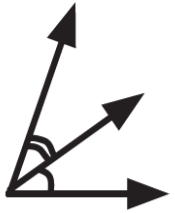

Equation 9x - 4 = 7x
Expression 9x - 4
Formula $a^2 + b^2 = c^2$
Function f(x)
Inequality 9x - 4 > 6x
C


Quadrilaterals
Kite
Rhombus
Parallelogram
Square
Rectangle
Trapezoid
A

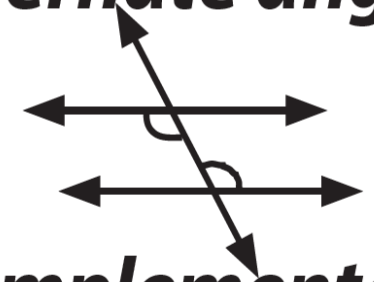

Acute angle
Obtuse angle
Right angle
Straight angle
B


Acute triangle
Equilateral triangle
Obtuse triangle
Isosceles triangle
Right triangle
Scalene triangle
C

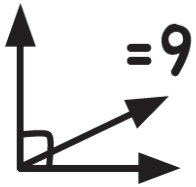
Adjacent angles



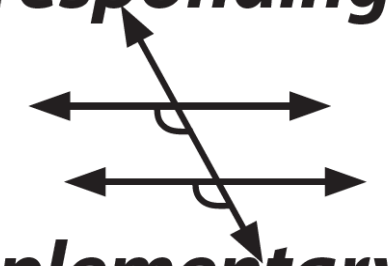
Alternate angles



Complementary angles

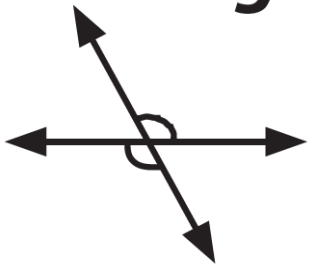


Corresponding angles



Supplementary angles

Vertical angles

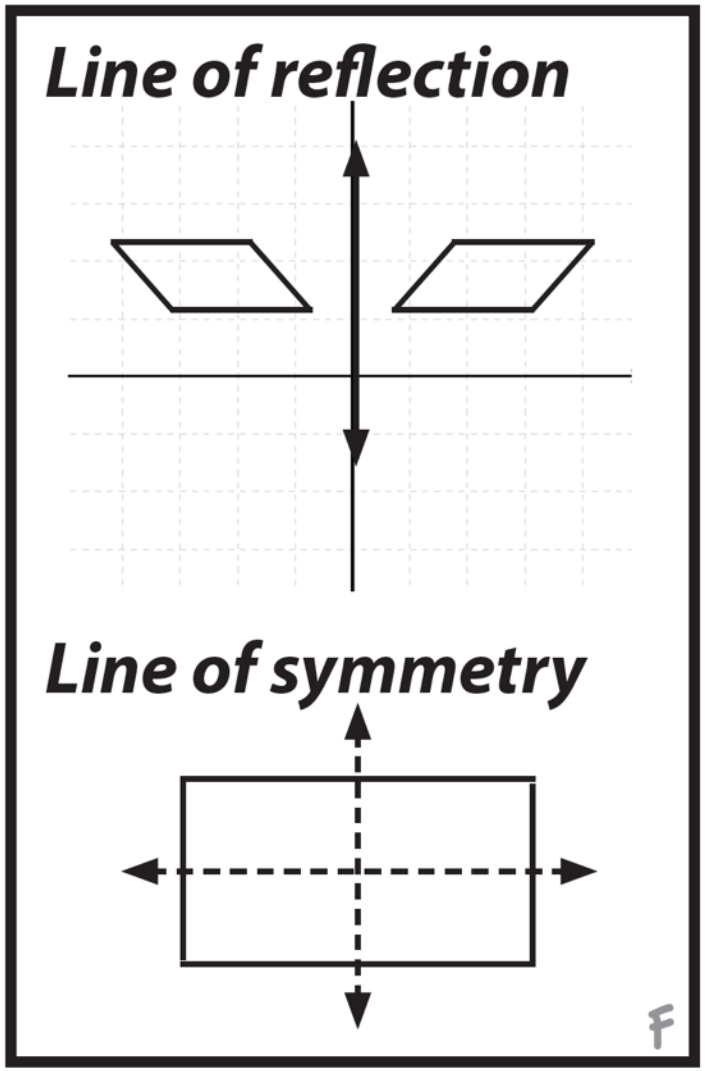


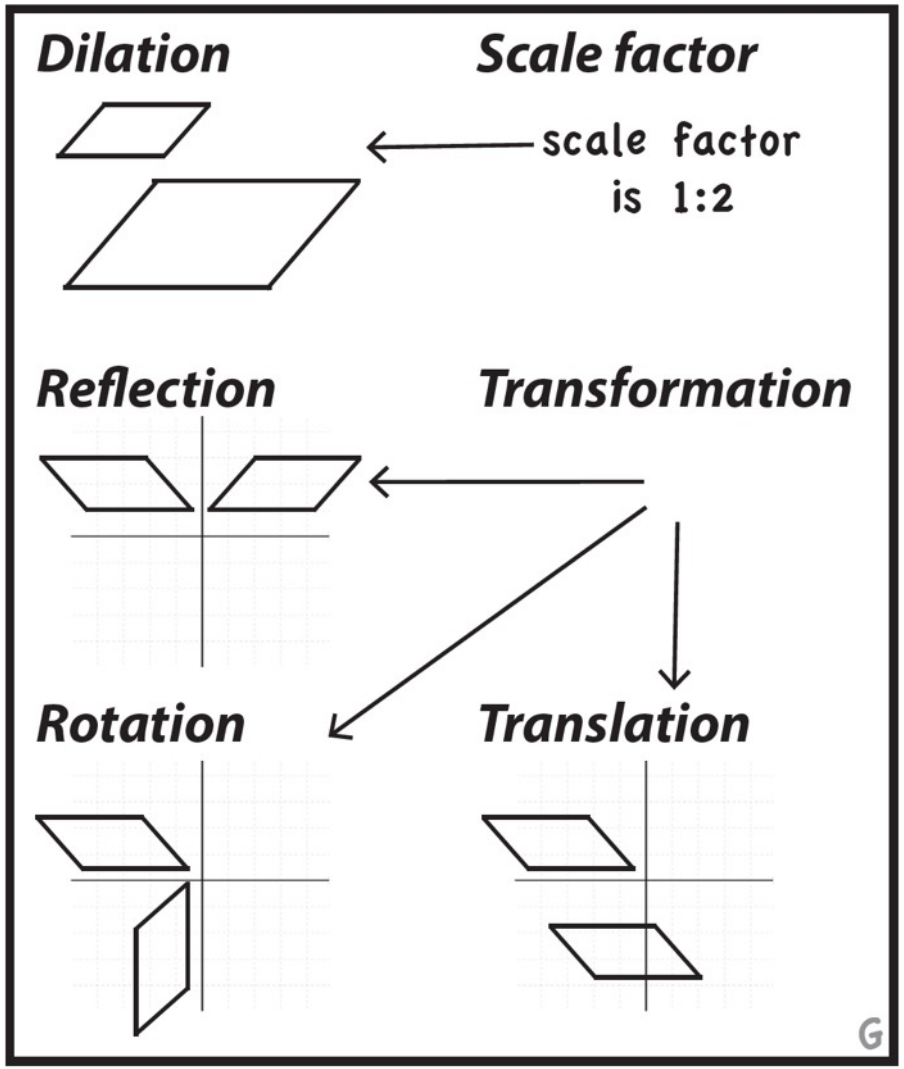
D

Congruent figures

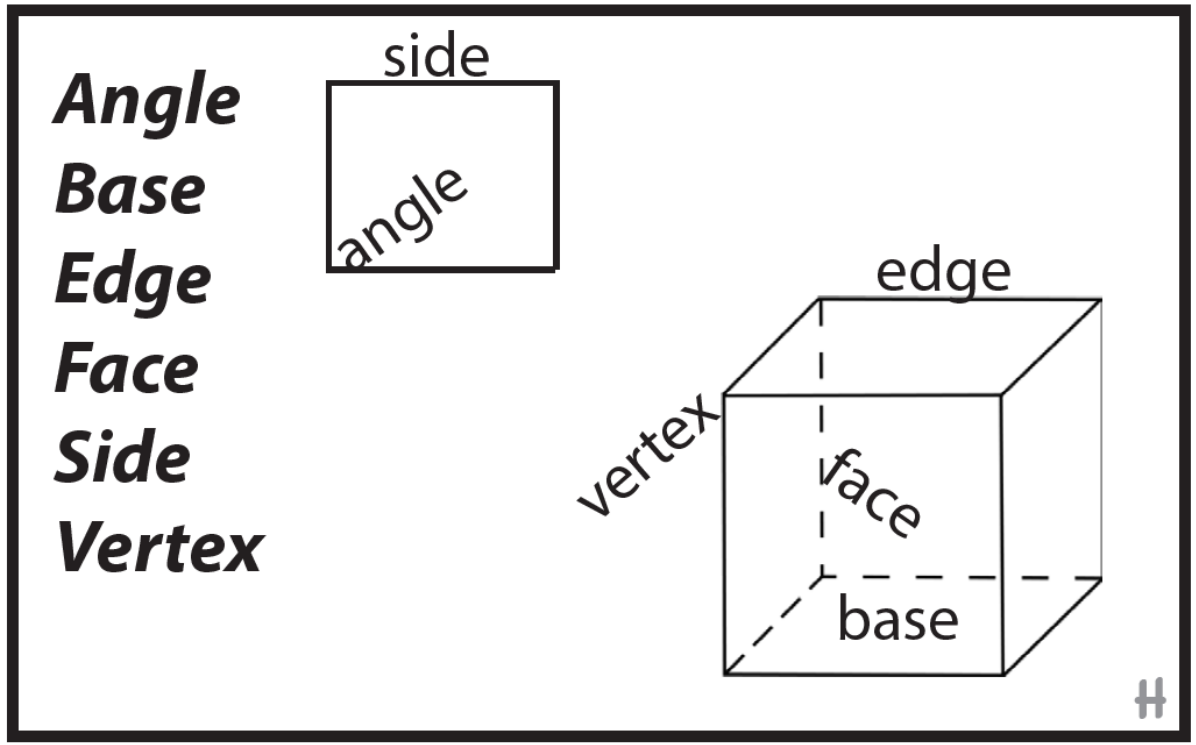
Similar figures

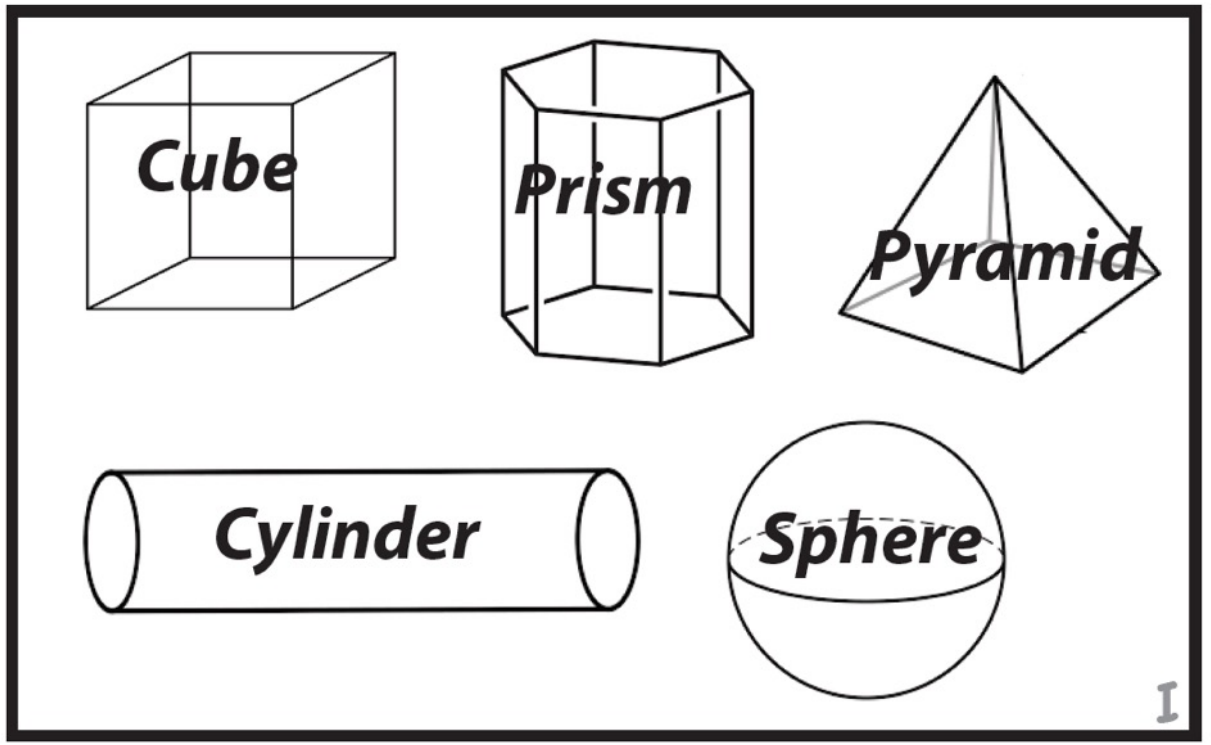
E


Line of reflection
Line of symmetry
F

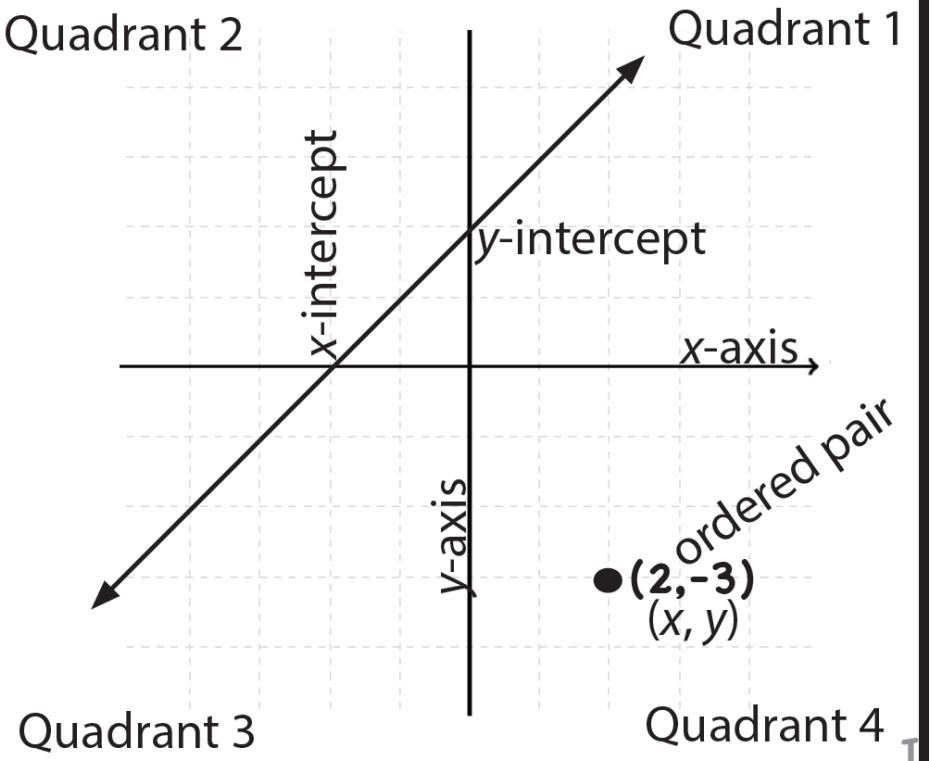

Dilation
Scale factor
scale factor is 1:2
Reflection
Transformation
Rotation
Translation
G

Angle
Base
Edge
Face
Side
Vertex




Cube
Prism
Pyramid
Cylinder
Sphere

Coordinate plane
Ordered pair
Quadrants
x
x-axis
x-intercept
y
y-axis
y-intercept



Discuss terms you want your students to use with precision.

Use formal math language

Use terms precisely

Use semantic maps

Term	Definition	Example	Nonexample
integer	(... -3, -2, -1, 0, 1, 2, 3...)	15	$\frac{1}{3}$
denominator	The equal parts of a whole or set.	$\frac{5}{8}$ 8 is the denominator 	$\frac{5}{8}$ 
numerator	The equal parts of a given fraction.	$\frac{5}{8}$ 5 is the numerator 	$\frac{5}{8}$ 

(Stevens et al., 2022)

Use word walls

difference

The result of subtracting or the result when comparing two numbers.

5 – 4 = 1

1 is the difference

horizontal line

A straight line that goes from left to right or right to left.

equivalent

Two numbers that have the same value.

$\frac{2}{4}$ **is equivalent to** $\frac{1}{2}$

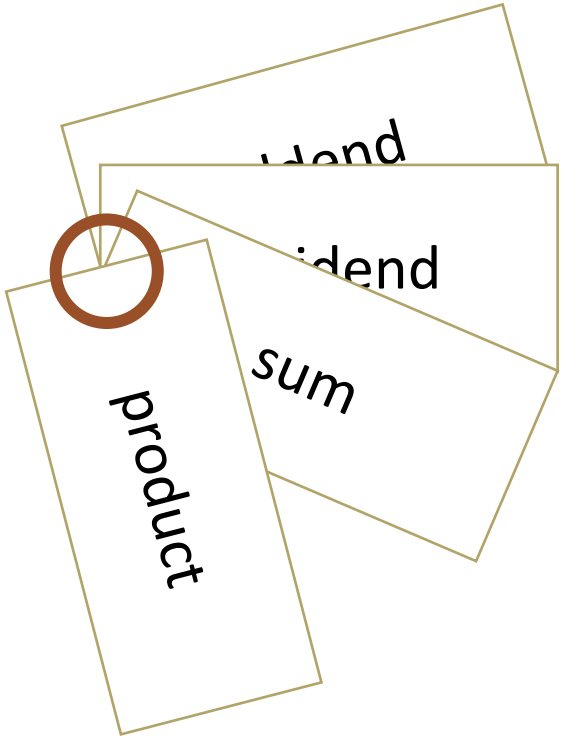
total

The result or sum when adding numbers.

5 + 7 = 12

Use flash cards

addend	quotient
divisor	dividend
factor	sum
product	difference

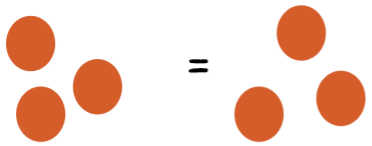


(Petersen-Brown et al., 2019)

Use glossaries

E

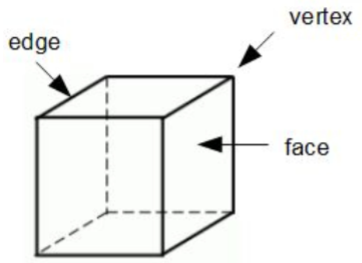
equal - with the same value



equation - expressions with an equal sign

$$5 = 2 + 3$$

edge - line segment between faces



Glosario de Matemáticas

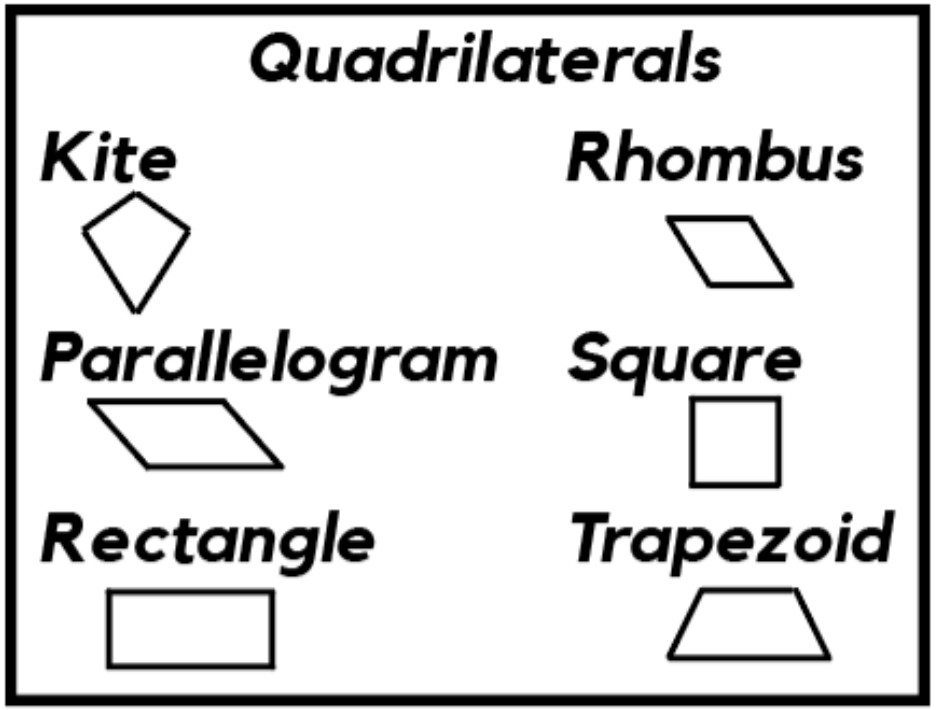
Término	Definición
adición *(add)*	Juntar o agregar.
algoritmo *(algorithm)*	Es una serie de pasos organizados que describe el proceso que se debe seguir, para dar solución a un problema específico.
alinear *(array)*	Un conjunto de objetos, imágenes o números alineados en columnas y filas.
ancho *(width)*	La medida de un lado de un objeto, generalmente el lado más corto.
ángulo *(angle)*	Dos semirrectas o segmentos de línea recta que comparten un punto final.
ángulo agudo *(acute angle)*	Un ángulo que mide menos de 90°.
ángulo obtuso *(obtuse angle)*	Un ángulo que mide entre 90° y 180°.
ángulo recto *(right angle)*	Un ángulo que mide exactamente 90°.
área *(area)*	La cantidad de unidades cuadradas que cubre una figura geométrica cerrada.
balance presupuestario *(balance the budget)*	Un presupuesto es cuando la cantidad total de dinero gastado, ahorrado y compartido es igual al ingreso total.
báscula *(balance scale)*	Instrumento de medición que se utiliza para medir el peso o la carga.
base (computación) *(base (computation))*	Un número que se multiplica por un exponente.
base (geometría) *(base (geometry))*	La forma inferior de una figura tridimensional.

SPIRAL
Specialized Math Intervention to Reach All Learners

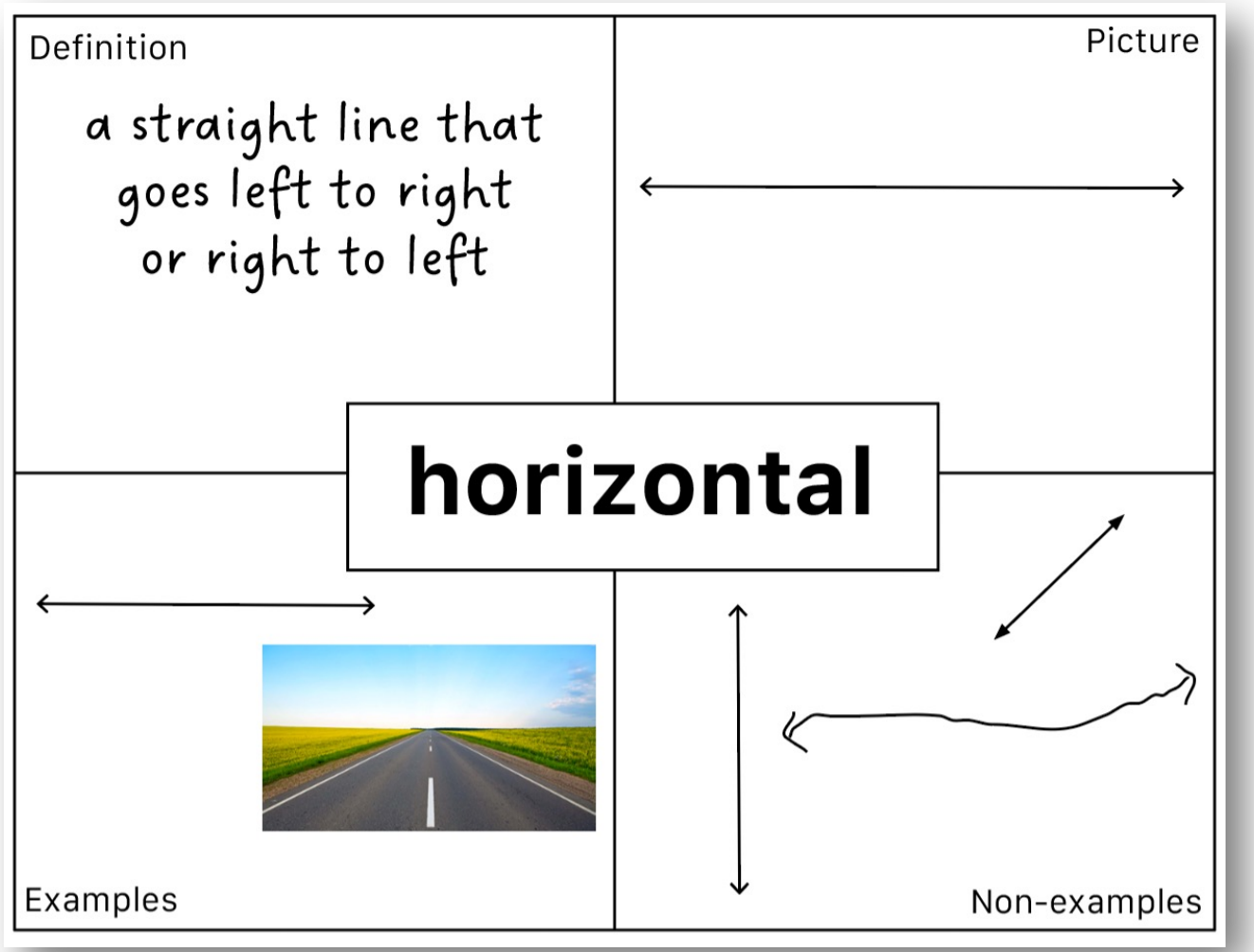
Use anchor charts

Addition Computation

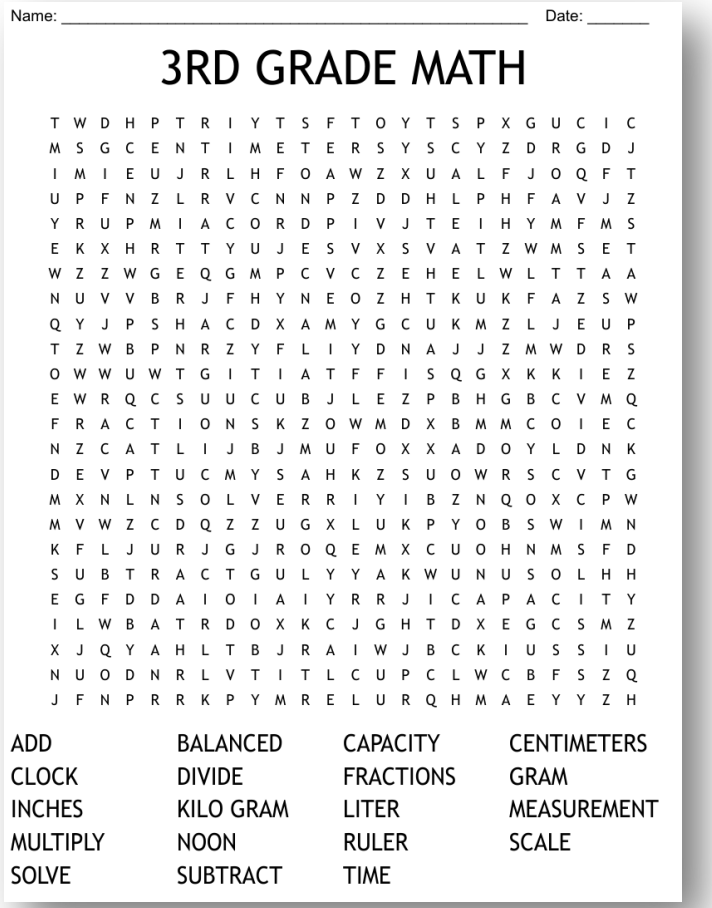
```
  1
 17  ← addend
+59  ← addend
---
 76  ← sum
```



Use graphic organizers



Use games



Name: _____ Date: _____

3RD GRADE MATH

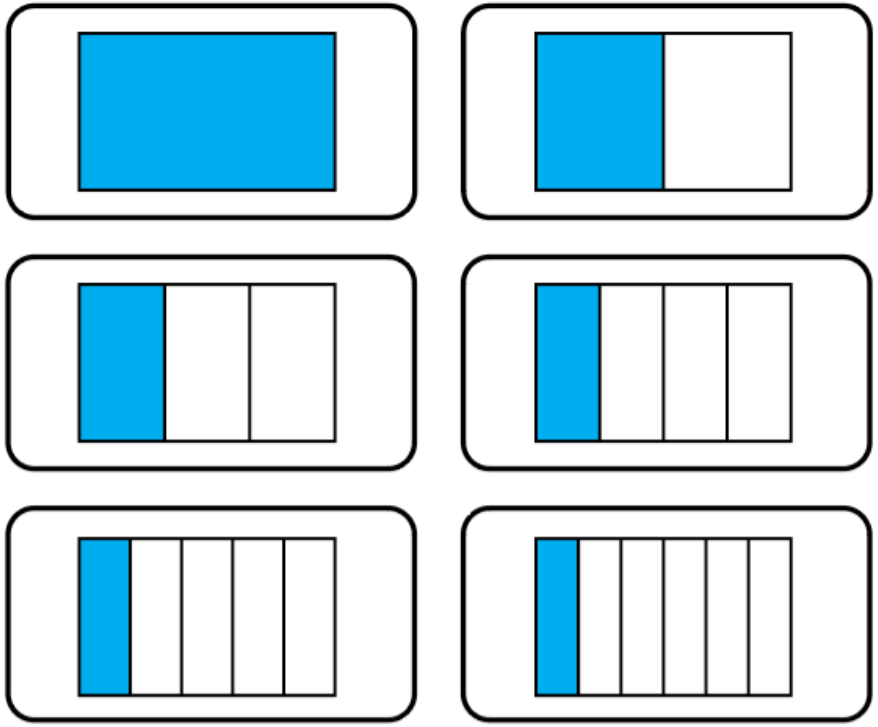
T W D H P T R I Y T S F T O Y T S P X G U C I C
M S G C E N T I M E T E R S Y S C Y Z D R G D J
I M I E U J R L H F O A W Z X U A L F J O Q F T
U P F N Z L R V C N N P Z D D H L P H F A V J Z
Y R U P M I A C O R D P I V J T E I H Y M F M S
E K X H R T T Y U J E S V X S V A T Z W M S E T
W Z Z W G E Q G M P C V C Z E H E L W L T T A A
N U V V B R J F H Y N E O Z H T K U K F A Z S W
Q Y J P S H A C D X A M Y G C U K M Z L J E U P
T Z W B P N R Z Y F L I Y D N A J J Z M W D R S
O W W U W T G I T I A T F F I S Q G X K K I E Z
E W R Q C S U U C U B J L E Z P B H G B C V M Q
F R A C T I O N S K Z O W M D X B M M C O I E C
N Z C A T L I J B J M U F O X X A D O Y L D N K
D E V P T U C M Y S A H K Z S U O W R S C V T G
M X N L N S O L V E R R I Y I B Z N Q O X C P W
M V W Z C D Q Z Z U G X L U K P Y O B S W I M N
K F L J U R J G J R O Q E M X C U O H N M S F D
S U B T R A C T G U L Y Y A K W U N U S O L H H
E G F D D A I O I A I Y R R J I C A P A C I T Y
I L W B A T R D O X K C J G H T D X E G C S M Z
X J Q Y A H L T B J R A I W J B C K I U S S I U
N U O D N R L V T I T L C U P C L W C B F S Z Q
J F N P R R K P Y M R E L U R Q H M A E Y Y Z H

ADD
BALANCED
CAPACITY
CENTIMETERS
CLOCK
DIVIDE
FRACTIONS
GRAM
INCHES
KILO GRAM
LITER
MEASUREMENT
MULTIPLY
NOON
RULER
SCALE
SOLVE
SUBTRACT
TIME

square	**circle**	**decagon**	**triangular prism**	**cone**
cylinder	**cube**	**sphere**	**line**	**pyramid**
parallelogram	**octagon**	**FREE**	**trapezoid**	**oval**
kite	**pentagon**	**cylinder**	**rectangle**	**line segment**
hexagon	**rhombus**	**triangle**	**quadrilateral**	**rectangular prism**

https://wordmint.com/public_puzzles/13055

Use games



one

one-half

one-third

one-fourth

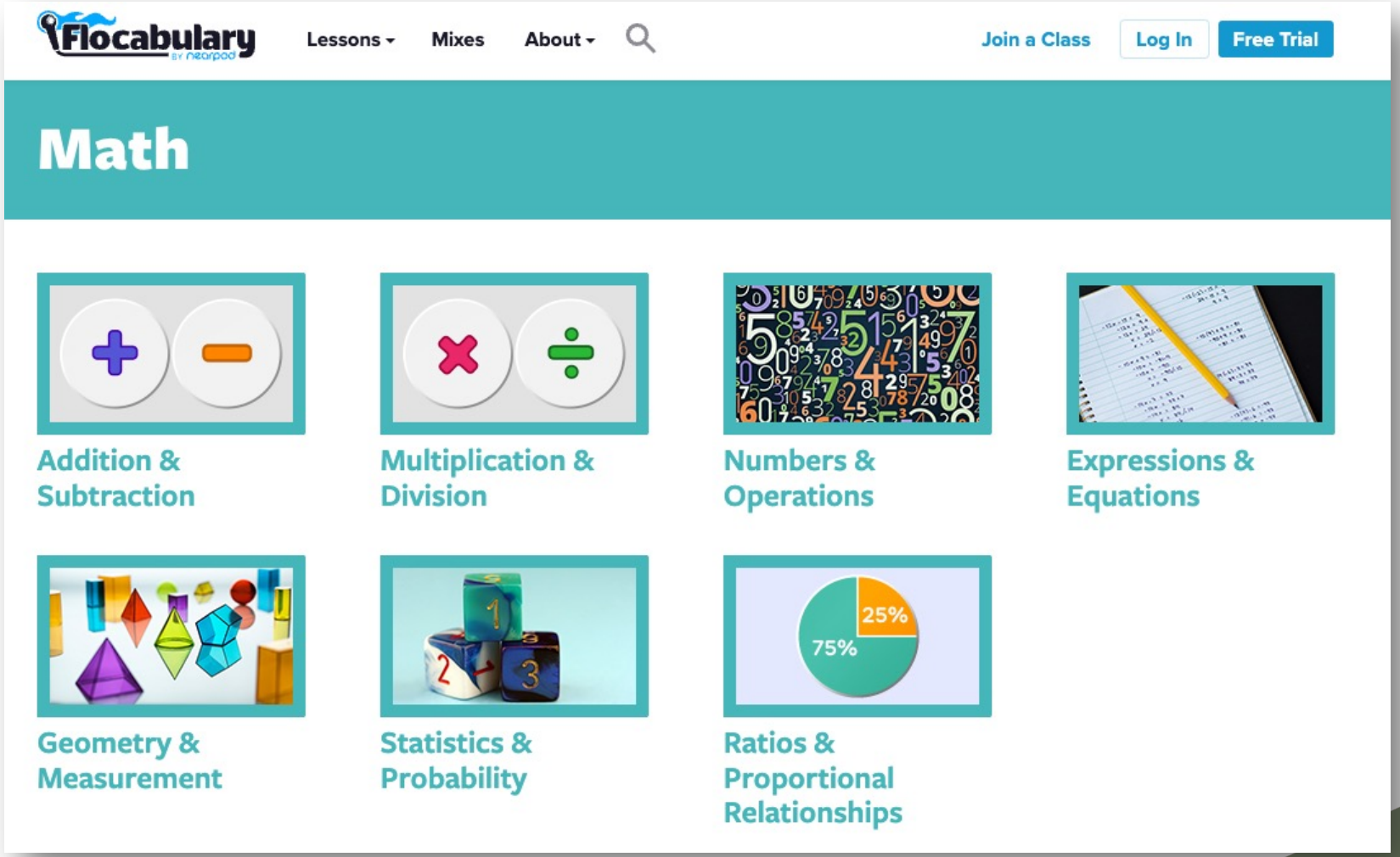
one-fifth

one-sixth

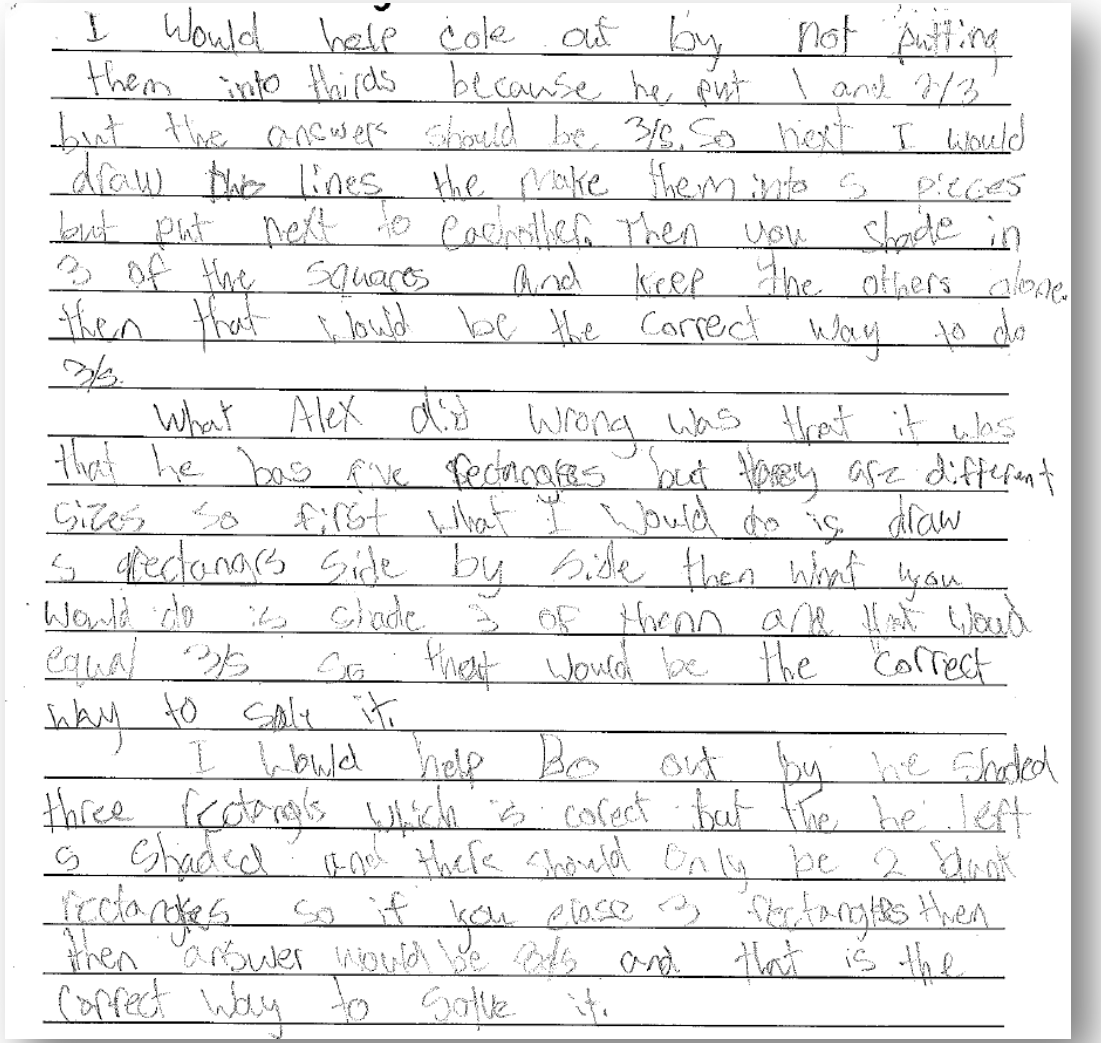
Use discussion

add
change
compare
decrease
difference
increase
part
put together
subtract
total

Use technology



Use math writing

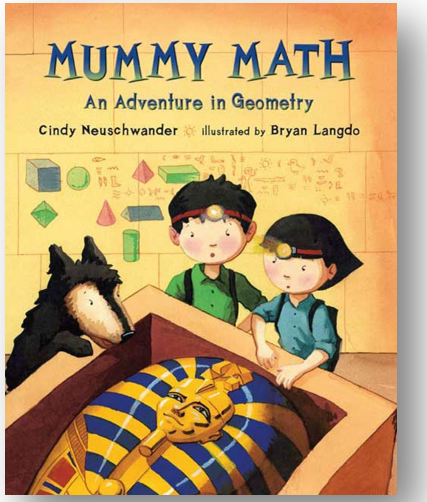


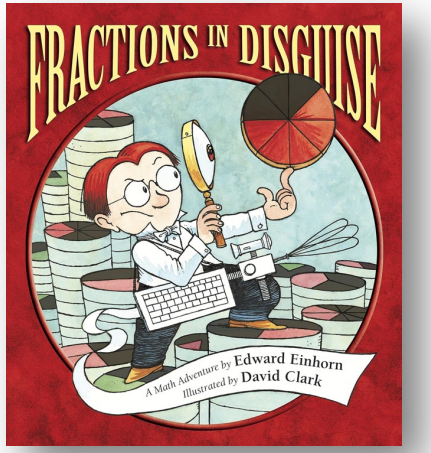
I would help Cole out by not putting them into thirds because he put 1 and 2/3 but the answers should be 3/5. So next I would draw the lines the make them into 5 pieces but put next to eachother. Then you shade in 3 of the squares And keep the others alone. then that would be the correct way to do 3/5.

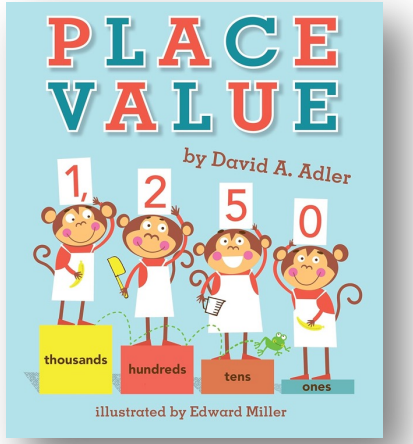
What Alex did wrong was that it was that he has five rectangles but they are different sizes so first what I would do is draw 5 rectangles side by side then what you would do is shade 3 of them and that would equal 3/5 so that would be the correct way to solve it.

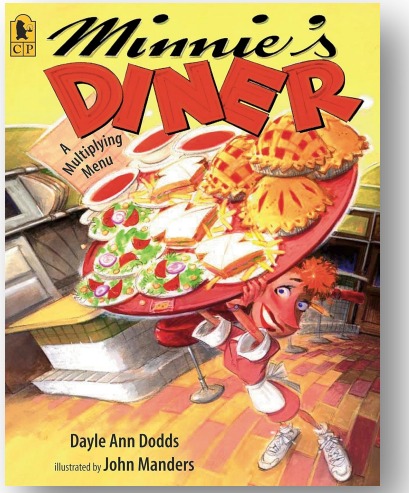
I would help Bo out by he shaded three rectangls which is correct but the he left 5 shaded and there should only be 2 blank rectangles so if you count 5 rectangles then then answer would be 3/5 and that is the correct way to solve it.

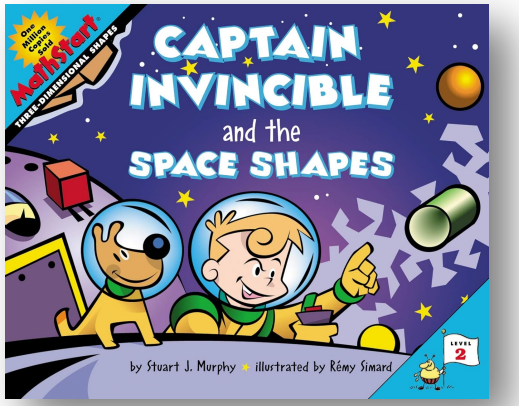
Use read-alouds

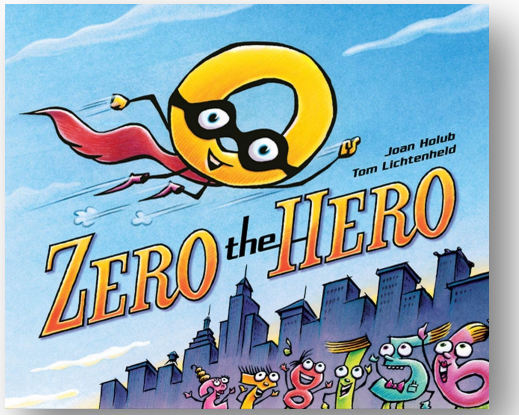












(Purpura et al., 2017)

Model and practice

Teacher	**Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?**
Students	Put together.
Teacher	**When we combine, we put things together. When you cook, you put ingredients together. For example, to make macaroni and cheese, you combine what?**
Students	Macaroni noodles and cheese!
Teacher	**That's right. You combine macaroni and cheese! Now, let's think about combining numbers. Look at this problem.** (Show problem.)

(Powell & Driver, 2015; Stevens et al., 2022)

Make sure students use mathematics vocabulary as much as adults do!

What are your strengths with vocabulary?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

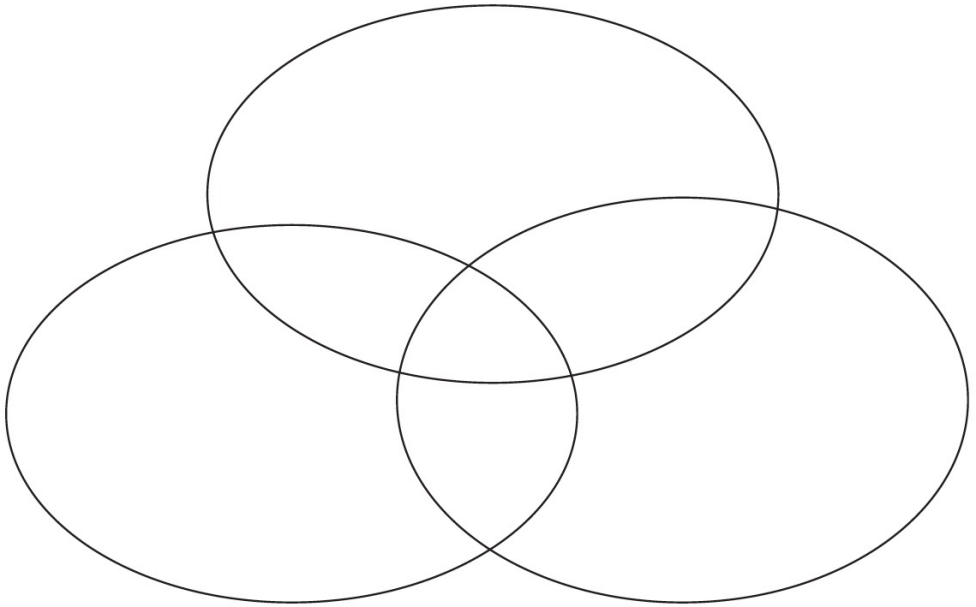
Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Representations

REPRESENTATIONS

Research and Information



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MATH

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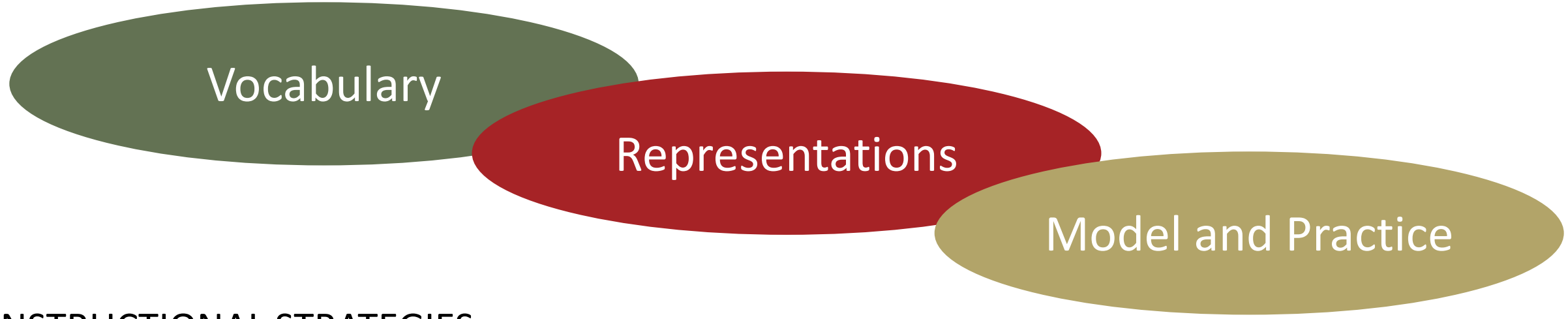
Instructional Platform

INSTRUCTIONAL DELIVERY

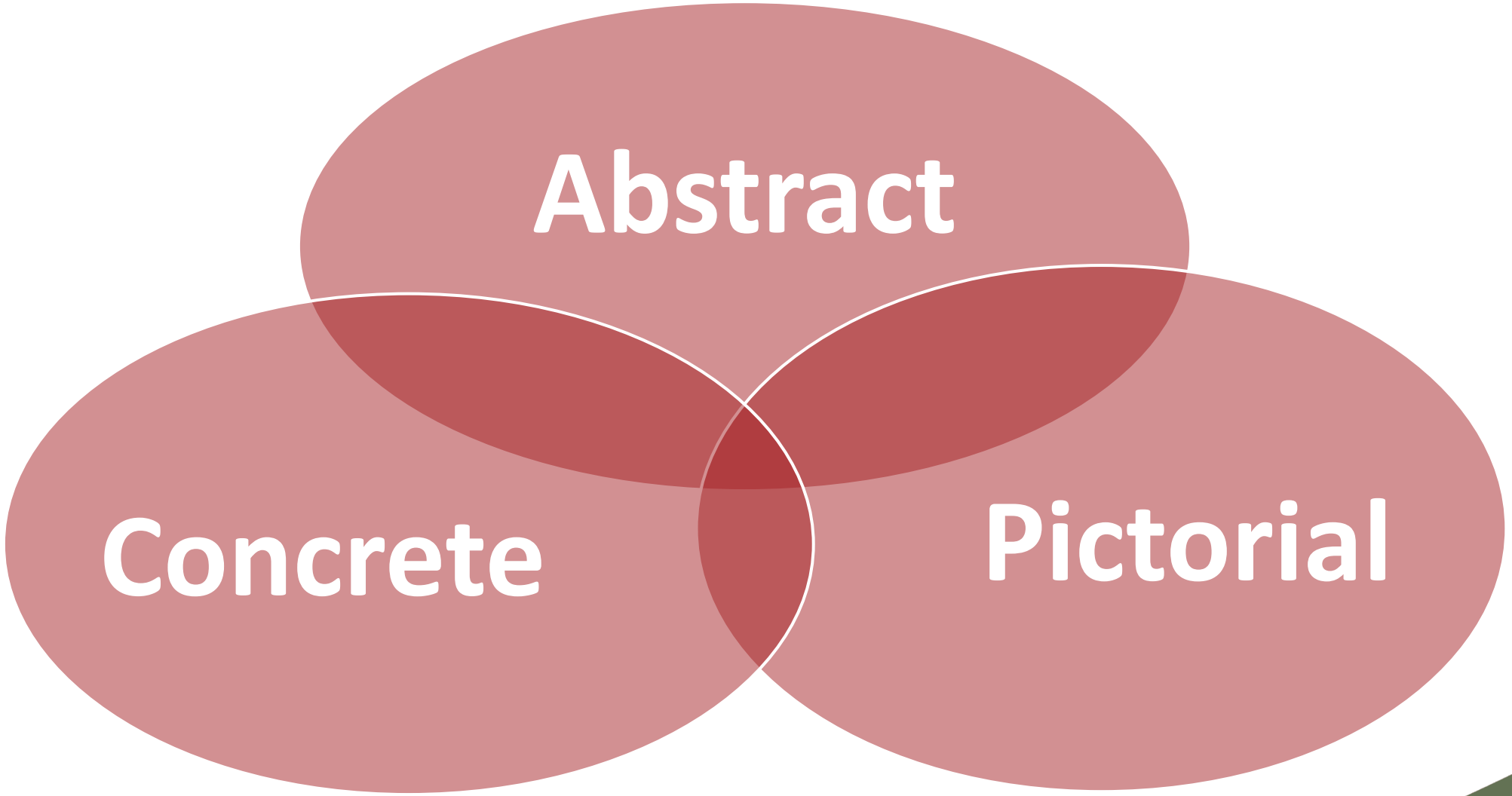


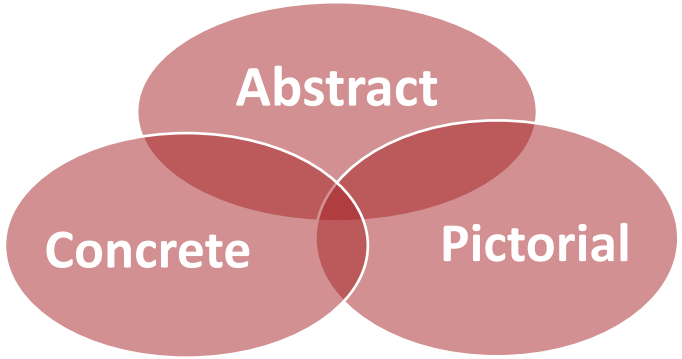
INSTRUCTIONAL STRATEGIES

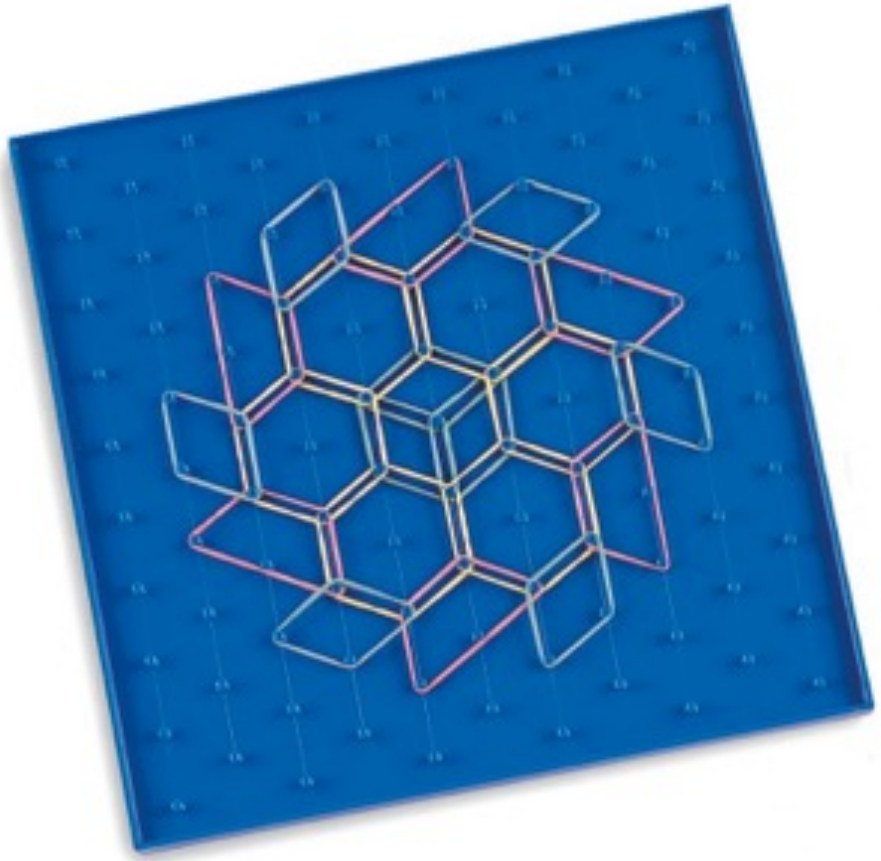
Hands-on manipulatives contribute to increases in mathematics performance.
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

Virtual manipulatives contribute to increases in mathematics performance.
(Bouck et al., 2020; Satsangi et al., 2016)

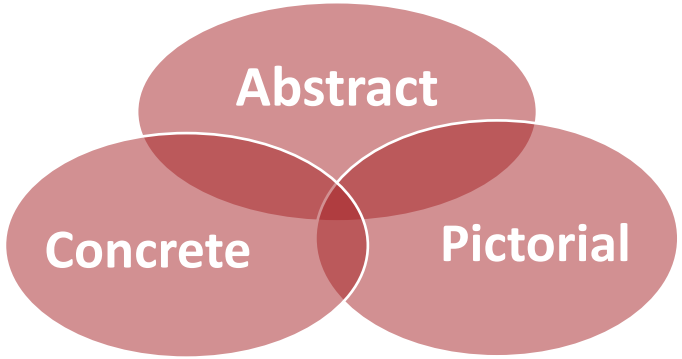
Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

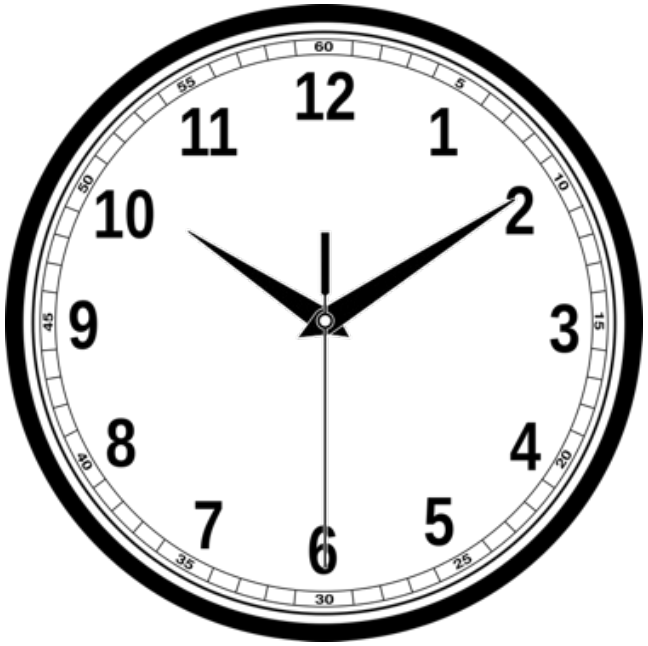

Abstract
Concrete
Pictorial

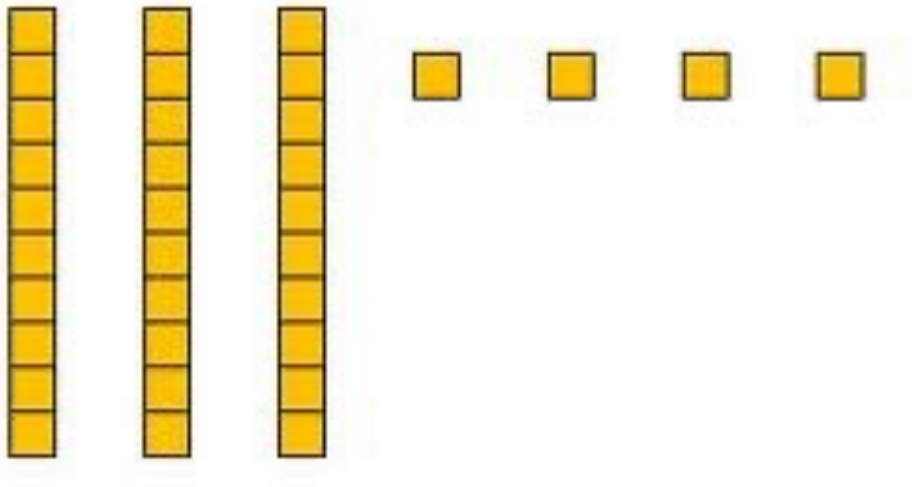

Abstract
Concrete
Pictorial

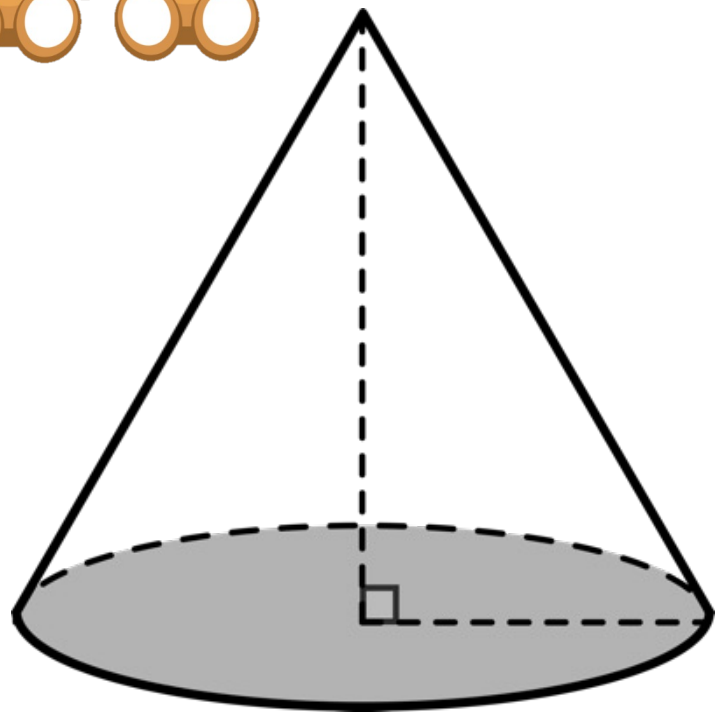


Describe your favorite hands-on materials.









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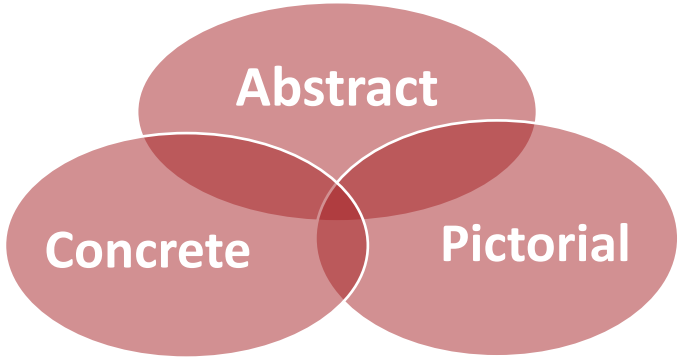
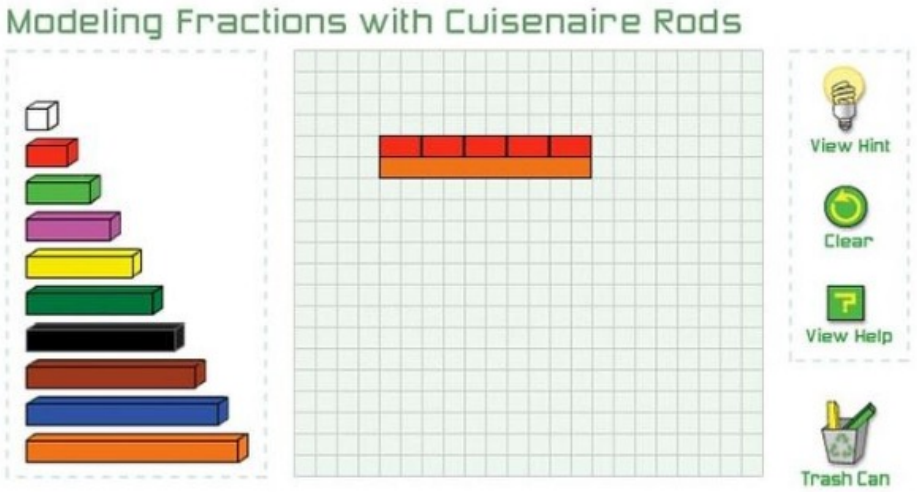
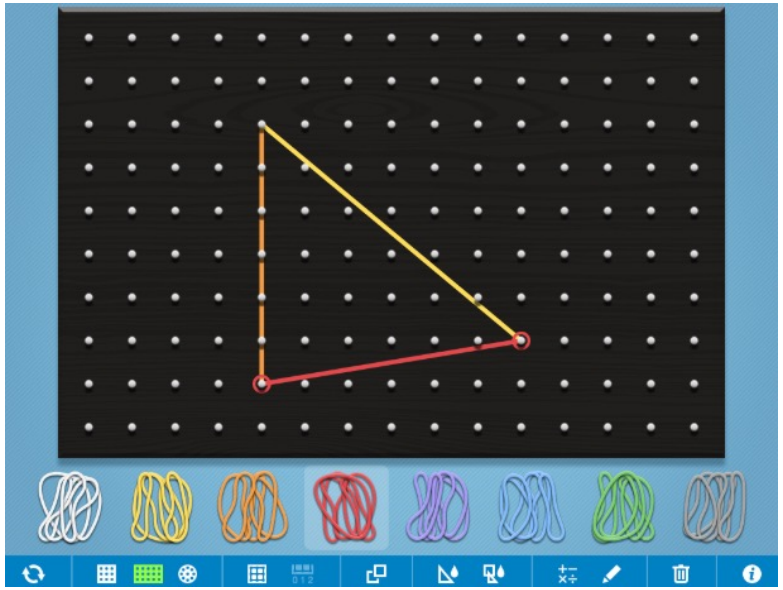
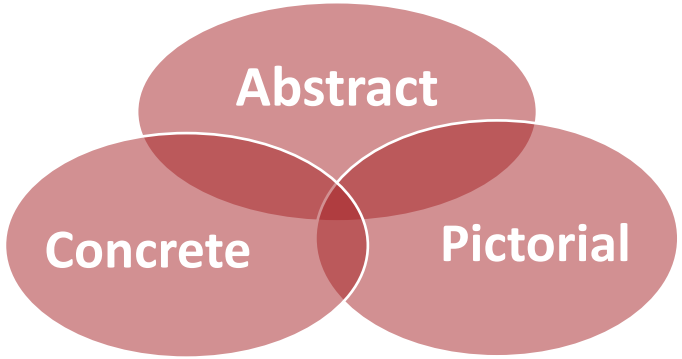

Abstract
Concrete
Pictorial


Modeling Fractions with Cuisenaire Rods
View Hint
Clear
View Help
Trash Can




Abstract
Concrete
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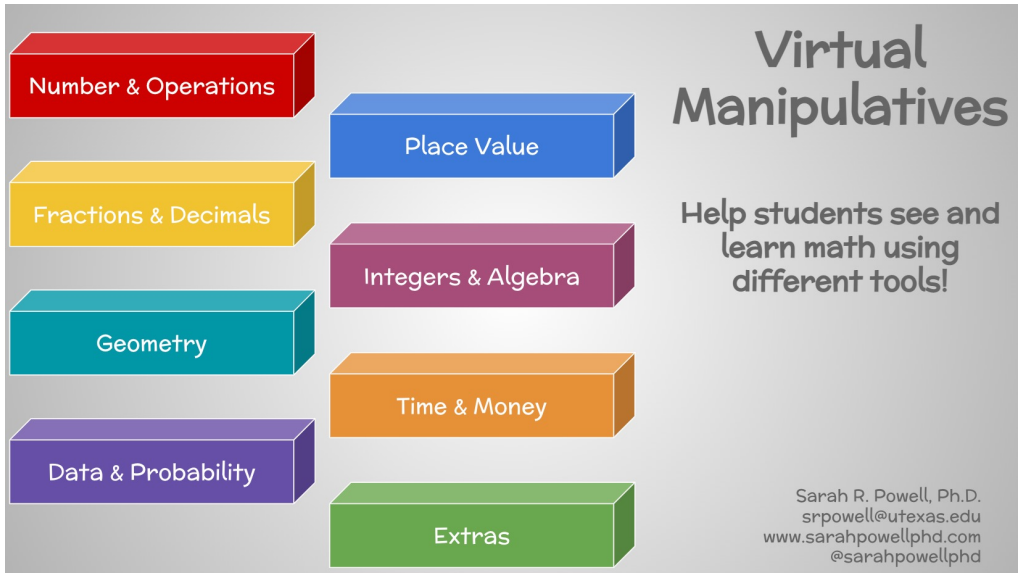
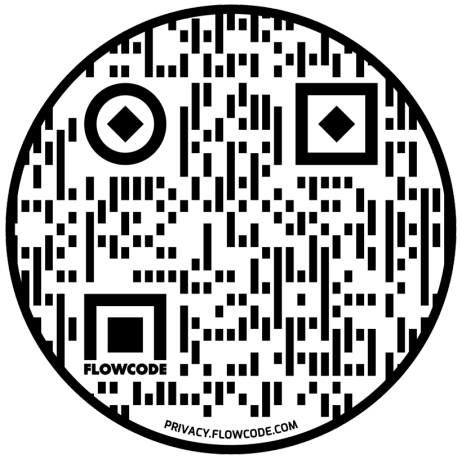

Number & Operations
Place Value
Fractions & Decimals
Integers & Algebra
Geometry
Time & Money
Data & Probability
Extras
Virtual Manipulatives
Help students see and learn math using different tools!
Sarah R. Powell, Ph.D.
srpowell@utexas.edu
www.sarahpowellphd.com
@sarahpowellphd


FLOWCODE
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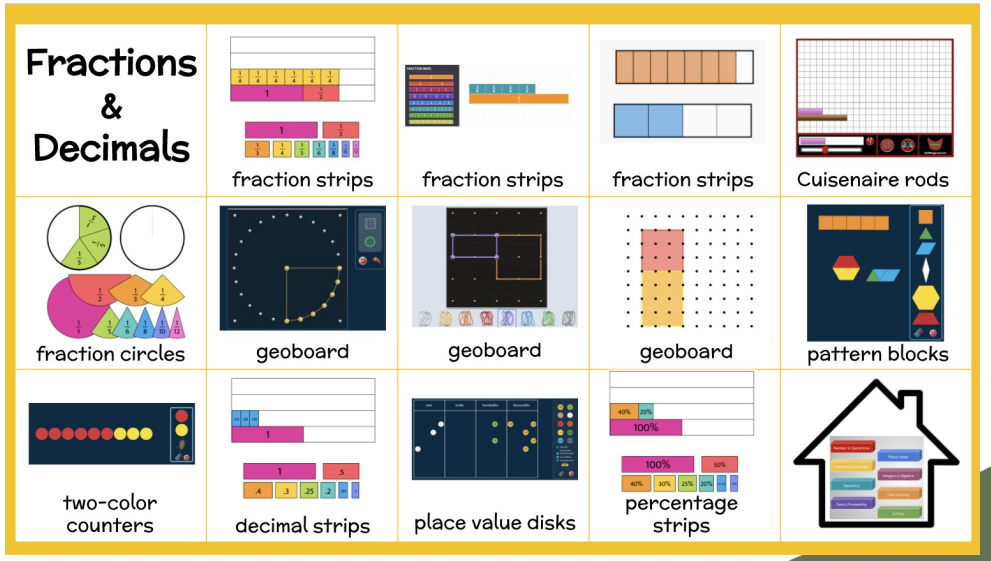
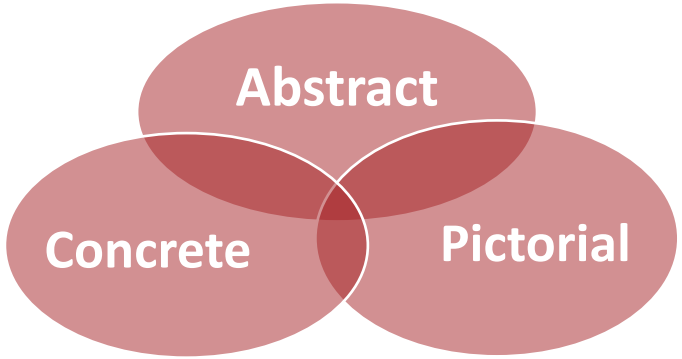

Fractions & Decimals
fraction strips
fraction strips
fraction strips
Cuisenaire rods
fraction circles
geoboard
geoboard
geoboard
pattern blocks
two-color counters
decimal strips
place value disks
percentage strips

#CEC2024

Describe your favorite virtual manipulatives.



2 + 8 = 10

34 = 3 tens and 4 ones

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

$$\begin{array}{r} 4{,}179 \\ +\ \ 569 \\ \hline \end{array}$$

REPRESENTATIONS

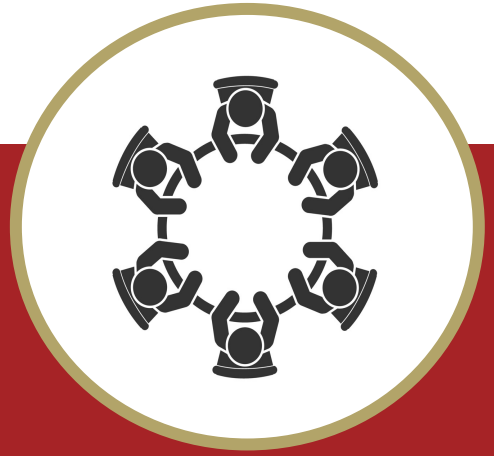
Representations

Item	Representations

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?



What are your strengths with representations?

What are your opportunities for growth?

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Next month?
Next year?

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Model and Practice

MODEL AND PRACTICE

Research and Information

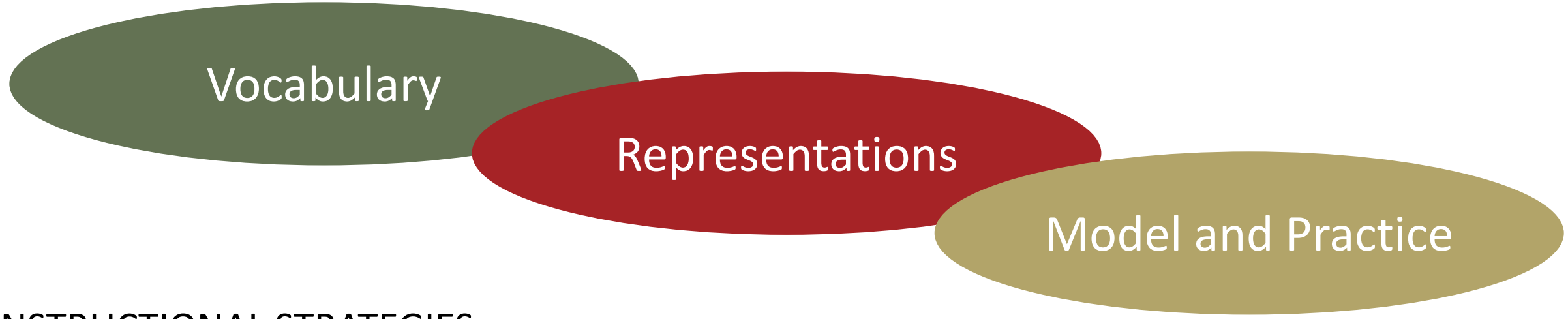
MODELING

PRACTICE

SUPPORTS

Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

Over a half century of research supports explicit (i.e., direct, systematic) instruction.
(Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.
(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics.
(Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

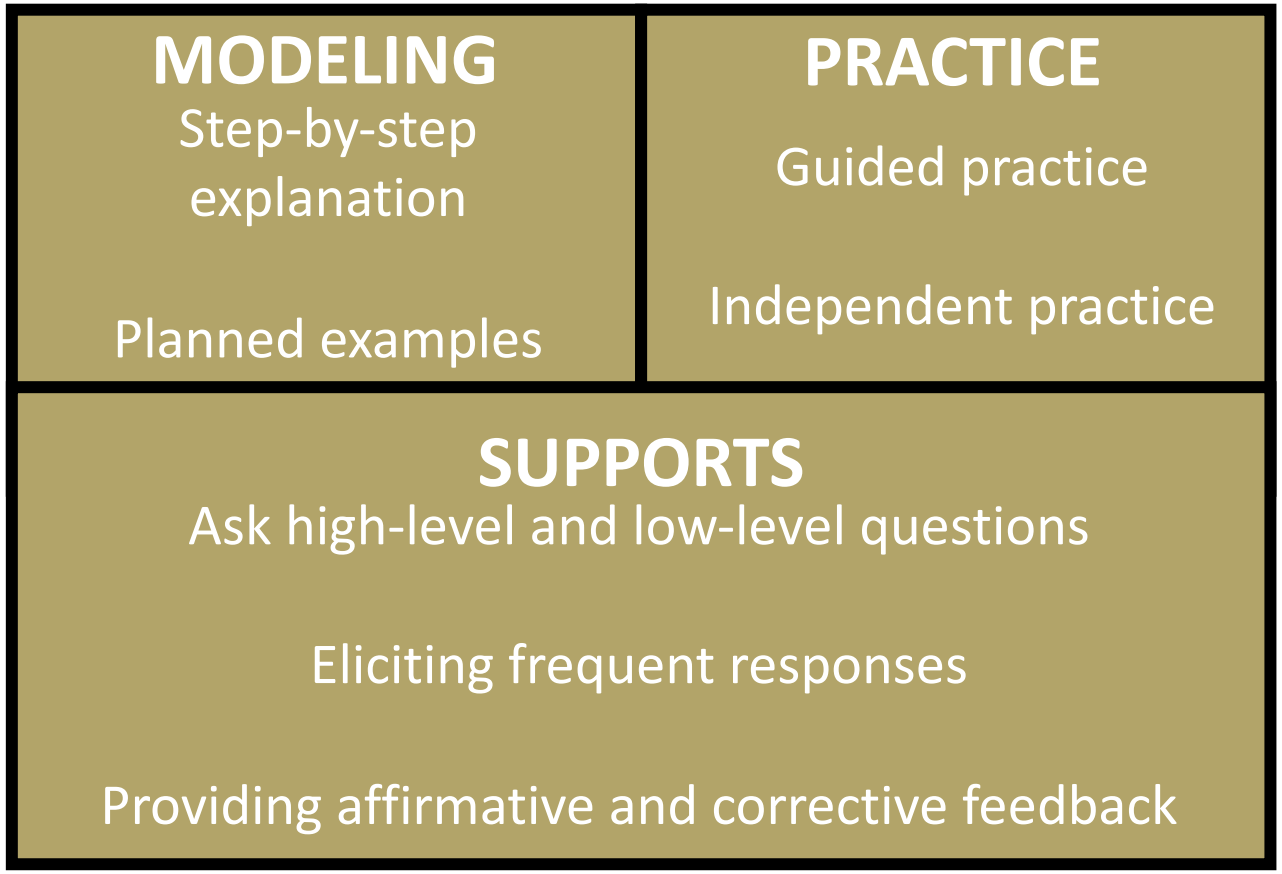
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

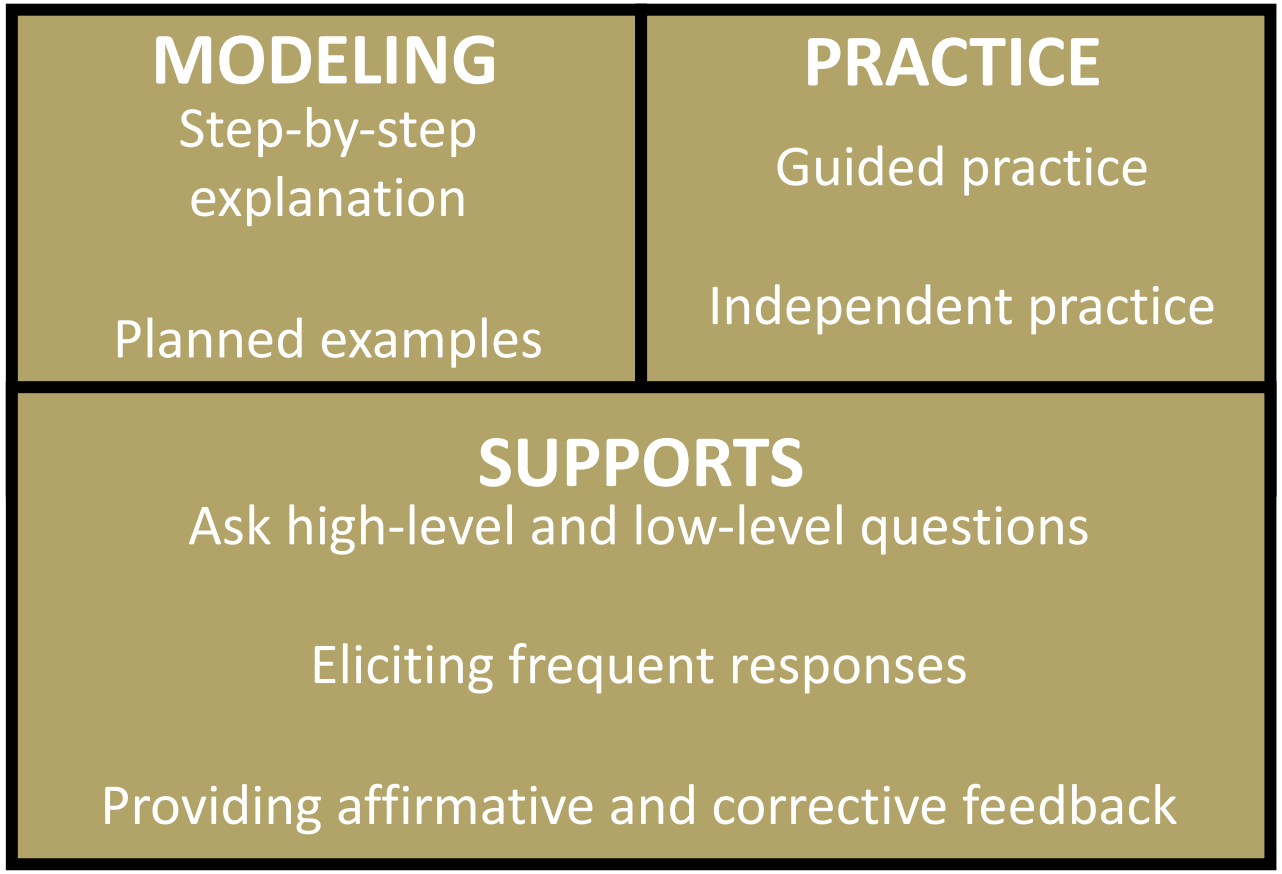
Providing affirmative and corrective feedback

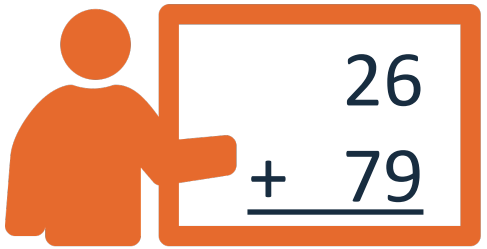
Modeling is a dialogue between the teacher and students.



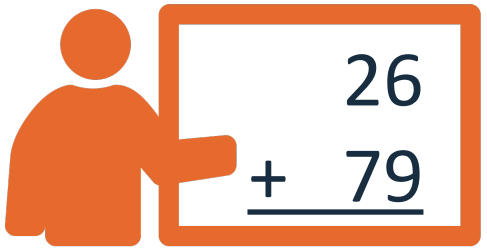
Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.





“Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether.”



"Let's solve this problem. What's the problem?"

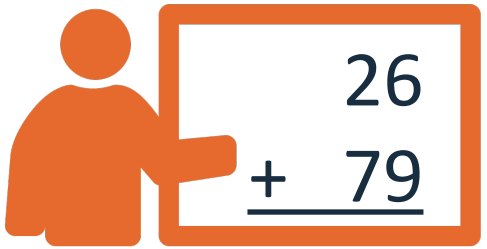
"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."



"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

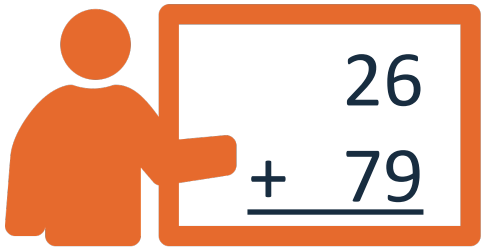
"Partial sums."

"What might partial mean?"

"Part of."

"We'll find parts – or partial sums - then add them together. With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."



"So, let's add the tens. What's 20 plus 70? Use your base-10 blocks or other tools."

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90 is the partial sum when you add the tens. What does 90 represent?"

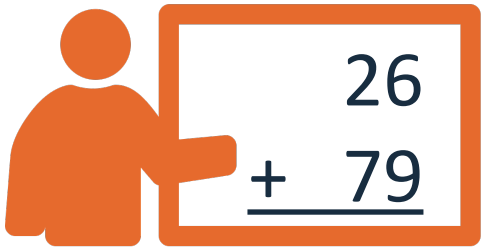
"Now, let's add the ones. What should we add?"

"90."

"90."

"It's the partial sum of adding 20 plus 70."

"6 plus 9."



"6 plus 9 equals what? Use your base-10 blocks or other tools."

"How did you get 15?"

"Let's write 15 below the 90. Where do we write the 15?"

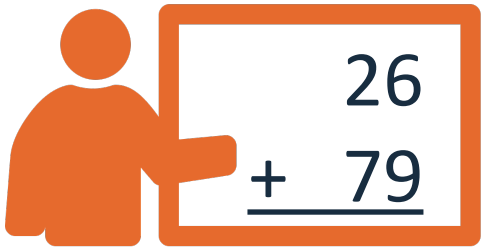
"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"15."

"We knew we had 9, then we added on 6."

"Below the 90."

"90 plus 15."



"What's 90 plus 15? Use your go-to strategy."

"105."

"How did you add those addends?"

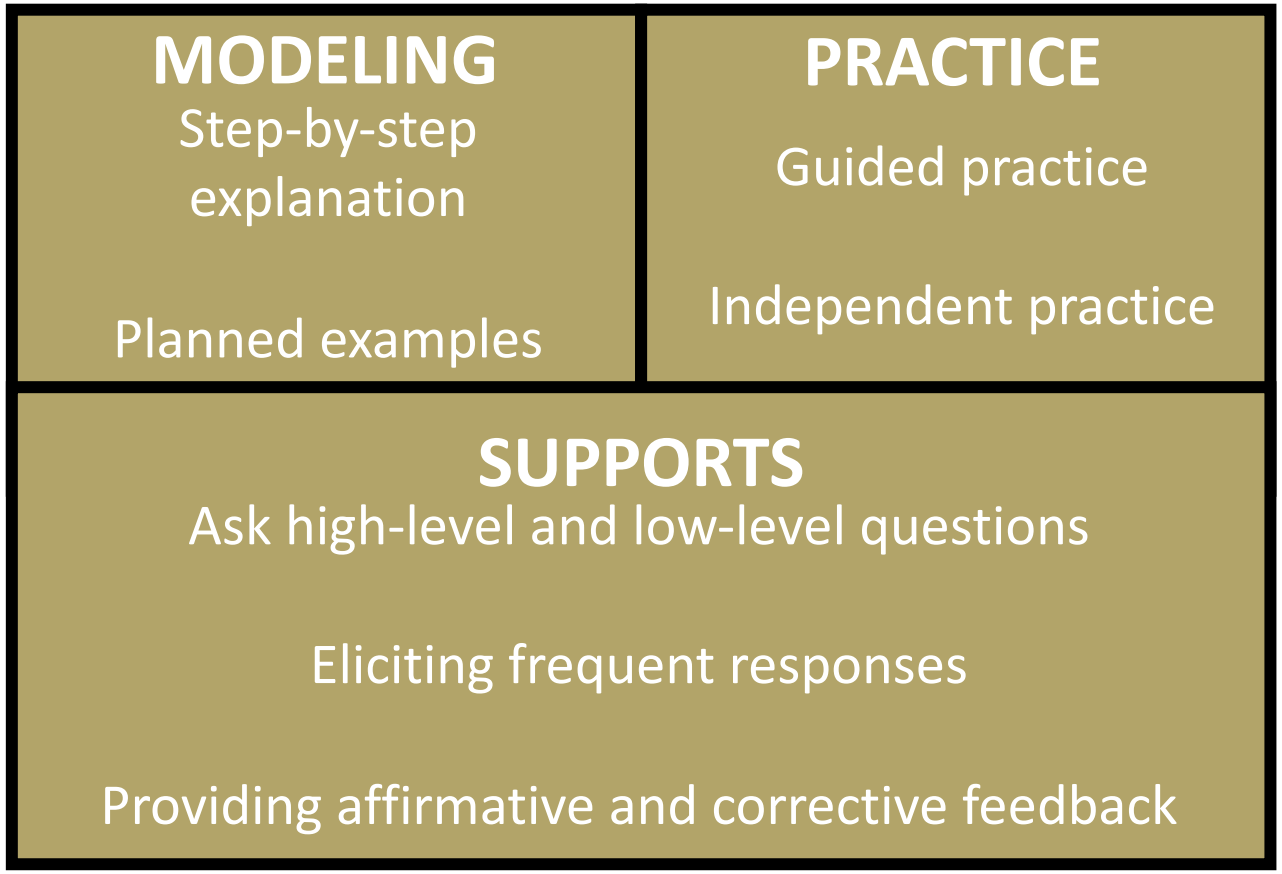
"I added 90 plus 10 then added 5 more."

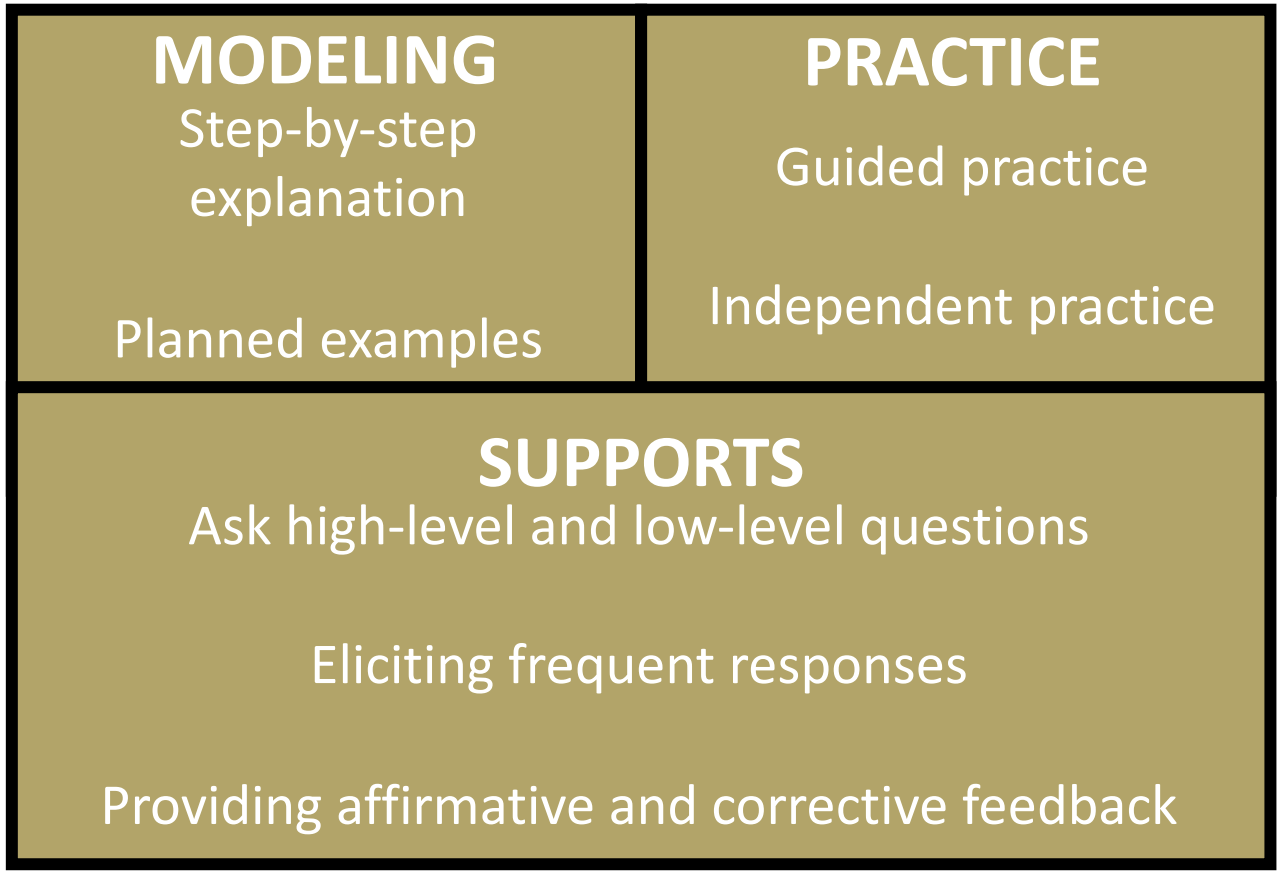
"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."

Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.

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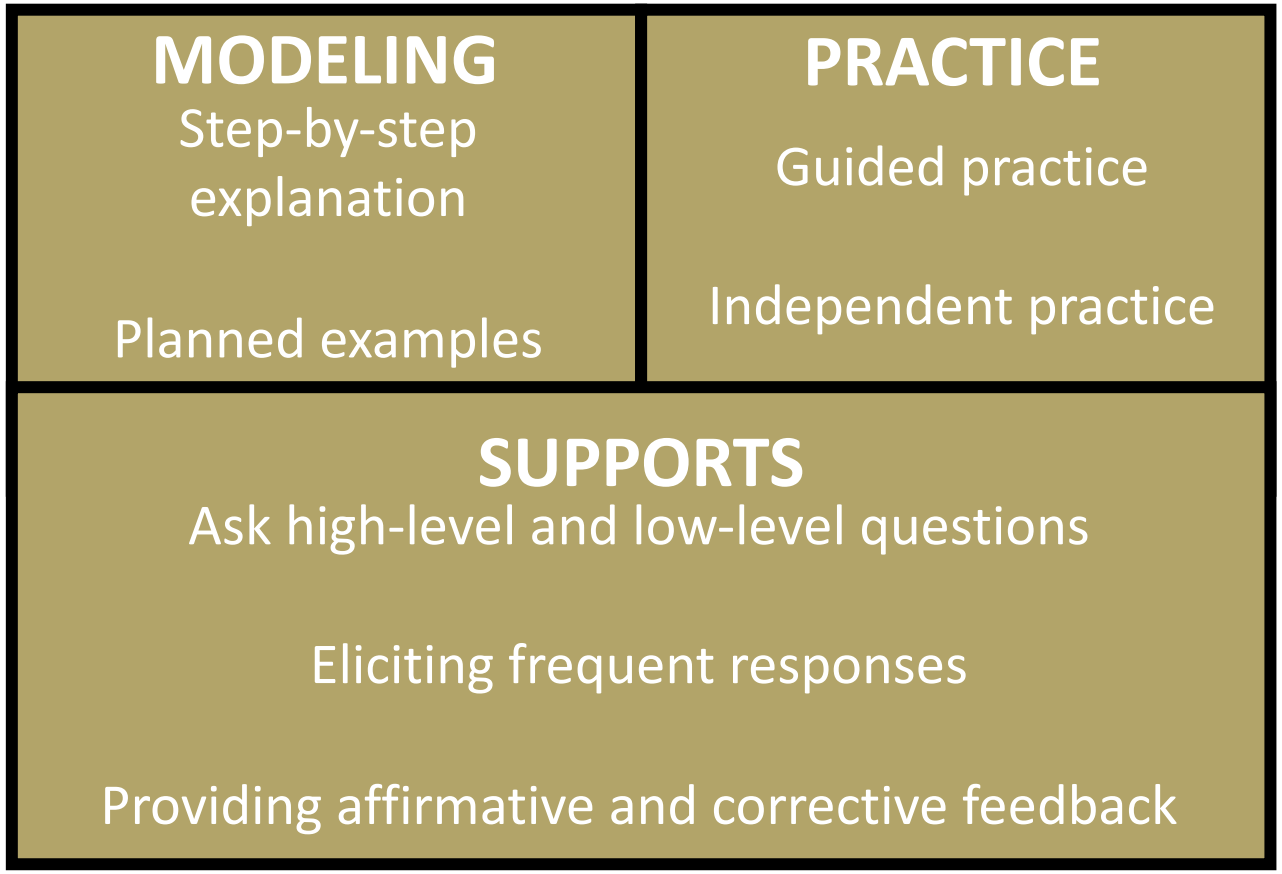
Eliciting frequent responses
Providing affirmative and corrective feedback



Practice continues as a dialogue between the teacher and students.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.

“Let’s work on a problem together.”

MODELING	PRACTICE
Step-by-step explanation Planned examples	Guided practice Independent practice

SUPPORTS

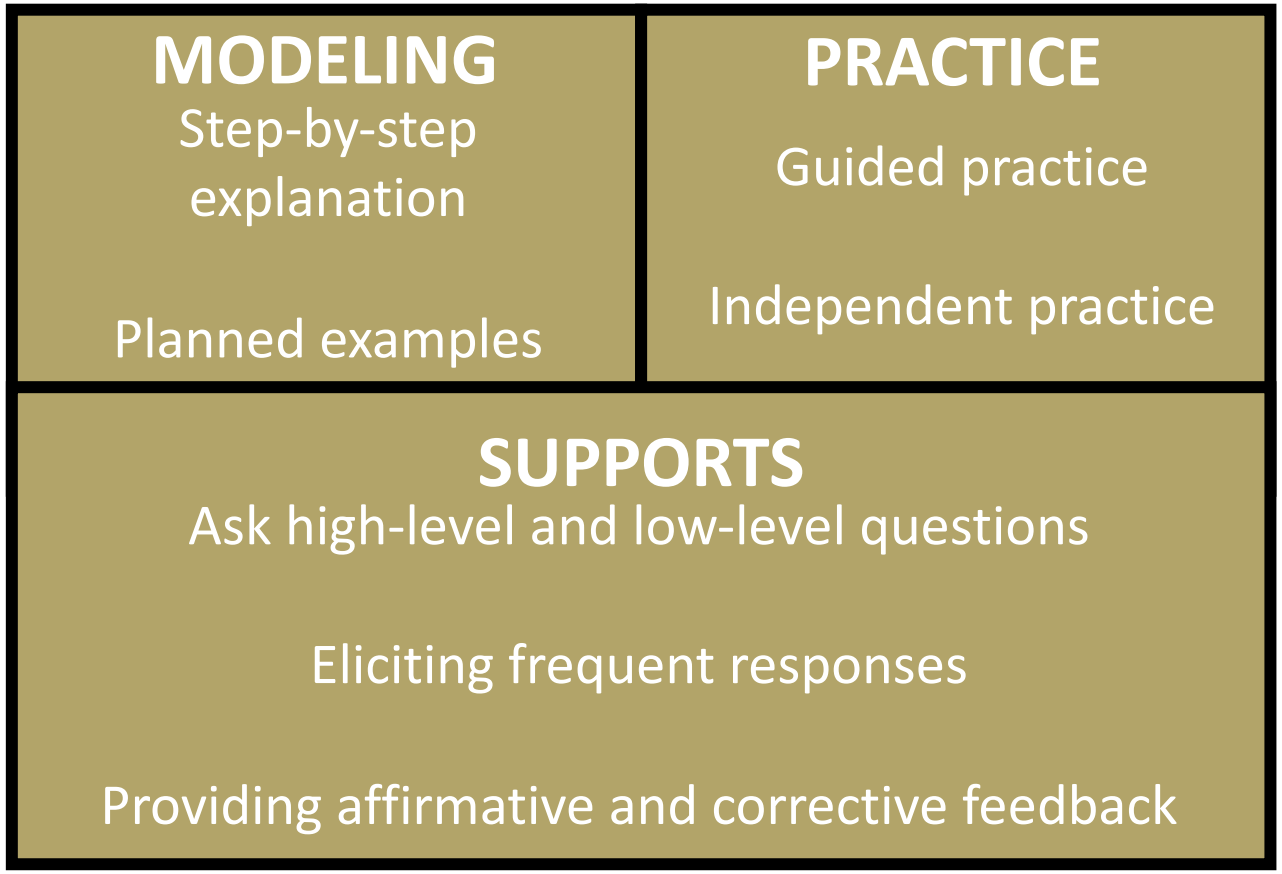
Ask high-level and low-level questions

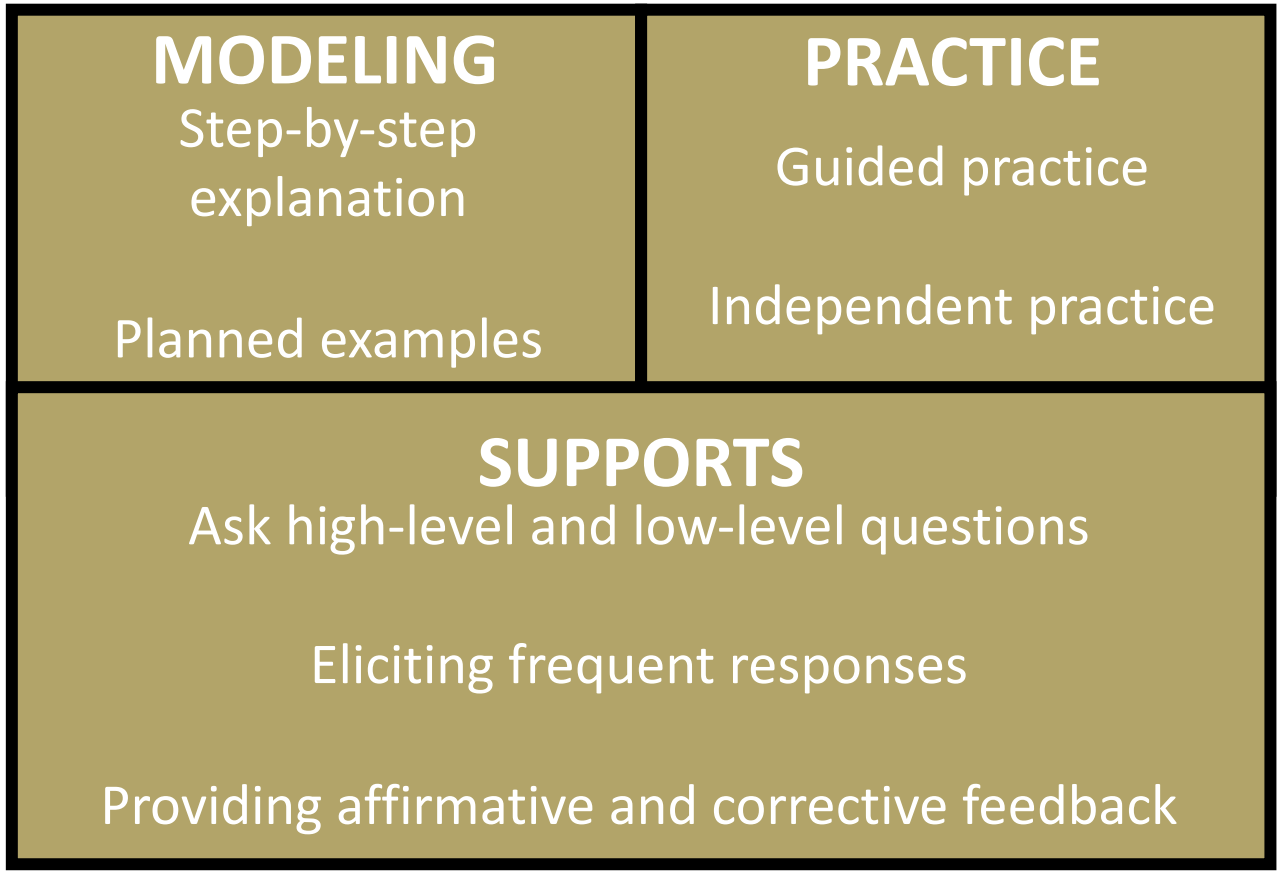
Eliciting frequent responses

Providing affirmative and corrective feedback

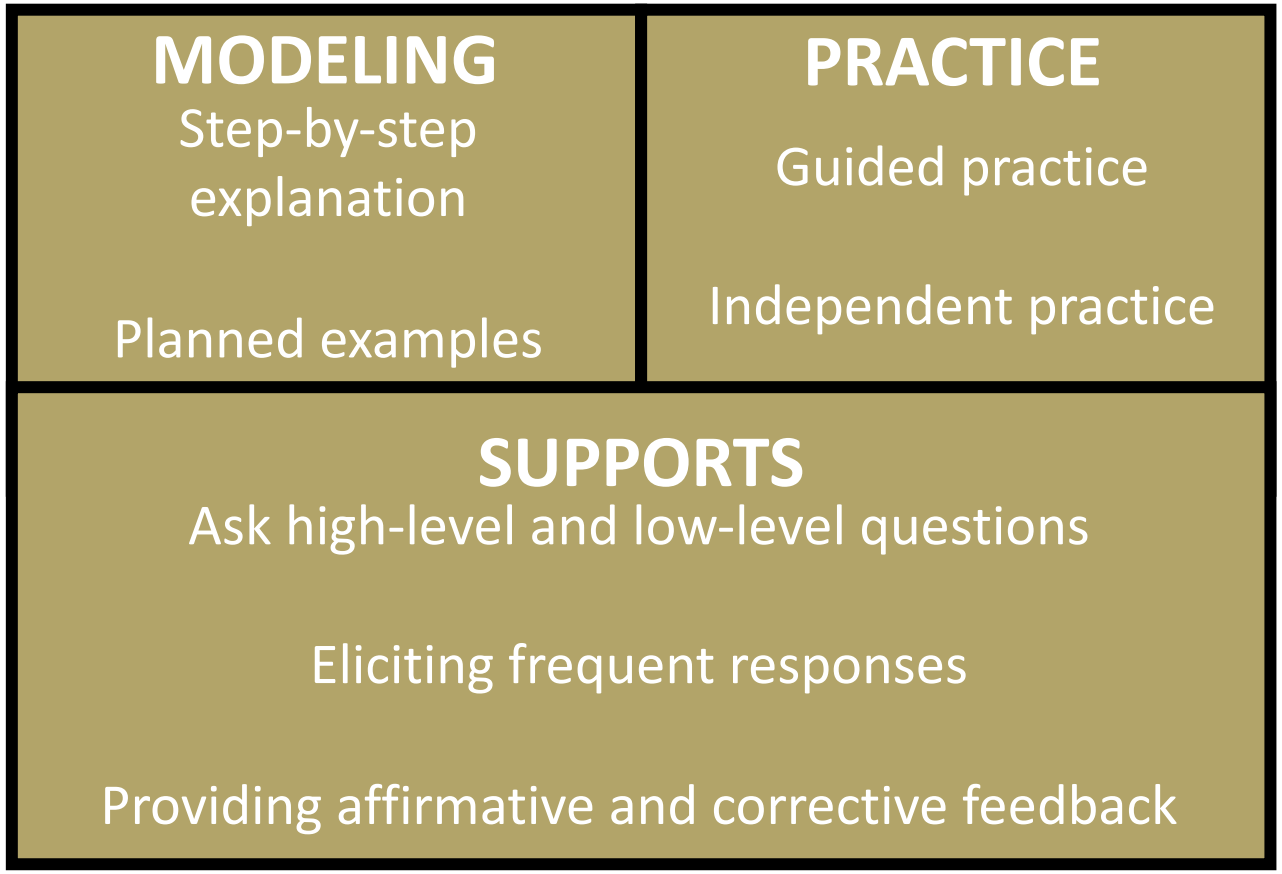
Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"


MODELING
Step-by-step explanation
Planned examples
PRACTICE
Guided practice
Independent practice
SUPPORTS
Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback



These **Supports** should be used in both **Modeling** and **Practice.**



During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

“What is 7 times 9?”

“63.”

MODELING
Step-by-step explanation

Planned examples

PRACTICE
Guided practice

Independent practice

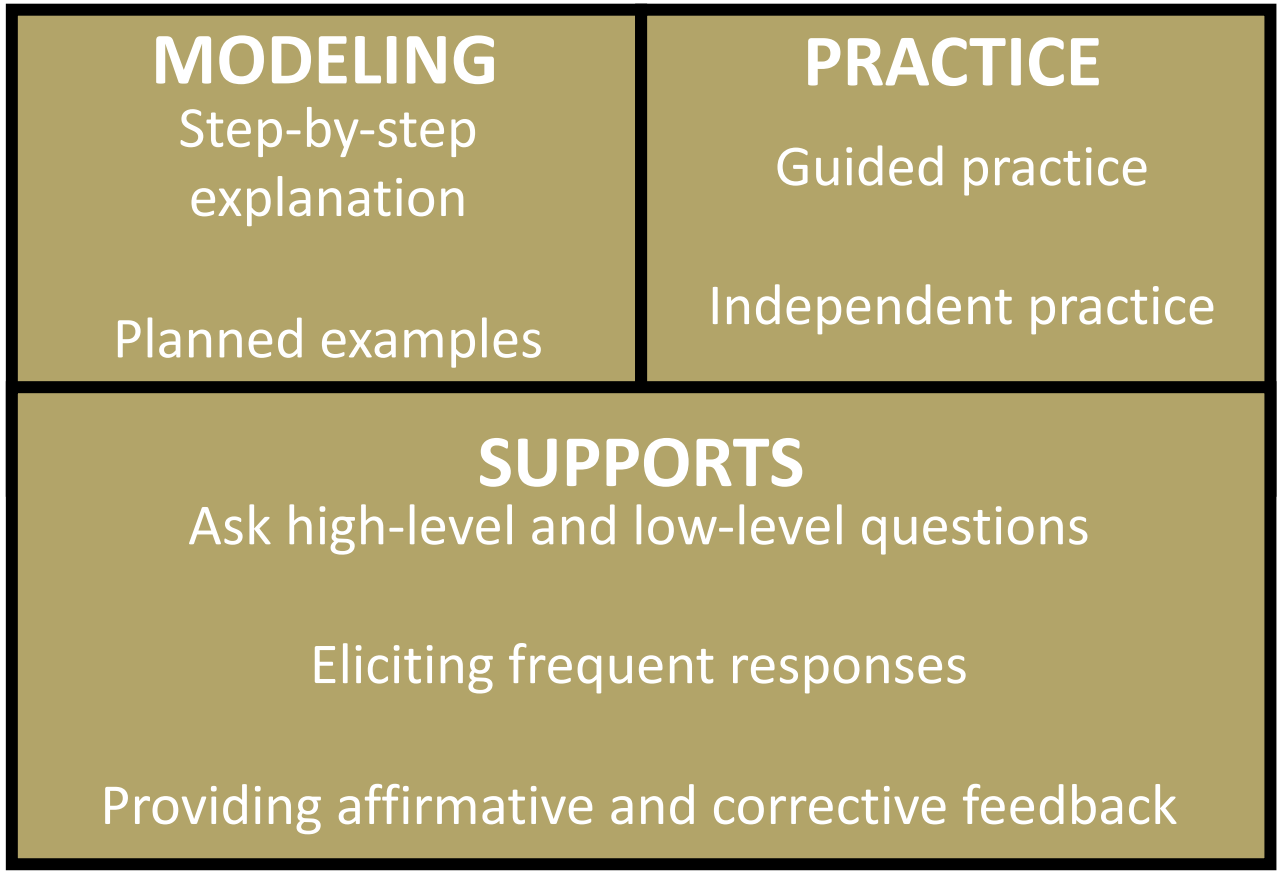
SUPPORTS
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."



During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

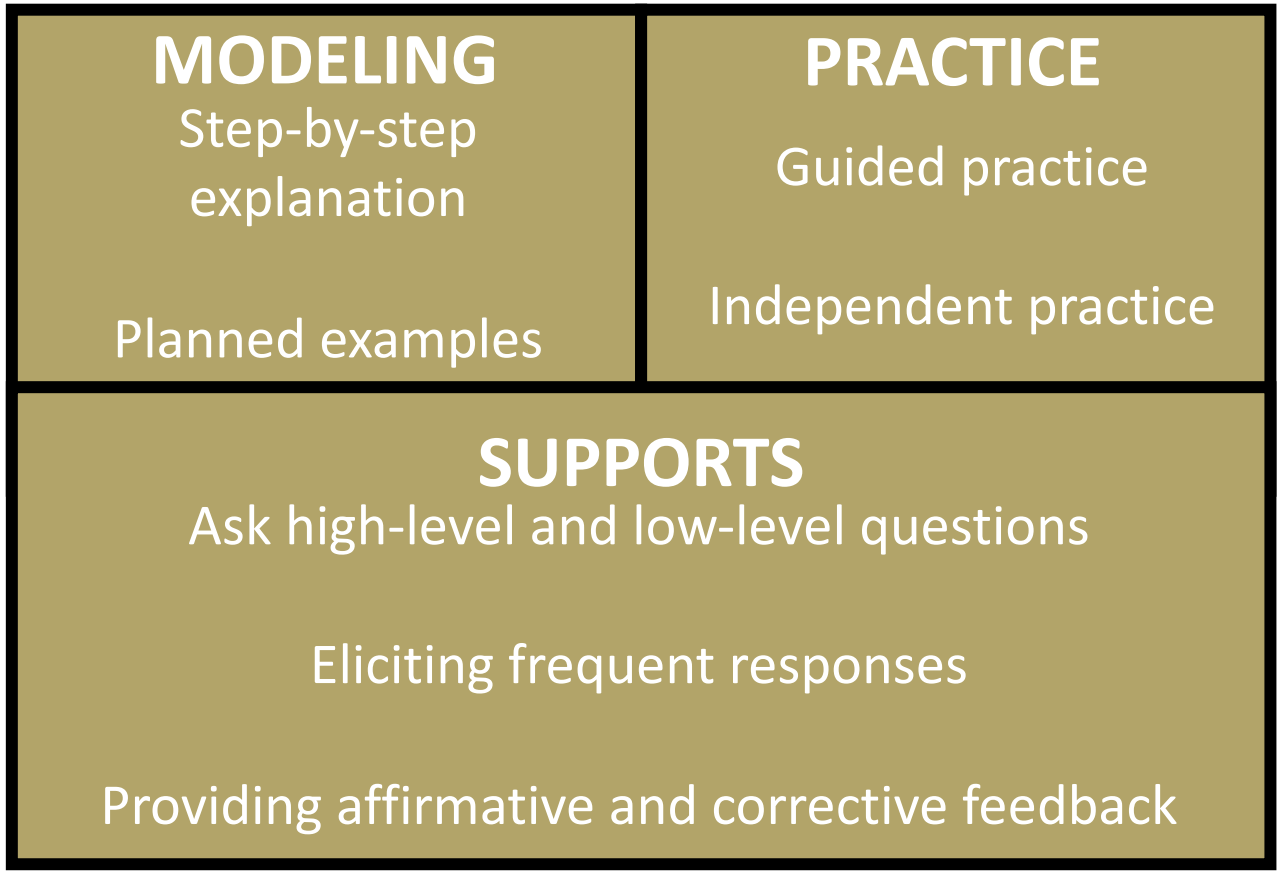
Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

MODELING	PRACTICE
Step-by-step explanation Planned examples	Guided practice Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

“Nice work using your word problem attack strategy.”

MODELING
Step-by-step explanation

Planned examples

PRACTICE
Guided practice

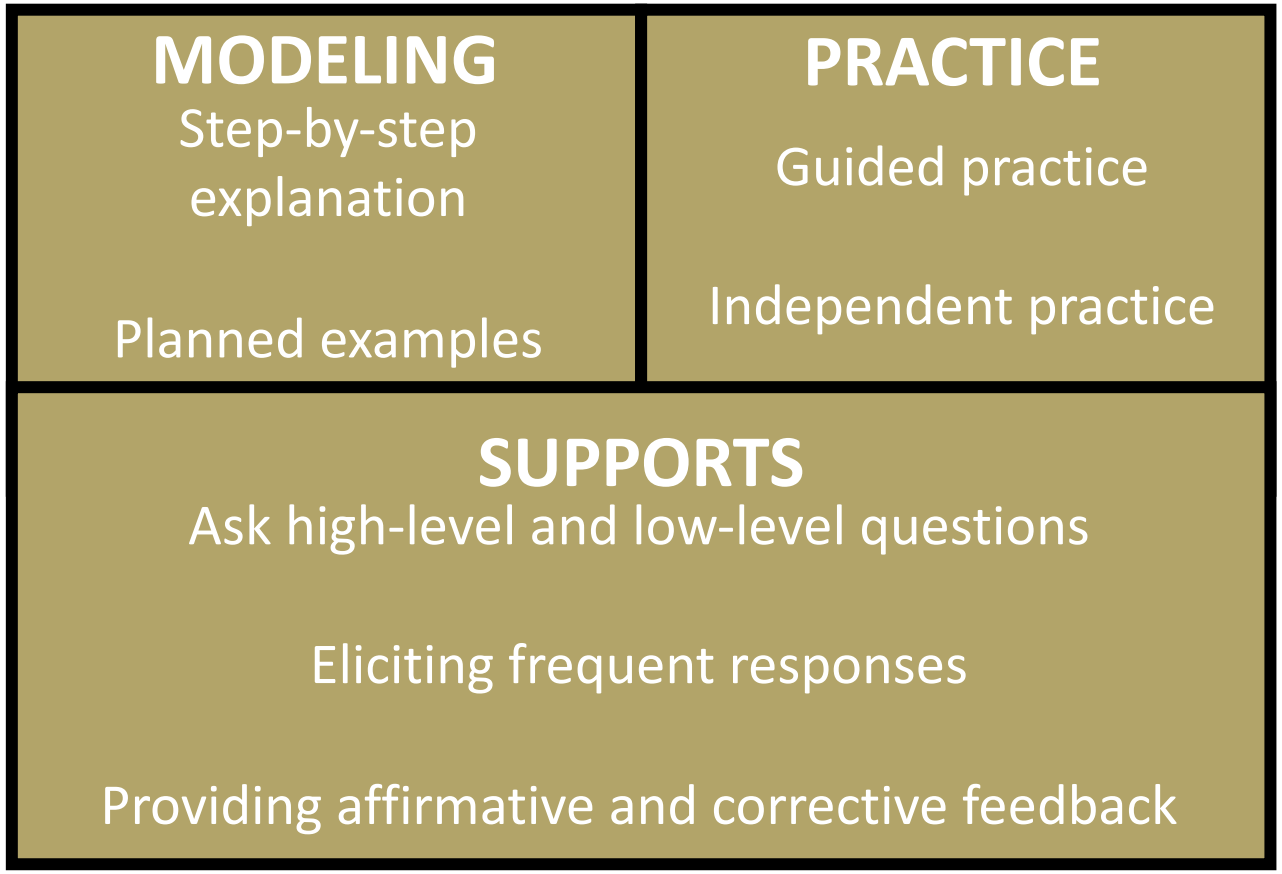
Independent practice

SUPPORTS
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."


MODELING
Step-by-step explanation
Planned examples
PRACTICE
Guided practice
Independent practice
SUPPORTS
Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback

MODEL AND PRACTICE

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with modeling and practice?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Fluency

FLUENCY

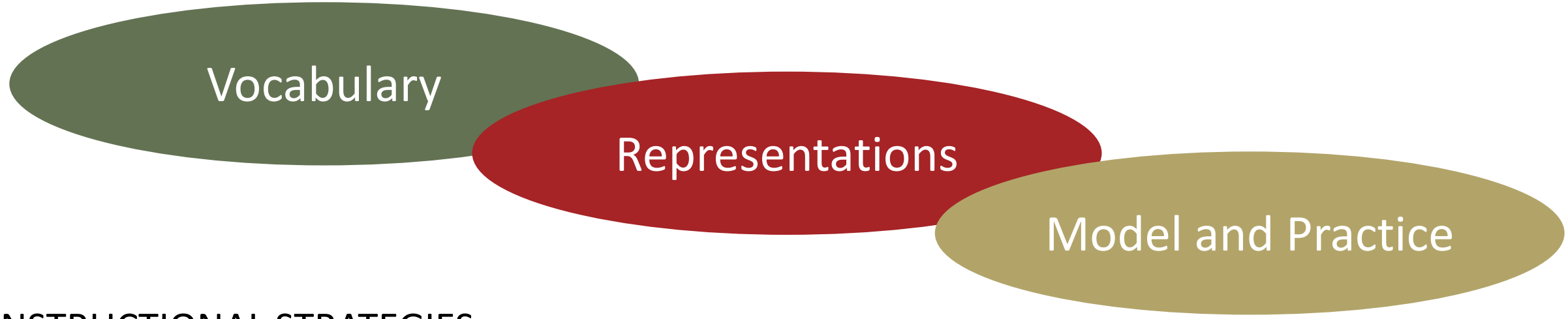
Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No

Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.

(Bailey et al., 2012; Cirino et al., 2019; Koponen et al., 2007, 2017; Vukovic et al., 2014

Fact fluency practice improves mathematics fact performance.

(Burns et al., 2010; Codding et al., 2011; McCallum et al., 2004; Nelson et al., 2013; Poncy et al., 2010; Schutte et al., 2015; Stocker & Kubina, 2017)

Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.

Addition	Subtraction
Multiplication	Division

Counting

Comparison

Money

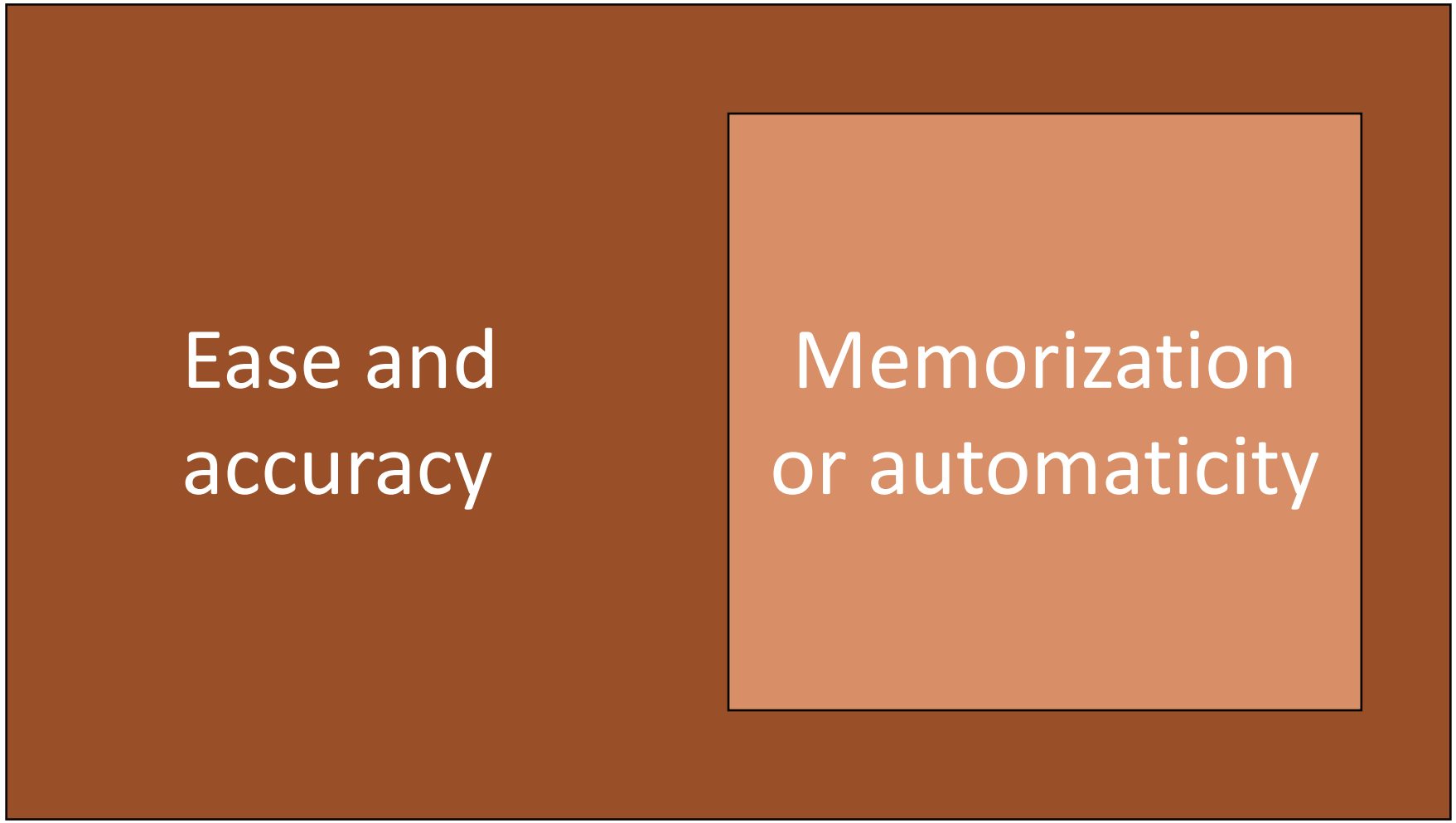
Time

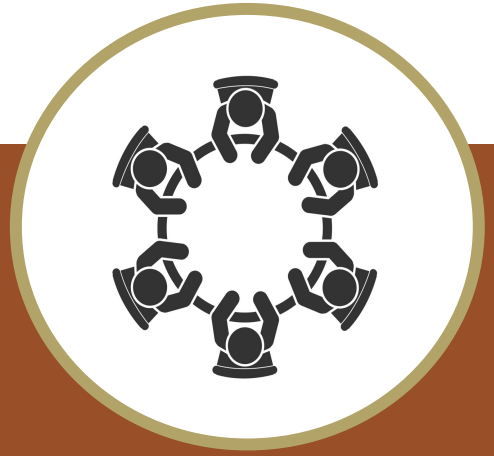
Fractions

Geometry

Measurement

Algebra


Ease and accuracy
Memorization or automaticity



List different types of fluency.

Discuss whether they require memorization.

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation.

$$\begin{array}{r} 15 \\ +\ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation.

$$\begin{array}{r} 1.4 \\ +\ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ 0.14 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation.

$$\begin{array}{r} 1.4 \\ +\ -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$-135 \div 2 =$

FLUENCY

Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)

Dice

Roll the Dice

8 + 7 = 15

____ + ____ = ____

____ + ____ = ____

____ − ____ = ____

____ − ____ = ____

____ + ____ = ____

____ + ____ = ____

____ − ____ = ____

____ − ____ = ____

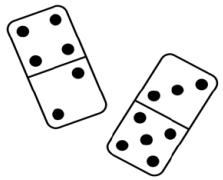
Beach Ball

4 plus 6 equals 10.

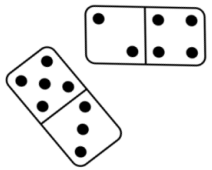
7 plus 6 equals 13.

2 plus 2 equals 4.

Dominoes



Dominoes



4 + 6 = 10

____ + ____ = ____

____ − ____ = ____

____ − ____ = ____

____ × ____ = ____

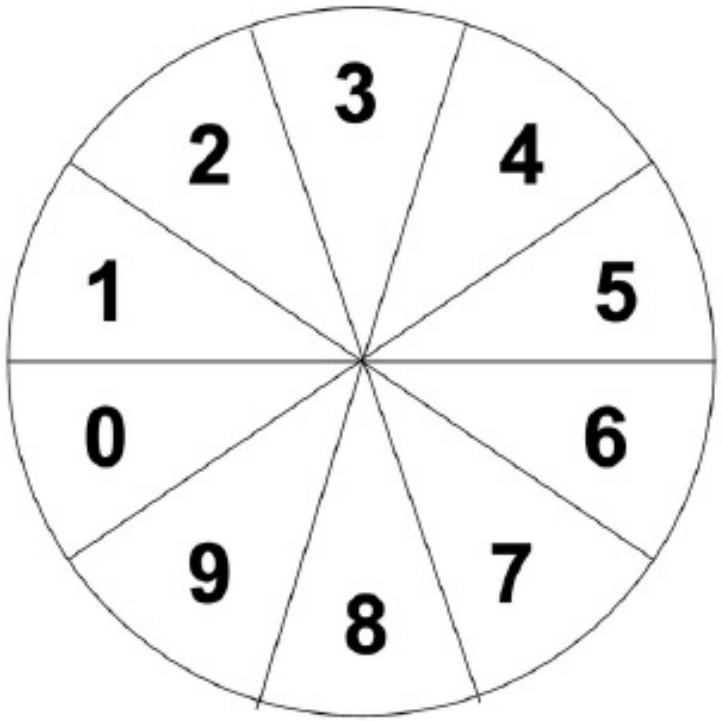
____ × ____ = ____

Spinner

2 times 4 equals 8.

6 times 9 equals 54.

7 times 1 equals 7.



Playing Cards

Cards 

____ + ____ = ____

____ + ____ = ____

6 ____ – 2 ____ = 4 ____

____ – ____ = ____

____ + ____ = ____

____ + ____ = ____

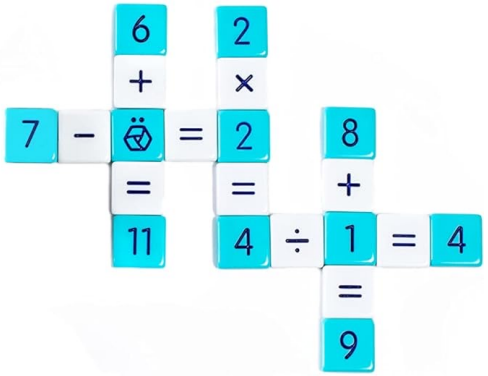
____ – ____ = ____

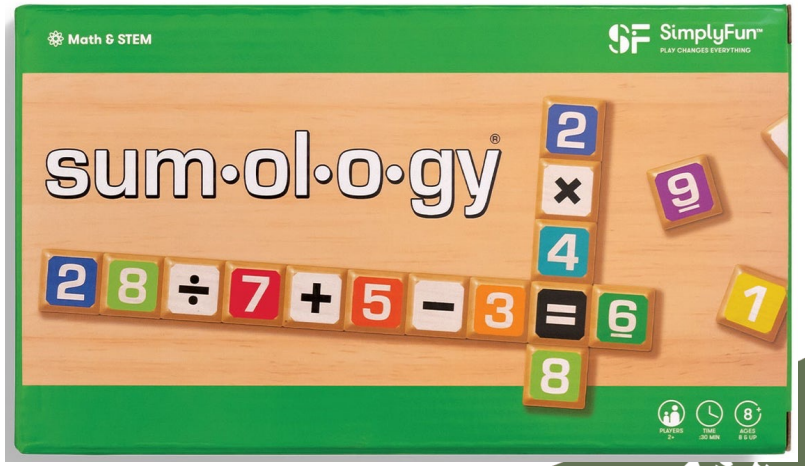
____ – ____ = ____

Wrap-Ups

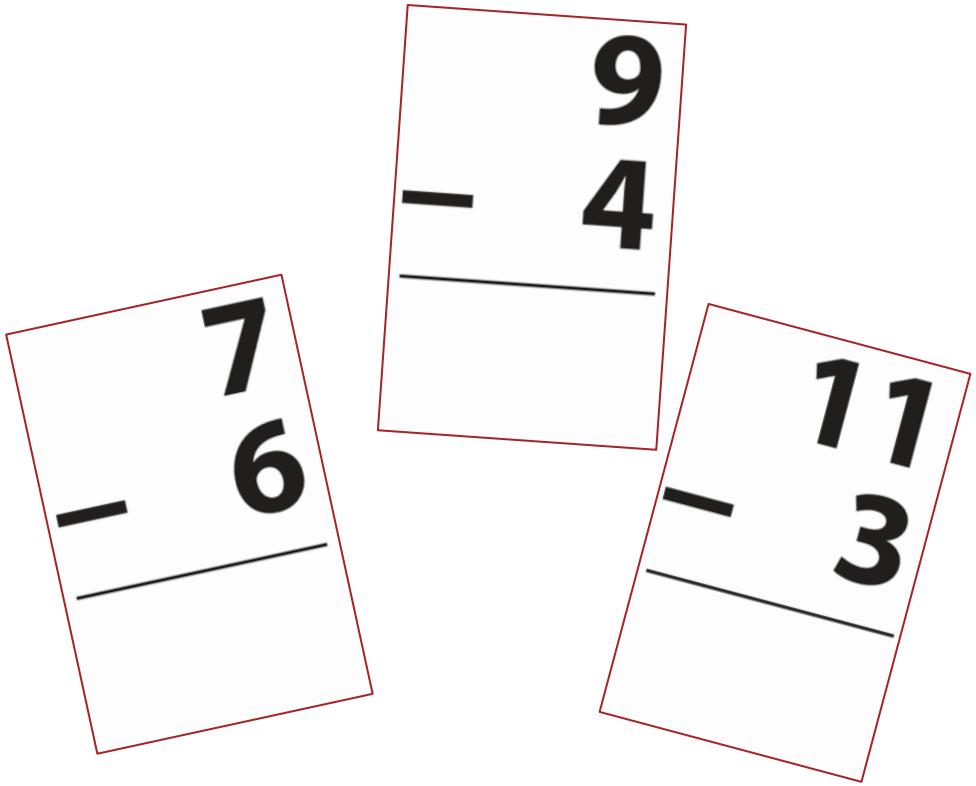
Mobi Math

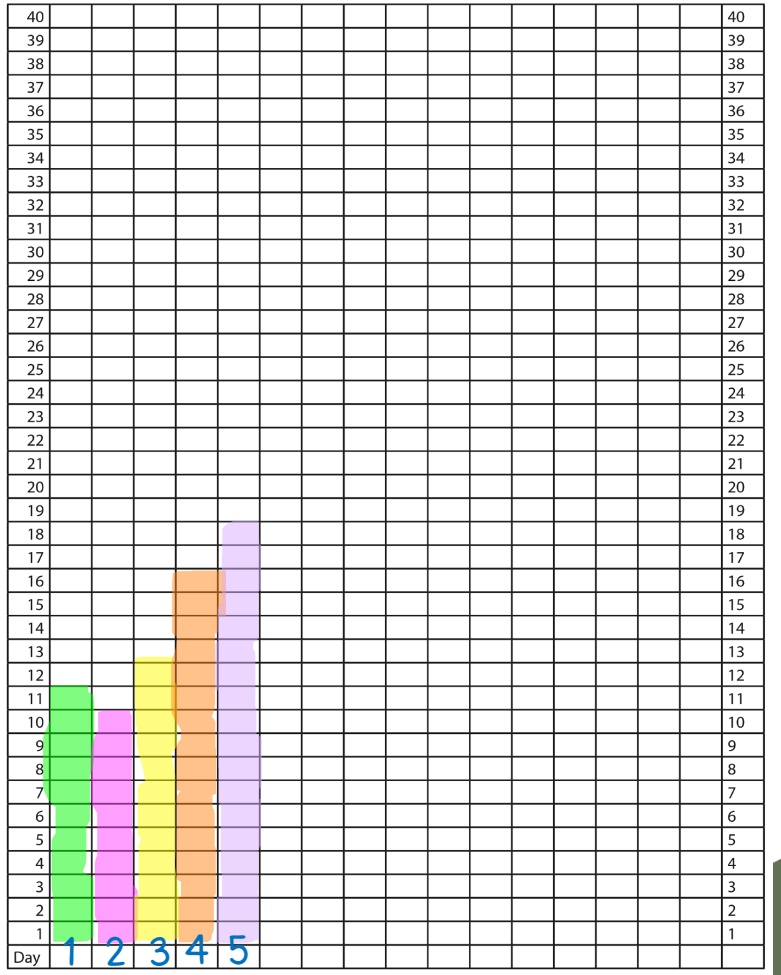

mobi.


6
+
7 − = 2
=
11
2
×
=
8
+
4 ÷ 1 = 4
=
9


Math & STEM
SimplyFun
sum·ol·o·gy
2 8 ÷ 7 + 5 − 3 = 6
2
×
4
8
9
1

Flash Cards

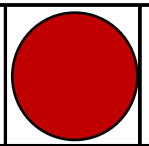
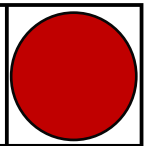
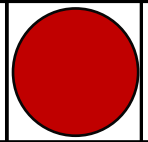
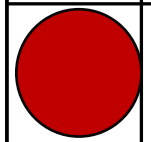

7
− 6
9
− 4
11
− 3


40
39
38
37
36
35
34
33
32
31
30
29
28
27
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
Day
1
2
3
4
5

Bingo

Math Bingo

Your teacher will call out a math problem. Quickly solve the problem. If you have the answer on your bingo card, cover it with a chip. The middle space is free!
The first person to finish the pattern your teacher decides wins!
(5 in a row, 4 corners, etc.)

12		24	100	
15	0	42	16	20
8	35		6	4
	2	40	27	7
50	10	30	48	14

8 times 10 equals…

3 times 1 equals…

2 plus 3 equals…

Magic Squares

Magic Squares Board

1. Place the sum or product in the bottom right corner.
2. In the bottom row, create a fact with a sum or product of the bottom right corner.
3. In the right column, create a fact with a sum or product of the bottom right corner.
4. Create two columns with a sum or product of the bottom number.
5. Create two rows with a sum or product of the right column number.
6. Write the created facts below.

0	2	2
5	4	9
5	6	11

0 + 2 = 2

5 + 4 = 9

2 – 0 = 2

9 – 5 = 4

4	5	9
2	0	2
6	5	11

7	3	10
1	0	1
8	3	11

6	1	7
3	2	5
9	3	12

4	4	8
2	2	4
6	6	12

5	1	6
4	3	7
9	4	13

5	1	6
3	4	7
8	5	13

6	3	9
2	3	5
8	6	14

1	5	6
6	2	8
7	7	14

6	2	8
3	4	7
9	6	15

4 5 2 0

5 6 9 11 2

#CEC2024

Cover, Copy, Compare

Cover, Copy, Compare

	9 x 6 54	8 × 6 48	
7 × 8 56		6 × 5 30	
9 × 9 81		7 × 9 63	
6 × 7 42		8 × 5 40	
8 × 8 64		7 × 7 49	

File Folder

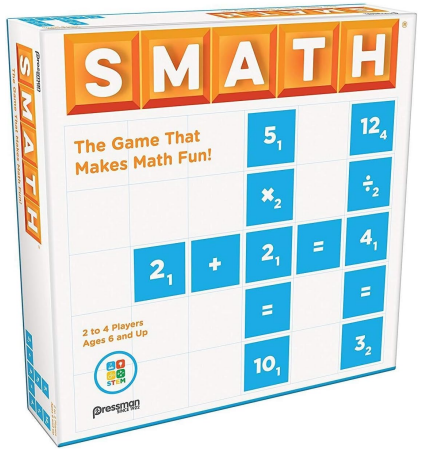
6 + 3 =	9
1 + 7 =	8
6 + 4 =	10
7 + 3 =	
2 + 7 =	
5 + 6 =	
4 + 7 =	
7 + 8 =	
6 + 7 =	
7 + 9 =	
7 + 6 =	
8 + 7 =	
7 + 0 =	
9 + 6 =	
6 + 0 =	
6 + 8 =	

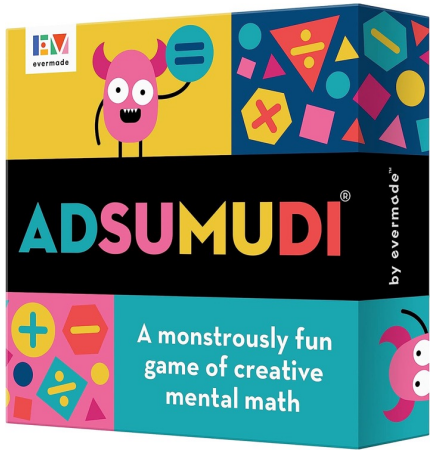
Taped Problems

Taped Problems

8 × 8 = 64	7 × 7 = 49	8 × 7 =
6 × 5 =	6 × 7 =	6 × 8 =
5 × 7 =	5 × 5 =	5 × 6 =
6 × 6 =	8 × 6 =	7 × 6 =
7 × 8 =	8 × 5 =	7 × 5 =

Games





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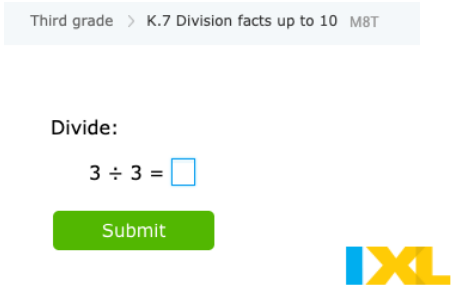
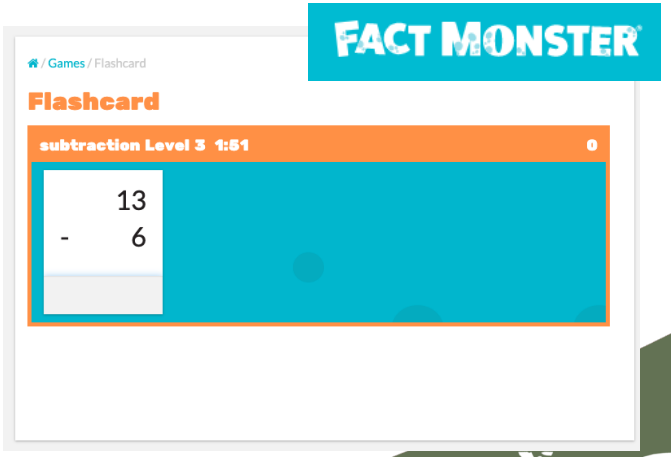
Technology





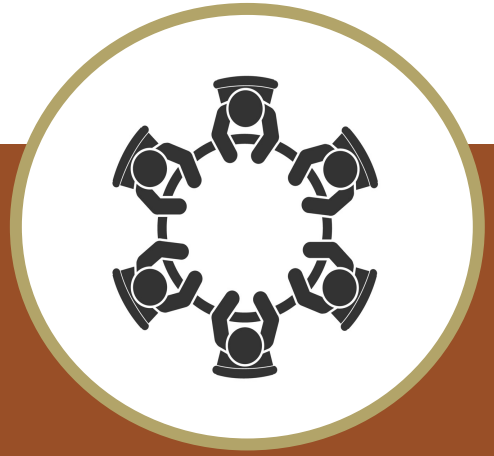
#CEC2024

DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)



What are your strengths with fluency?

What are your opportunities for growth?

What are your plans for next Monday?

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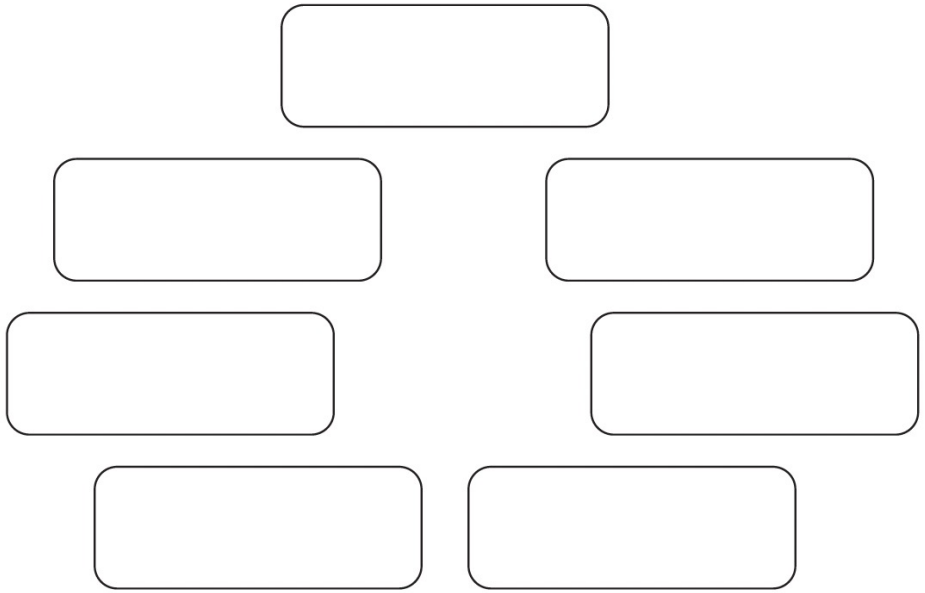
Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Word Problems

WORD PROBLEMS

Research and Information



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Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

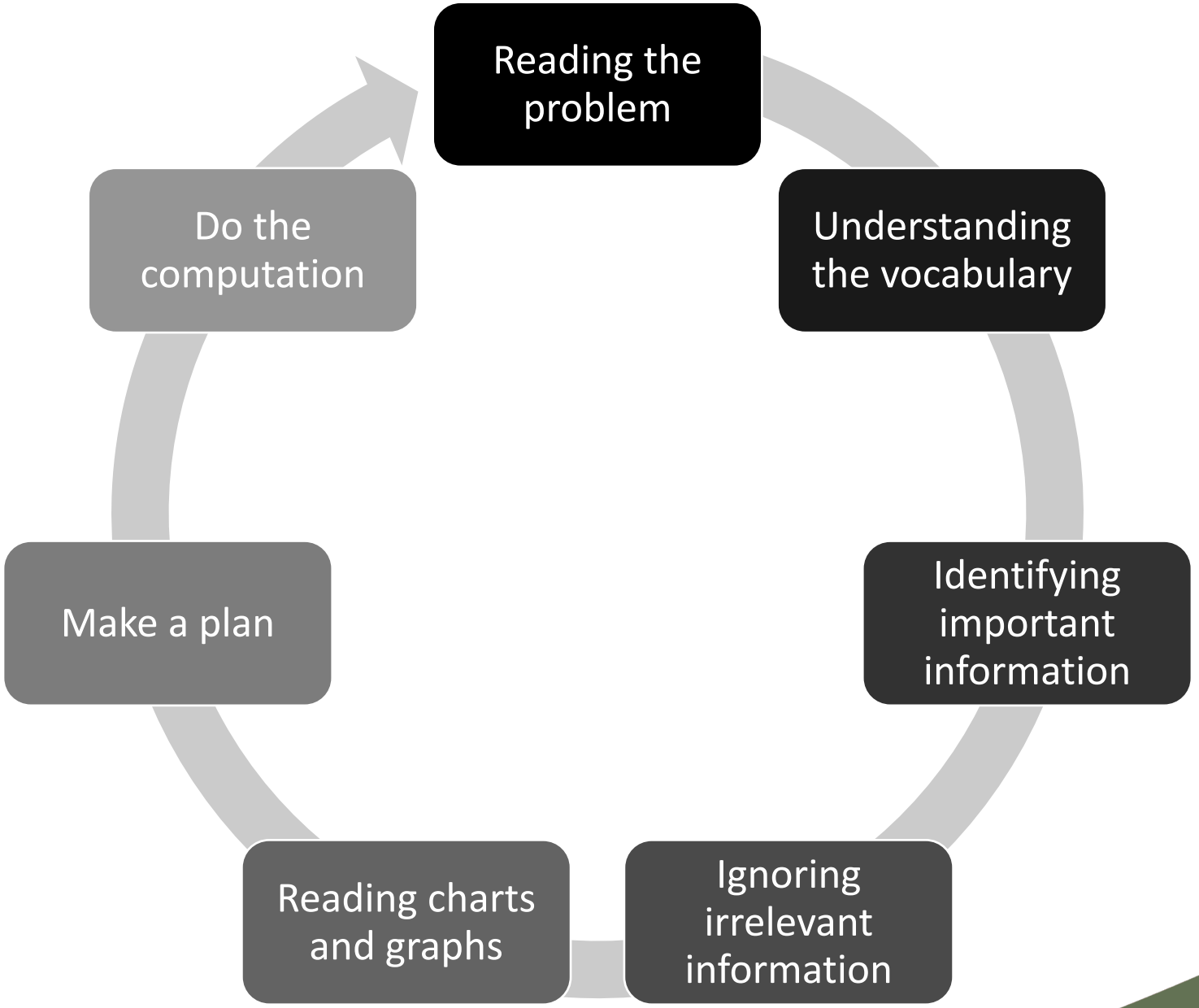
Fluency

Word Problems

Key words tied to operations is an ineffective word-problem strategy.
(Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves word-problem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

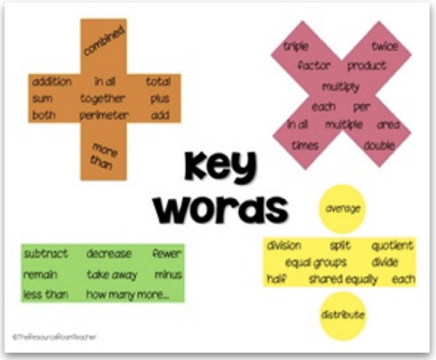
A focus on schemas improves word-problem performance.
(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)


Reading the problem
Understanding the vocabulary
Identifying important information
Ignoring irrelevant information
Reading charts and graphs
Make a plan
Do the computation

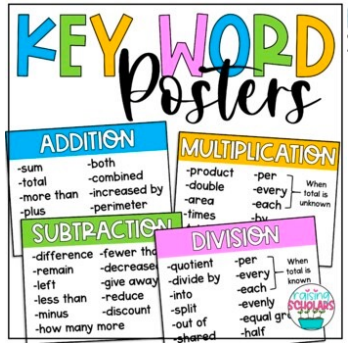
1. Keywords tied to operations

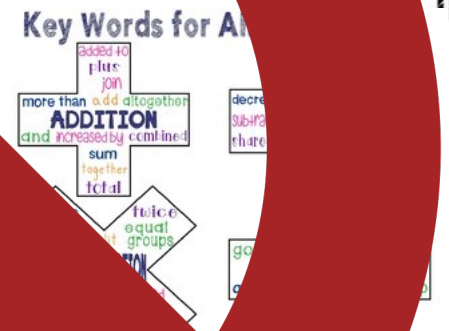
Carmelita had 8 pencils **fewer** than Jenny. If Jenny had 18 pencils, how many pencils did Carmelita have?

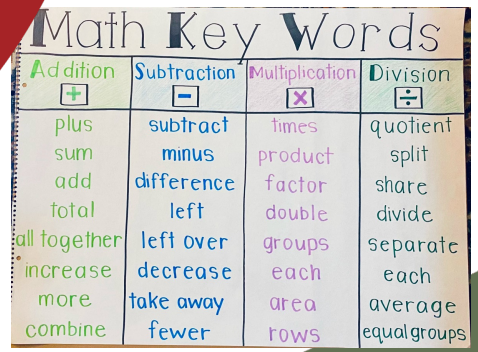
Carmelita had 8 pencils **fewer** than Jenny. If Carmelita had 18 pencils, how many pencils did Jenny have?

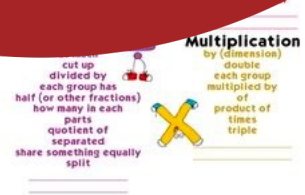


Math Posters!









CEC 2024

m Words Poster Set

(4.1)

75%

7 Reviews
Write a Review

pondents would
nend this to a friend

do you want to know about
oduct?

wer in about 48 hours

th 3 monthly payments of
just $4.33. No fees.
Learn more

Quantity

1

Add to Wish list

#CEC2024

Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Description of Multi-Step Word Problems (n = 84)

	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
Schema	*n*	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

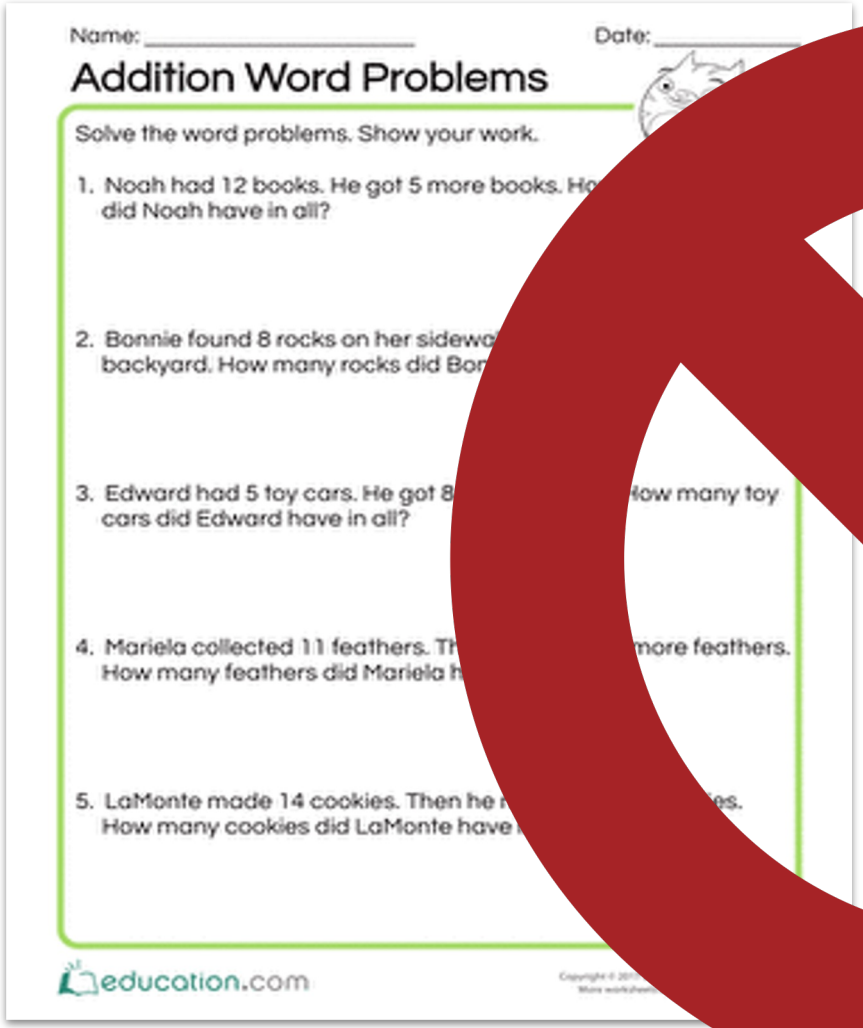
Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!

2. Presenting Problems by Operation


Name: _____ Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. Ho
did Noah have in all?

2. Bonnie found 8 rocks on her sidewa
backyard. How many rocks did Bon

3. Edward had 5 toy cars. He got 8 ow many toy
cars did Edward have in all?

4. Mariela collected 11 feathers. Th more feathers.
How many feathers did Mariela h

5. LaMonte made 14 cookies. Then he
How many cookies did LaMonte have

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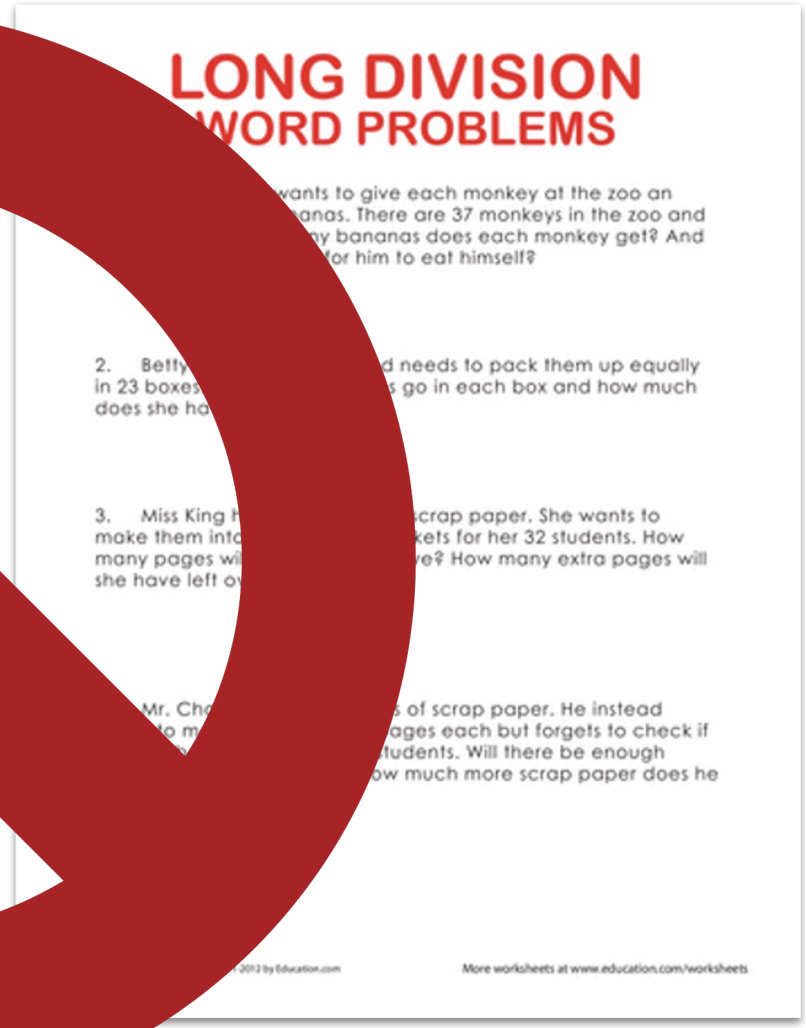
LONG DIVISION WORD PROBLEMS

wants to give each monkey at the zoo an
anas. There are 37 monkeys in the zoo and
y bananas does each monkey get? And
or him to eat himself?

2. Betty d needs to pack them up equally
in 23 boxes s go in each box and how much
does she ha

3. Miss King h crap paper. She wants to
make them into kets for her 32 students. How
many pages wi ve? How many extra pages will
she have left o

Mr. Ch of scrap paper. He instead
to m ages each but forgets to check if
udents. Will there be enough
w much more scrap paper does he

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CEC 2024

WORD PROBLEMS

Attack Strategies

SOLVE

- Study the problem
- Organize the information
- Line up a plan
- Verify the plan
- Examine the answer

R-CUBES

- Read the problem
- Circle key numbers
- Underline the question
- Box action words
- Evaluate steps
- Solve and check

UPS Check

- Understand
- Plan
- Solve
- Check

MATH

Teach an attack strategy

Teach about schemas

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve
the problem.

Review your answer.

SOLVE

Study the problem.

Organize the information.

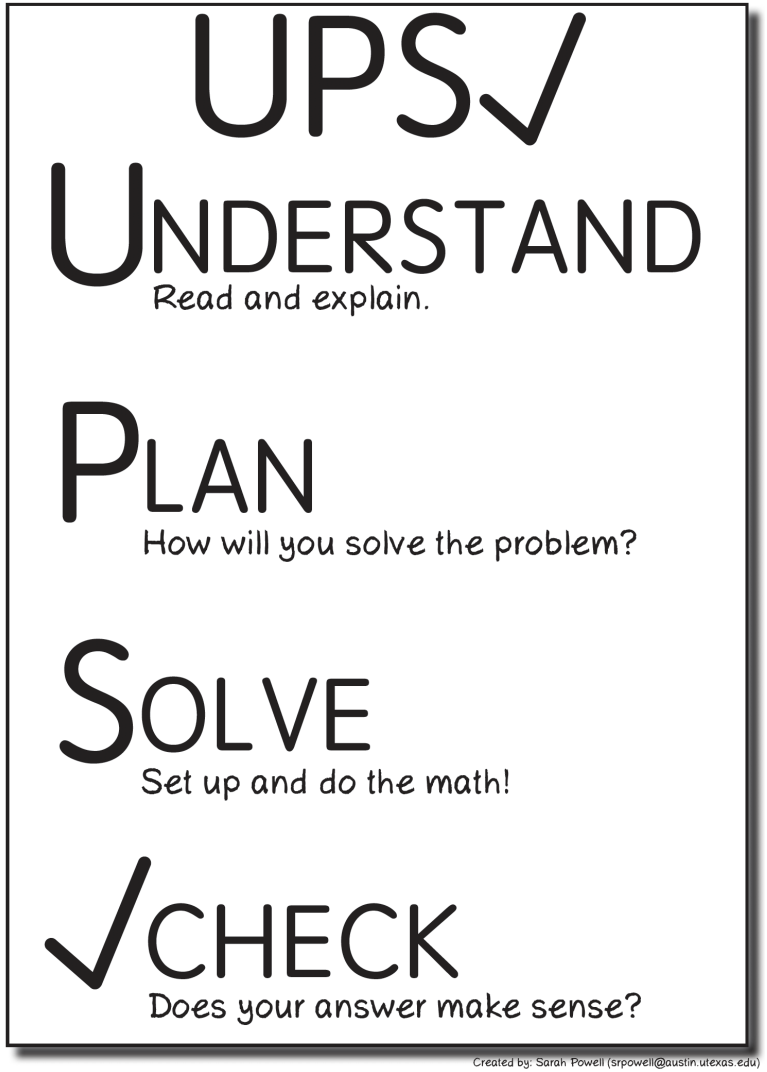
Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.



#CEC2024

Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

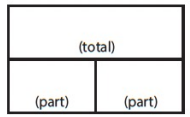
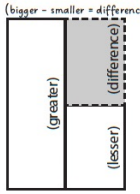
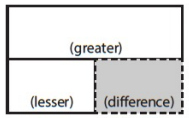
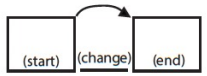
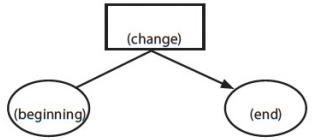
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B - s = D (bigger - smaller = difference)  G - L = D (greater - less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/- C = E (start +/- change = end)   	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	

Powell & Fuchs (2018).
Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.

Total

Part-part-whole
Combine

Parts put together into a **total**

Mandy saw **3** canoes and **8** kayaks. How many boats did Mandy see?

Total

Mandy saw **11** boats. If **3** of the boats were canoes, how many were kayaks?

Part

Mandy saw **11** boats. **8** of the boats were kayaks, how many were canoes?

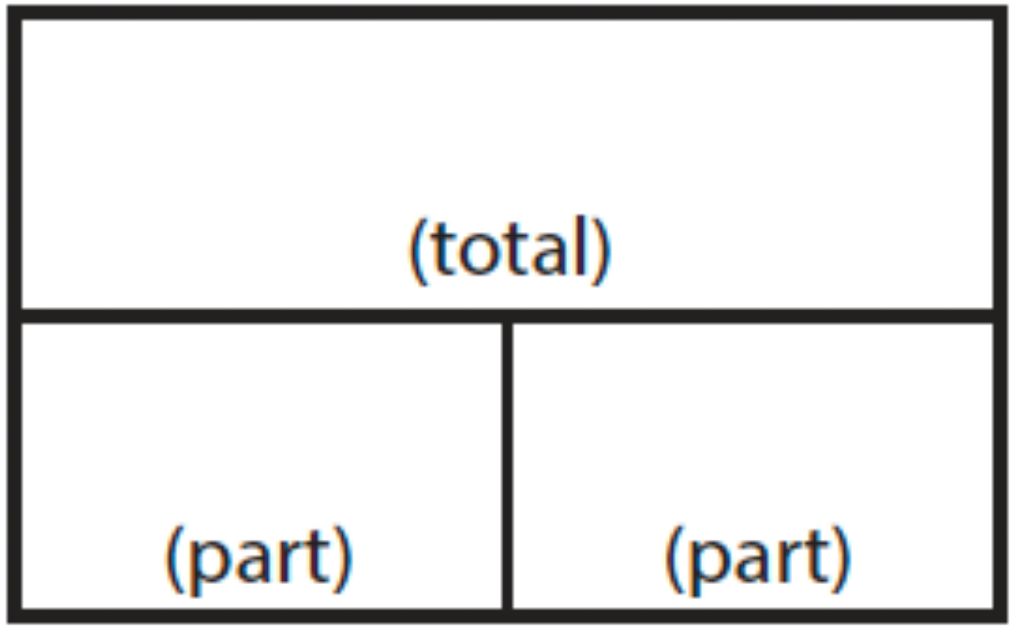
Part

Total

"Are parts put together for a total?"

Total

P1 + P2 = T



Total

WORD PROBLEMS: TOTAL

A.
An artist poured 6.09 kilograms of orange sand and 14.26 kilograms of blue sand into a mixing container for a project. What was the total amount of sand the artist poured into the container in kilograms?

B.
A movie theater has 710 seats.
- 158 seats are red.
- 247 seats are black.
- 119 seats are yellow.
- The rest of the seats are green.

How many seats are green?

C.
The frequency table shows the number of visitors a park had on three different days.

Park Visitors

Day	Number of Visitors
Friday	𝍸 𝍸 |||
Saturday	𝍸 𝍸 𝍸 𝍸 𝍸 |
Sunday	𝍸 𝍸 𝍸 ||

What was the total number of visitors the park had over those three days?

Your TOTAL problem:

MATH

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Difference

Compare

Greater and **lesser** amounts compared for a **difference**

L’Tanya has **10** pencils. Vickie has **4** pencils. How many more pencils does L’Tanya have?

Difference

L’Tanya has **6** more pencils than Vickie. If Vickie has **4** pencils, how many does L’Tanya have?

Greater amount

Vickie has **6** fewer pencils than L’Tanya. L’Tanya has **10** pencils. How many pencils does Vickie have?

Lesser amount

Total

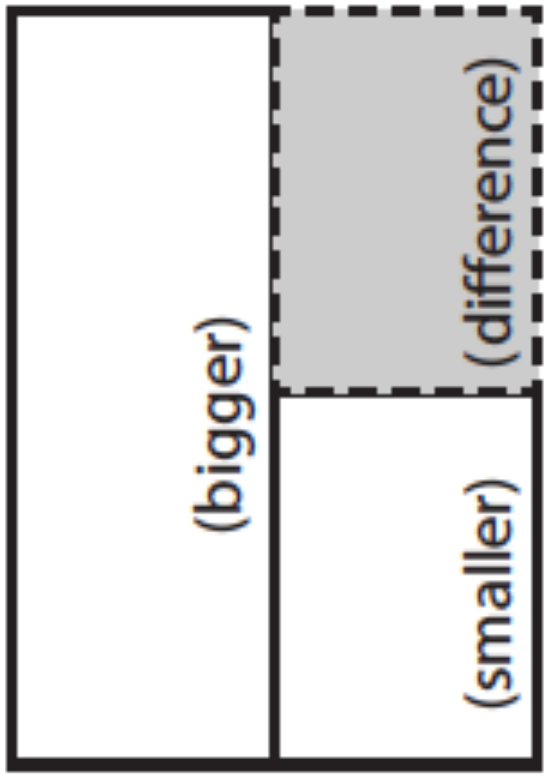
"Are parts put together for a total?"

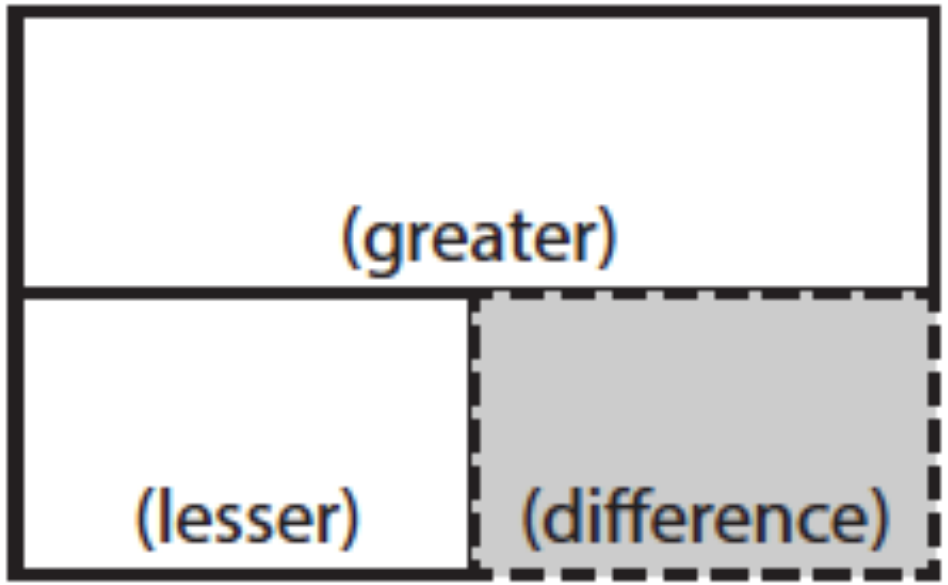
Difference

"Are amounts compared for a difference?"

Difference

G – L = D





Difference

WORD PROBLEMS: DIFFERENCE

D.
There are two lions at a zoo. The weight of the younger lion is 379 pounds. The weight of the older lion is 514 pounds. What is the difference in pounds between these two weights?

E.
The frequency table shows the number of movies watched last month by each student in Mr. Westley's class.

Movies Watched Last Month

Number of Movies	Number of Students
1	𝍸 ||
2	𝍸 𝍸 ||
3	||||
4	||

What is the difference between the number of students who watched 2 movies last month and the number of students who watched one movie last month?

F.
Samantha, Gordon, and Diego each brought an ice chest to a picnic.
- The weight of Samantha's ice chest was 83 pounds.
- The weight of Gordon's ice chest was 28 pounds.
- The weight of Diego's ice chest was 37 pounds.

What was the difference in pounds between the weight of Samantha's ice chest and the combined weight of Gordon's and Diego's ice chests?

Your DIFFERENCE problem:

MATH

#CEC2024

CEC 2024

Change

Join

An amount that **increases** or decreases

Annette had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Annette have now?

End amount

Annette had 6 notebooks. Then, she bought a few more notebooks. Now, Annette has 9 notebooks. How many notebooks did she buy?

Change amount

Annette had some notebooks. Then, she bought 3 notebooks. Now, Annette has 9 notebooks. How many notebooks did she have to start with?

Start amount

Change

Separate

An amount that increases or **decreases**

Jenny baked **9** cookies. Then, she ate **3** of the cookies. How many cookies does Jenny have now?

End amount

Jenny baked **9** cookies. Then, she ate some of the cookies. Now, she has **6** cookies. How many cookies did Jenny eat?

Change amount

Jenny baked some cookies. She ate **3** of the cookies and has **6** cookies left. How many cookies did Jenny bake?

Start amount

Total

"Are parts put together for a total?"

Difference

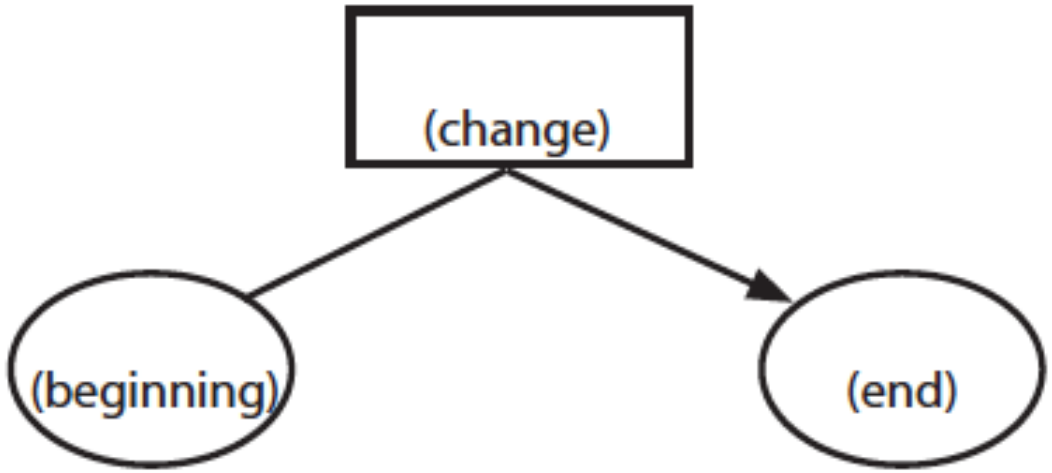
"Are amounts compared for a difference?"

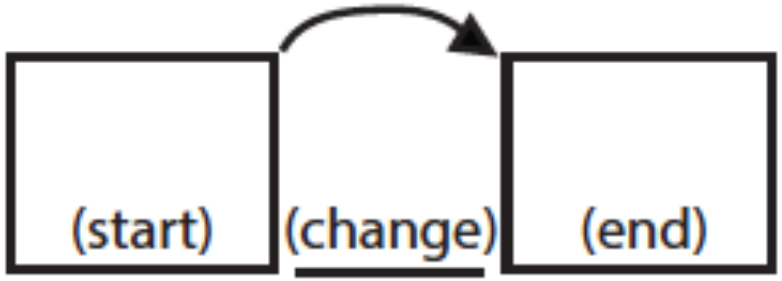
Change

"Does an amount increase or decrease?"

Change

ST +/– C = E





Change

WORD PROBLEMS: CHANGE

G. Landon had one string that was 10 meters long. He used 6.275 meters of this string for a project. What was the length of string in meters that Landon had left?	H. A parade began at 11:30 a.m. and ended at 2:18 p.m. How long did the parade last?
I. Ms. Fitzgerald had gallons of fruit punch. She served gallon of the fruit punch to her family at lunch. How many gallons of fruit punch did Ms. Fitzgerald have left after lunch?	Your CHANGE problem:

MATH

Total

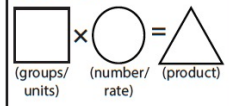
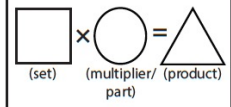
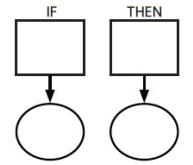
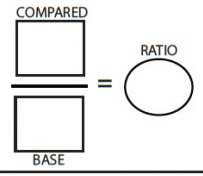
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units	GR x N = P  	Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	S x T = P  	Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Ratios/ Proportions (Percentages; Unit Rate) Relationships among quantities Ratio	  	Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.

Equal Groups

Array
Vary

Groups multiplied by **number in each group** for a **product**

Evan has **2** boxes. There are **6** muffins in each box. How many muffins does Evan have?

Product

Evan has **12** muffins. They want to place them equally into **2** boxes. How many muffins will Evan place in each box?

Number in each group

Evan has **12** muffins. They put them into boxes with **6** muffins each. How many boxes did Evan use?

Groups

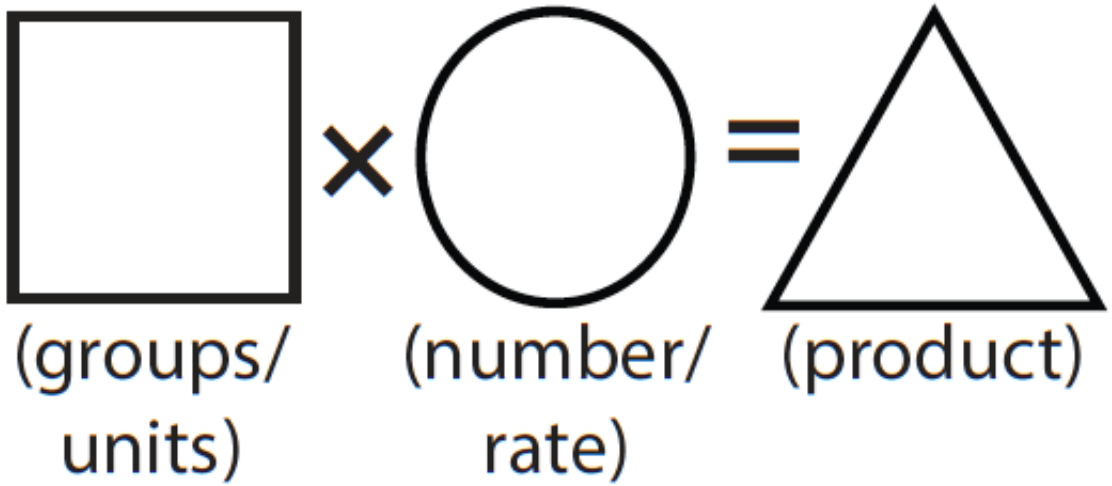
Equal Groups

"Are there groups with an equal number in each group?"

Equal Groups

GR × N = P

GR × E = P



Equal Groups

WORD PROBLEMS: EQUAL GROUPS

J. There are 4 erasers on each table in a classroom. There are 5 tables in the classroom. What is the total number of erasers on all of the tables in this classroom?	K. Carmine has 291 balloons. She put the same number of balloons into 3 groups. What is the best estimate of the number of balloons in each group? A. 90 B. 100 C. 75 D. 85
L. Kelsi spends $6.75 every Saturday for breakfast. What is the total amount of money Kelsi spends on breakfast for 14 Saturdays?	Your EQUAL GROUPS problem:

MATH

Comparison

Set multiplied by a number of **times** for a **product**

Joan ran **6** minutes. L'Tanya ran **4** times longer than Joan. How many minutes did L'Tanya run?

Set

Number of times

Product

Equal Groups

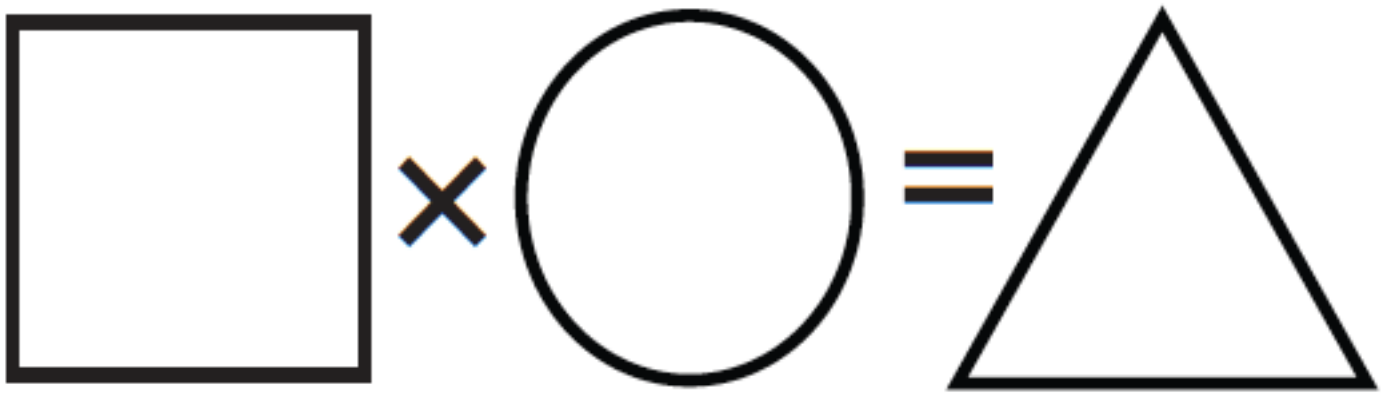
"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

Comparison

S × T = P



Comparison

WORD PROBLEMS: COMPARISON

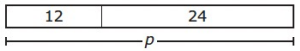
M.
Jonathan and Elizabeth are comparing the masses of their rocks.
•Jonathan's rock has a mass of 0.2 kilogram.
•Elizabeth's rock has a mass 8 times the mass of Jonathan's rock.

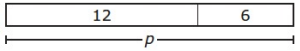
What is the mass of Elizabeth's rock in kilograms??

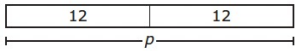
N.
Haruko did 9 sit-ups in P.E. class. The number of sit-ups Tom did can be represented by this expression. 2×9

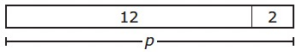
Which statement is true?
F. Tom did 2 times as many sit-ups as Haruko.
G. Haruko did 2 times as many sit-ups as Tom.
H. Tom did 2 more sit-ups than Haruko.
J. Haruko did 2 more sit-ups than Tom.

O.
Erin has 12 pictures from a field trip and some pictures from a vacation. She has twice as many pictures from the vacation as from the field trip. Which strip diagram represents p, the total number of pictures Erin has?

F 

G 

H 

J 

Your COMPARISON problem:

#CEC2024

Ratio/Proportion

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Equal Groups

"Are there groups with an equal number in each group?"

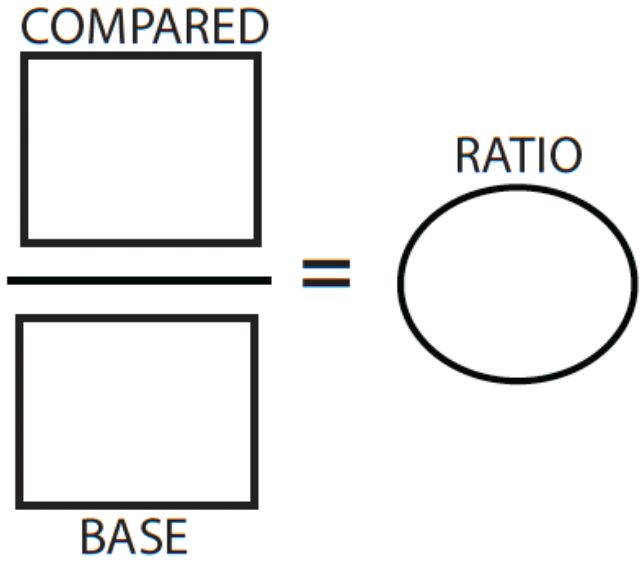
Comparison

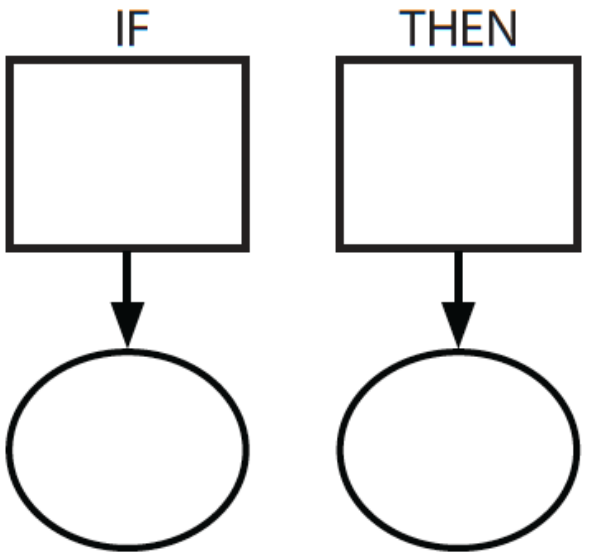
"Is a set compared a number of times?"

Ratio/Proportion

"Are there relationships among quantities - if this, then this?"

Ratio/Proportion





Ratio/Proportion

WORD PROBLEMS: RATIO or PROPORTION

P. Dennis made an extra $245.00 for selling furniture. The extra $245.00 was 7% of the total value of the furniture he sold. What was the total value of the furniture Dennis sold?	Q. A baseball traveled 330 feet in 5 seconds. Which rate is equivalent to the rate at which the baseball traveled? A. 55 feet per second B. 66 feet per second C. 55 seconds per foot D. 66 seconds per foot
R. During a 90-minute school play, the main character was on stage 80% of the time. What amount of time in minutes was the main character on stage?	Your RATIO or PROPORTION problem

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

WORD PROBLEMS

What are your strengths?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

What are your strengths with word-problem solving?

What are your opportunities for growth?

What are your plans for next Monday?
Next month?
Next year?

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

Model and Practice

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Resources


Pirate Math Equation Quest
About
Research
Individual
Small Group
STAAR
Videos
Welcome to Pirate Math Equation Quest!
Individual Word-Problem Intervention
Small-Group Word-Problem Intervention
Small-Group Word-Problem Intervention for STAAR
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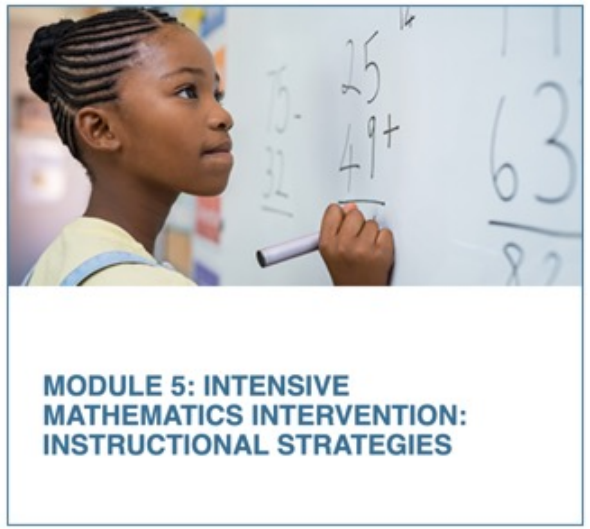
https://intensiveintervention.org



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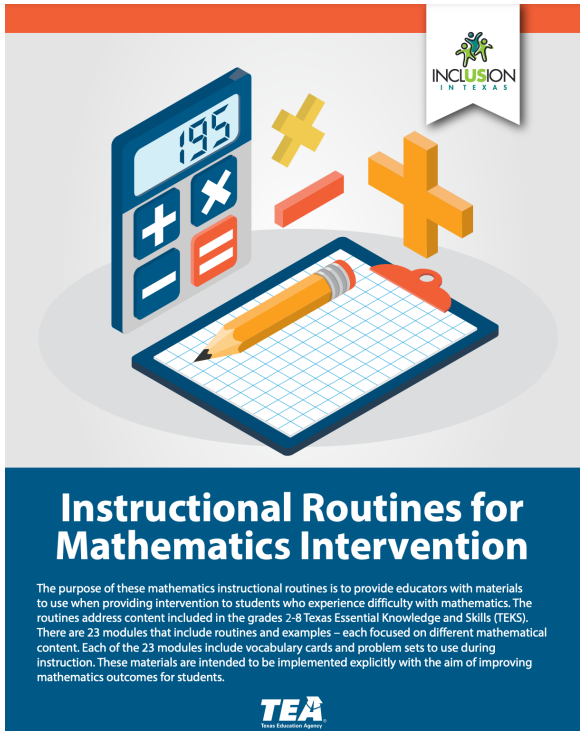
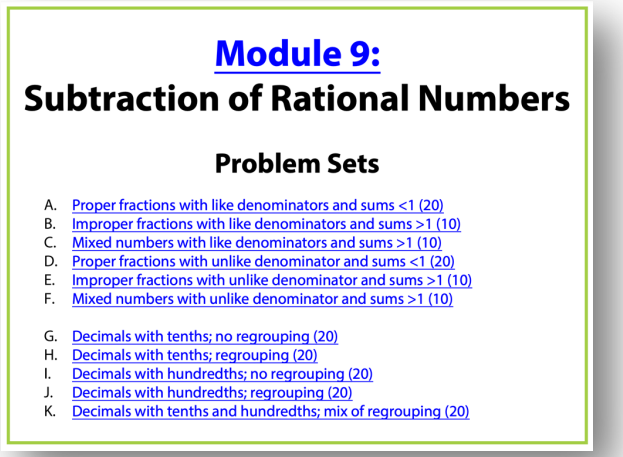


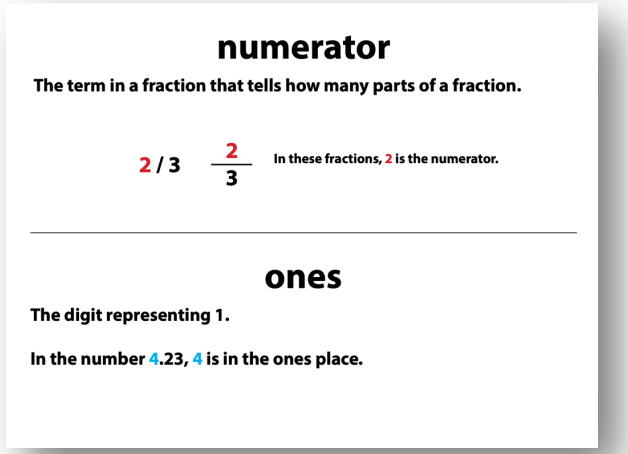


Module 9:
Subtraction of Rational Numbers

Problem Sets

A. Proper fractions with like denominators and sums <1 (20)
B. Improper fractions with like denominators and sums >1 (10)
C. Mixed numbers with like denominators and sums >1 (10)
D. Proper fractions with unlike denominator and sums <1 (20)
E. Improper fractions with unlike denominator and sums >1 (10)
F. Mixed numbers with unlike denominator and sums >1 (10)

G. Decimals with tenths; no regrouping (20)
H. Decimals with tenths; regrouping (20)
I. Decimals with hundredths; no regrouping (20)
J. Decimals with hundredths; regrouping (20)
K. Decimals with tenths and hundredths; mix of regrouping (20)



numerator

The term in a fraction that tells how many parts of a fraction.

2 / 3 $\frac{2}{3}$ **In these fractions, 2 is the numerator.**

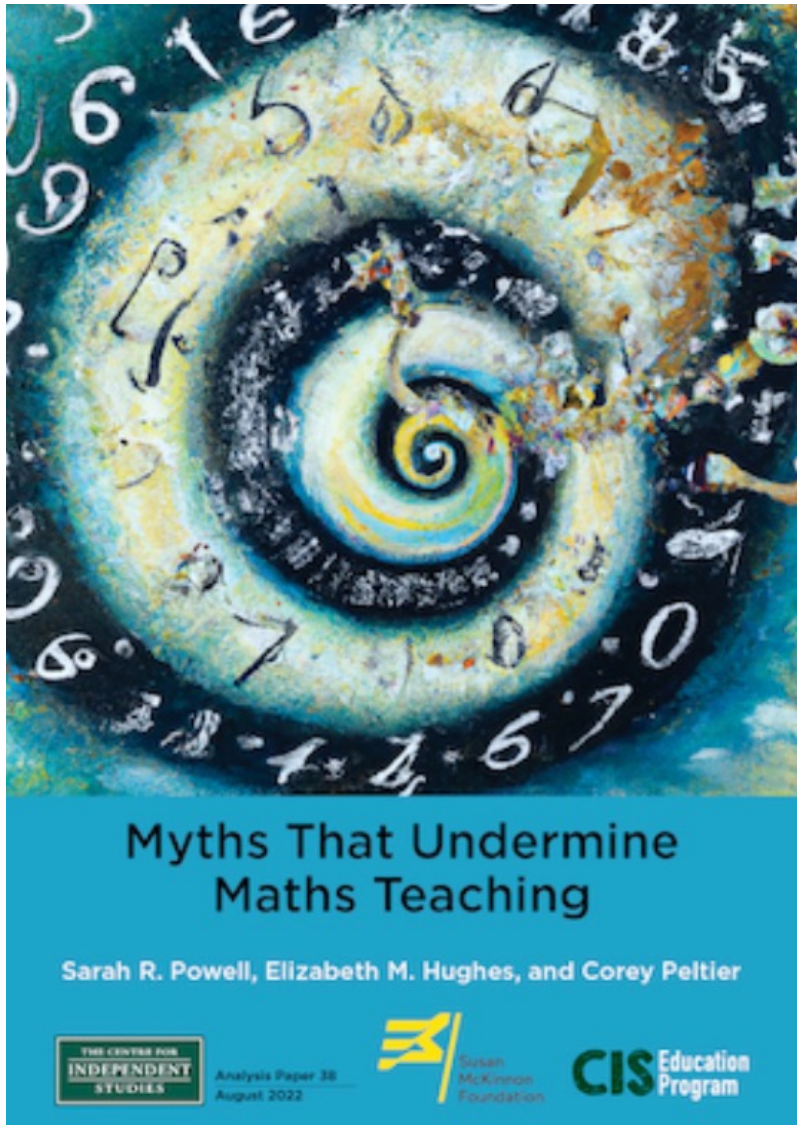
---

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

https://spedsupportstage.tea.texas.gov/resource-library/instructional-routines-mathematics-intervention



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Analysis Paper 62 | February 2024

MATHS PRACTICES YOU CAN COUNT ON

A Guide to Five Research-Validated Practices in Mathematics

Sarah R. Powell, Sarah G. King, and Sarah A. Benz

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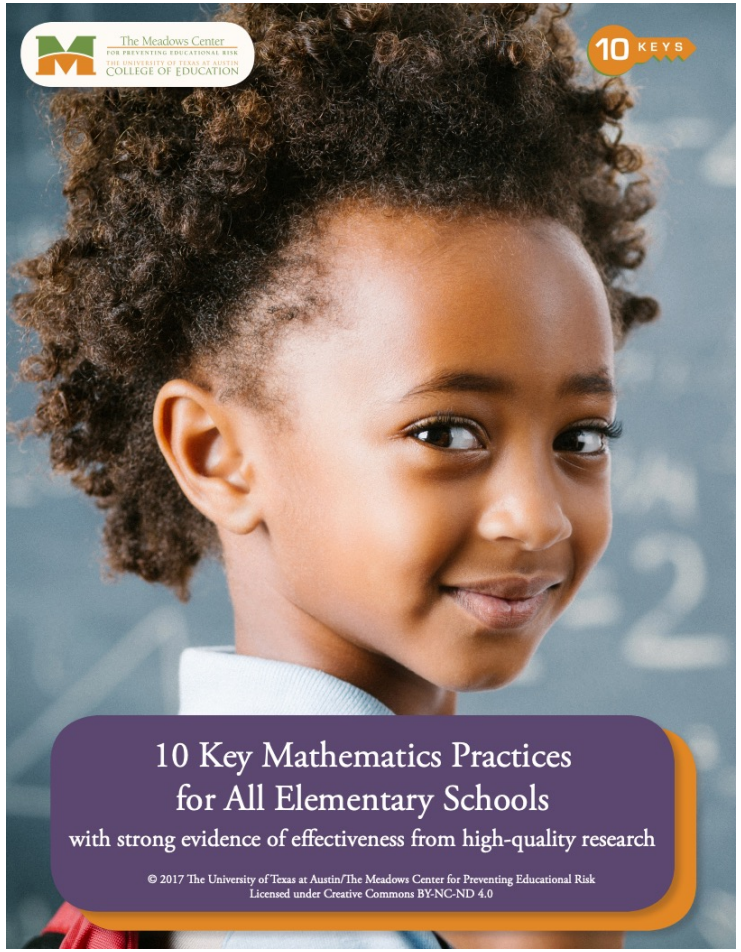

https://meadowscenter.org/resource/10-key-mathematics-practices-for-all-elementary-schools-with-strong-evidence-of-effectiveness-from-high-quality-research/

https://meadowscenter.org/resource/10-key-math-practices-for-all-middle-and-high-schools-with-strong-evidence-of-effectiveness-from-high-quality-research/

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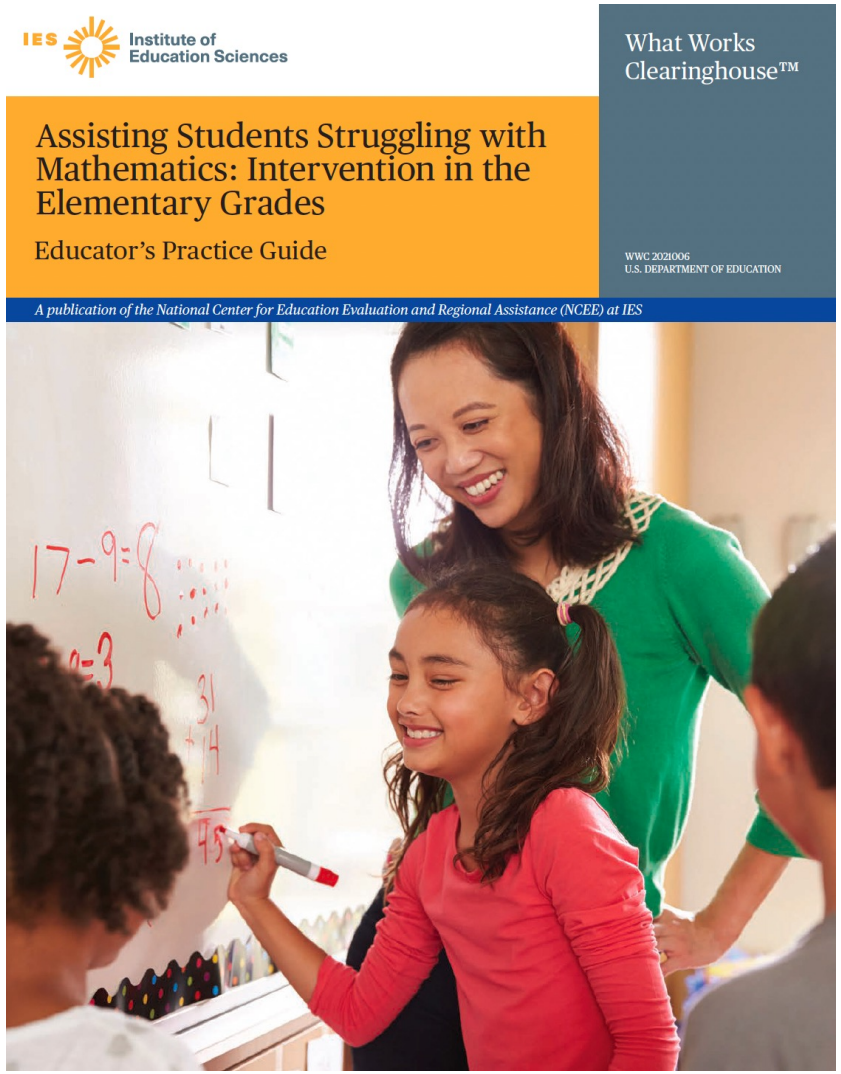
https://ies.ed.gov/ncee/wwc/PracticeGuide/26



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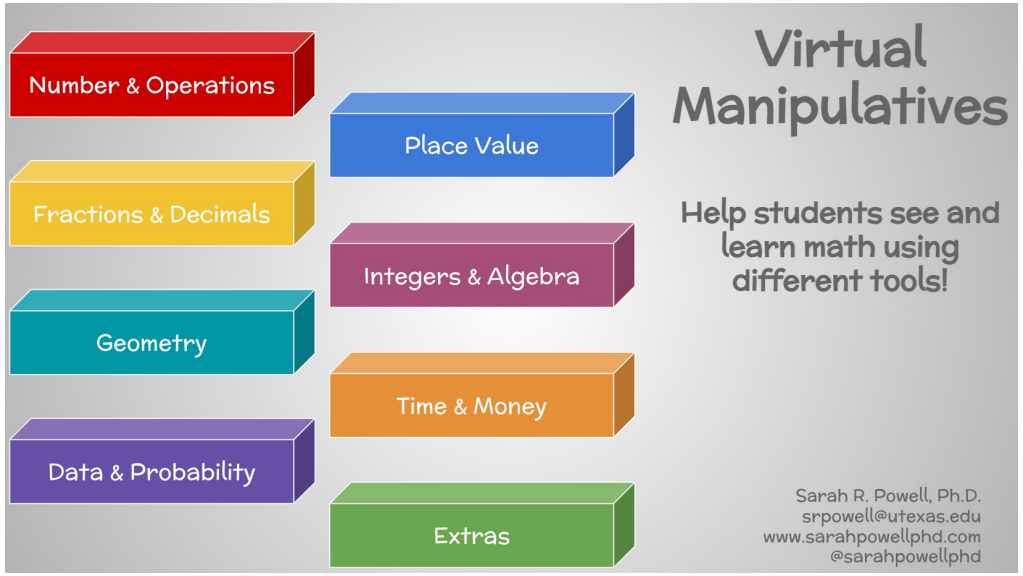
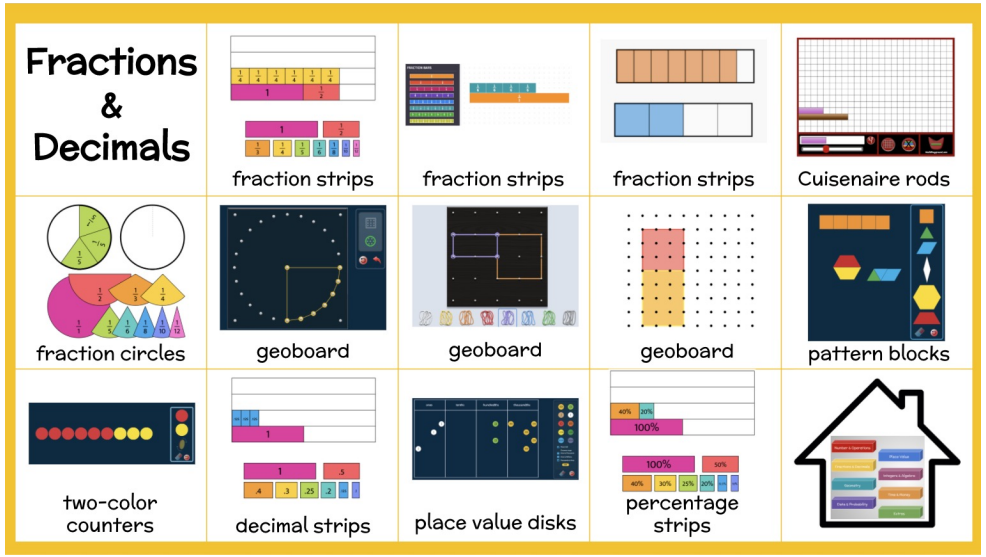
bit.ly/srpowell





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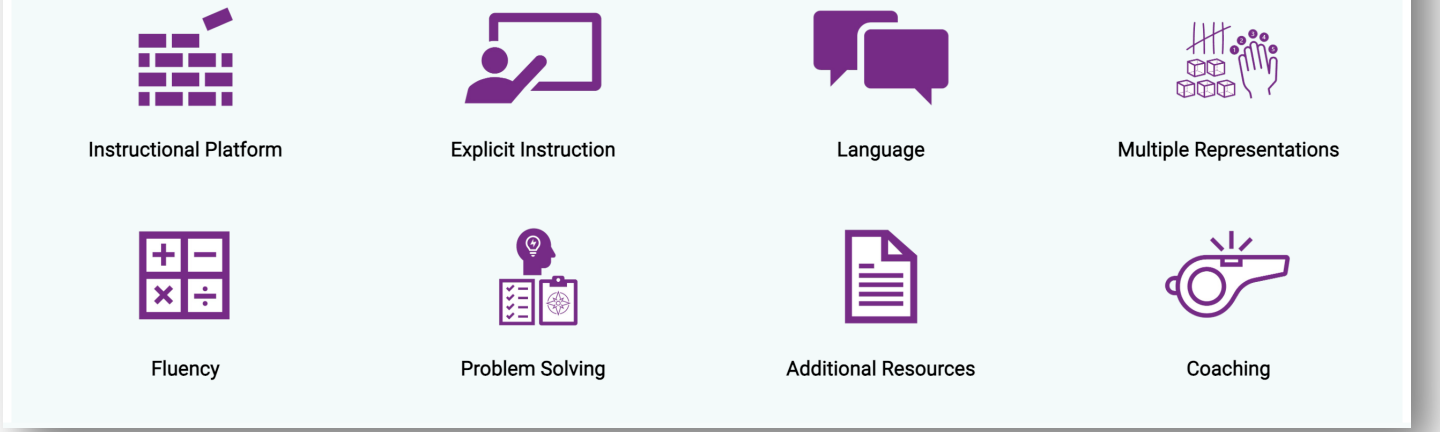
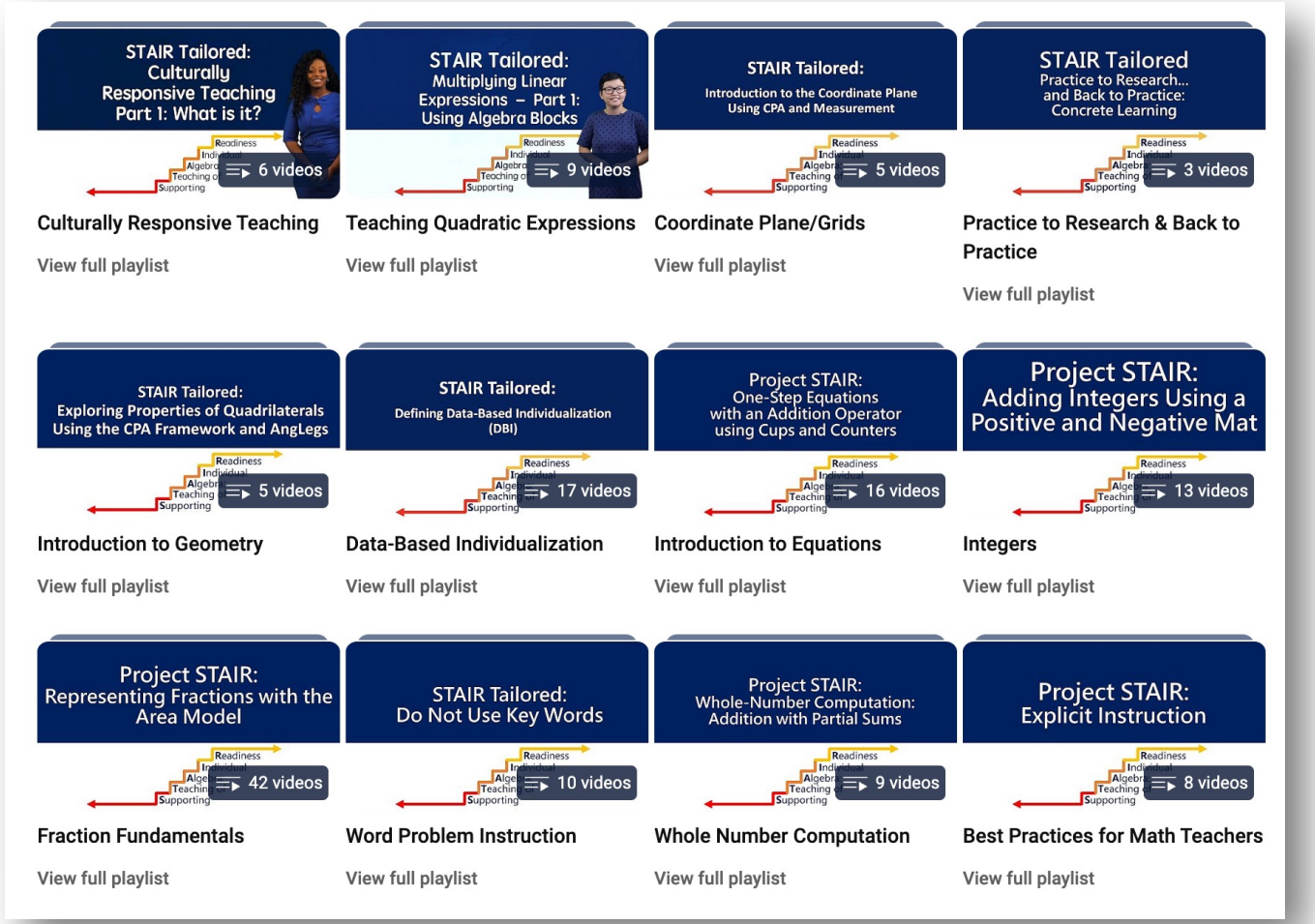
Resources



https://www.mathspiral.com



https://www.youtube.com/channel/UCE2puwDtUSNXFONIOhmYmvA

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