Significance Computer Oriented Numerical Analysis in Solving Problems using Interpolation and Approximation techniques and related applications

Mausumi Goswami
Computer Science and Engineering department, Christ (Deemed to be University), Bangalore
Sneha B. Antony
Computer Science and Engineering department, Christ (Deemed to be University), Bangalore
B.S Purkayastha
Computer Science department, Assam University

***Abstract*—**Interpolation is a very important concept in numerical techniques. Most of the time functions may not be available clearly. After that, find the value of the function at any given point which is not present in the table is called as interpolation. The purpose is to categorize the chosen techniques with their applications to know where these methods are used. This paper is a task of examination on the genuine constant significance of Numerical Methods in the interpolation and approximation.

***Keywords*—** *Numerical methods, solutions, equations, various categories applications, interpolation, approximation*

I. INTRODUCTION

Numerical analysis is the learning of algorithms that employ numerical approximation for mathematical analysis problems. A numerical technique is a mathematical tool intended to resolve numerical problems. The execution of a numerical technique with a suitable convergence check in an encoding language is called a numerical algorithm. In this section we mainly discuss about interpolation and approximation. Interpolation is said to be a technique of building fresh points of data inside the range of a discrete collection of acknowledged points of data. Numerical techniques for ordinary differential equations are techniques used for the solutions of ordinary differential equations to find numerical approximations. These are also well-known as "numerical integration", even though this word is occasionally taken to refer the computation of integrals.

The main categories of Numerical Method in the area of Interpolation and approximation are Lagrangian Polynomials, Divided differences, Newton's backward difference, Newton's forward interpolation.

In next section various methods are discussed . In this paper it is attempted to consolidate the various applications of these methods in engineering and Science-technology.

II. BACKGROUND

In this section we discuss mainly about Interpolation and approximation. The first method discussed is the Lagrangian Polynomials. Lagrange's polynomial is used for polynomial interpolation. For given distinct points X and Y, Lagrange polynomial is the polynomial of the lowest degree that assumes every value of Y with respect to X. The interpolation polynomial of the smallest degree is distinctive but since it can be obtained through multiple methods, it cannot be referred as "Lagrange polynomial" but can be referred as "Lagrange form" of that unique polynomial.

$$Y = f(X) = \sum_{i=0}^{n-1} T_i Y_i$$

Divided differences: This method was used in the past to create logarithmic tables and trigonometric functions.
A table is created to identify the unknown depending on its position, the values of X may be near to the first or last value, if unknown is closer to the first value, forward difference is used else backward difference is used.

$$f[x_0] = f(x_0)$$
$$f[x_0, x_1] = \frac{f(x_0)}{(x_0 - x_1)} + \frac{f(x_1)}{(x_1 - x_0)}$$
$$f[x_0, x_1, x_2] = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{f(x_2)}{(x_2 - x_0)(x_1 - x_1)}$$
$$f[x_0, x_1, x_2] = \sum_{j=0}^{n} \frac{f(x_j)}{\prod_{k\in\{0,\ldots,n\}\{j\}} x_j - x_k}$$

So,
$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \cdots + (x - x_0)(x - x_1) \ldots (x - x_n) f(x_0, x_1, x_2, \ldots, x_n)$$

Here, x is the given point for which $\boldsymbol{f(x)}$ has to be determined.

Newton's forward interpolation: This method is used for finding an unknown y value for a known set of equally distributed values of x and their corresponding y values. In this method, the unknown y is close to the first known value of y. Here we can first create a difference table.

i	x_i	f_i	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0	x_0	f_0				
			Δf_0			
1	x_1	f_1		$\Delta^2 f_0$		
			Δf_1		$\Delta^3 f_0$	
2	x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$
			Δf_2		$\Delta^3 f_1$	
3	x_3	f_3		$\Delta^2 f_2$		
			Δf_3			
4	x_4	f_4				

$\Delta f \;= f(x_1) - f(x_0)$
$\Delta^2 f = \Delta f_1 - \Delta f_0$
$\Delta^3 f = \Delta^2 f_1 - \Delta^2 f_0$

$\Delta^n f = \Delta^{n-1} f_1 - \Delta^{n-1} f_0$

Using these values of delta, we can get the value,

$$f(x) = f(x_0) + u\Delta f_0 + \frac{u(u-1)}{2!}\Delta^2 f_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 f_0 + \cdots + \frac{u(u-1)\ldots(u-(n-1))}{(n-1)!}\Delta^{(n-1)} f_0$$

Here,
$f(x) = y$
x is the point at which the value of y is to be obtained. Also x is close to x_0 (First given value of x)

$u = \frac{x - x_0}{h}$
$h = x_i - x_{i-1}$
This is used for 'n' number of points.

Newton's backward difference: Similar to the previous method, this method is also used for finding an unknown y value for a known set of equally distributed values of x and their corresponding y values. In backward difference, the unknown y is close to the last known value of y.

For backward interpolation, we can find the difference table first.

i	x_i	f_i	∇f	$\nabla^2 f$	$\nabla^3 f$	$\nabla^4 f$
0	x_0	f_0				
			∇f_0			
1	x_1	f_1		$\nabla^2 f_0$		
			∇f_1		$\nabla^3 f_0$	
2	x_2	f_2		$\nabla^2 f_1$		$\nabla^4 f_0$
			∇f_2		$\nabla^3 f_1$	
3	x_3	f_3		$\nabla^2 f_2$		
			∇f_3			
4	x_4	f_4				

$\nabla f \;= f(x_1) - f(x_0)$
$\nabla^2 f = \nabla f_1 - \nabla f_0$
$\nabla^3 f = \nabla^2 f_1 - \nabla^2 f_0$

$\nabla^n f = \nabla^{n-1} f_1 - \nabla^{n-1} f_0$

Using these values of delta, we can get the value,

$$f(x) = f(x_n) + v\nabla f_n + \frac{v(v+1)}{2!}\nabla^2 f_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 f_n + \cdots + \frac{v(v+1)\ldots(v+(n-1))}{(n-1)!}\nabla^{(n-1)} f_n$$

Here,
$f(x) = y$
x is the point at which the value of y is to be obtained. Also x is close to x_n(last given value of x)

$v = \frac{x - x_n}{h}$
$h = x_i - x_{i-1}$
This is used for 'n' number of points.

III. SIGNIFICANCE OF NUMERICAL METHOD IN THE SOLUTION OF EQUATIONS, INTERPOLATION AND APPROXIMATION

In this section significance of the above mentioned methods are discussed.

3.1 Significance of the Lagrangian Polynomials method is described below:

1. The pseudospectral Legendre technique meant for discretizing optimal control problems.
2. Advanced order interpolatory vector bases intended for computational electromagnetics.
3. Direct trajectory optimization by means of a pseudospectral method.
4. Coefficient Symmetries used for applying Arbitrary Order Lagrange Type Variable Fractional Delay Digital Filters.
5. Survey: interpolation techniques in medical image processing.
6. Multiresolution representations by means of the self corrected functions of densely supported wavelets.
7. A motion vector recovery algorithm meant for digital video by means of Lagrange interpolation.
8. Size reduction via interpolation in fuzzy rule bases.
9. Lagrange recreation based technique intended for the QoS routing problem.
10. IE-FFT Algorithm used for a Non conformal Equation for Volume Integral which is meant for Electromagnetic dispersion From Dielectric substances.

3.2 Significance of the Divided differences method is described below:

1. Ability of reproducing kernel spaces in learning theory.
2. Fixed Element Solution of Saturable Magnetic Field Problems.
3. Divided Difference Filtering based on huber.
4. Comparison of direct drive as well as concepts based on geared generator for wind turbines.
5. High-speed digital signal processing with control.
6. Architecture as well as algorithms used for an IEEE 802.11 based multi channel wireless mesh network.
7. Local Gabor binary pattern histogram sequence: A non steady model intended for facial depiction and identification.
8. A new well organized algorithm used to solve differential algebraic systems by means of implicit Newton’s backward differentiation formulas.
9. Divided Difference Methods used for Galois Switching Functions.
10. Analytic explanation of short channel consequences in completely exhausted double gate as well as surrounding gate MOSFETs.

3.3 Significance of the Newton’s forward interpolation technique is described below:

1. A customized Newton’s technique used for distribution of radial system with power flow analysis.
2. Training feedforward networks by means of the Marquardt algorithm.
3. Pairing of saturated systems which are electromagnetic in nature towards nonlinear power electronic devices.
4. Recursive linear smoothed Newton predictors used for polynomial extrapolation.
5. Algorithm baased on Time Domain Quasi Synchronous Sampling used for Analysis of Harmonics Based on Newton's Interpolation.
6. A learning on the execution of three phase current injection method Full Newton versus Forward and Backward Power Flow techniques used for allocation of larger Systems.
7. Nonlinear Microwave Imaging used for Breast Cancer Screening with the help of Gauss Newton's technique as well as the Algorithm based on CGLS Inversion .
8. General Relativity used for the Experimentalist.
9. Continuous Newton's technique meant for Power Flow Analysis.
10. Three phase flow of power calculations by means of current injection technique.

3.4 Significance of the Newton’s backward difference method is described below:

1. Optical Fiber Delay-Line Signal Processing.
2. FIR prediction by means of algorithm based on Newton's backward interpolation by means of smootshed successive differences.
3. Well-organized algorithms based on matrix values used to solve differential equations based on stiff Riccati .
4. Fast simulation algorithms used for RF circuits.
5. Latest Algorithm used for current transformer logged detection with reimbursement founded on Newton’s backward difference formulae and derivatives of secondary currents.
6. Accelerating relaxation algorithms used for circuit simulation by means of waveform Newton along with step size refinement.
7. Polynomial predictive filtering in control instrumentation.
8. Exact fractional-order differentiators used for polynomial signals.
9. Reat ime integration as well as differentiation of analog signals by means of digital filtering.
10. Efficient Minimization Method used for a Generalized Total Variation Functional.

IV. MERITS AND DEMERITS OF DIFFERENT METHODS

Lagrangian Polynomials	Simpler to calculate rather than non-polynomial estimation. The higher order problem are more precise.	Due to its rigidity, they have a tendency to over-fit the information.
Divided differences	Helps in the interpolation of data in unequal intervals. Structured calculation of delta values makes it easier to remember.	Accuracy decreases when the differences of dependent variables are not small. Accuracy depends on the number of dependent variables. Thus, this method increases time and complexity for more accuracy
Newton’s backward difference	This technique is appropriate and meant for the explanation of rigid differential equations. This method is broadly used in circuit replication.	This method possibly will involve the result of nonlinear equations. If the step-size h is constant, it is difficult to manage.

V. RESULTS OF APPLYING COMPUTER ORIENTED NUMERICAL METHOD IN THE SOLUTION OF EQUATIONS, INTERPOLATION AND APPROXIMATION

In this section few results obtained after applying the methods are shown.

	Method I	Method II	ERROR
x	f(x)= sinx	f(x)=sinx	
0	0	0	0
0.25	0	0.247403959	0.24740396
0.5	0	0.479425538	0.47942554
0.75	0.1	0.68163876	0.58163876
1	0.5	0.841470984	0.34147098
1.25	0.99	0.948984619	0.04101538
1.5	1	0.997494986	0.00250501
1.75	1	0.983985946	0.01601405
2	1	0.909297426	0.09070257

Figure 1 Results of Lagrange interpolation. This table includes the results of interpolation using Lagrange. It exhibits computation of y using Lagrange interpolation.

Method II	ERROR	percentage error
f(x)=sinx	Absolute Error	Relative Error
0	0	0%
0.247403959	0.24740396	100
0.479425538	0.47942554	100
0.68163876	0.58163876	85.32947275
0.841470984	0.34147098	40.58024465
0.948984619	0.04101538	4.322028005
0.997494986	0.00250501	0.251130485
0.983985946	0.01601405	1.627467757
0.909297426	0.09070257	9.975017129

Figure 2 Calculation of percentage of error in Lagrange method

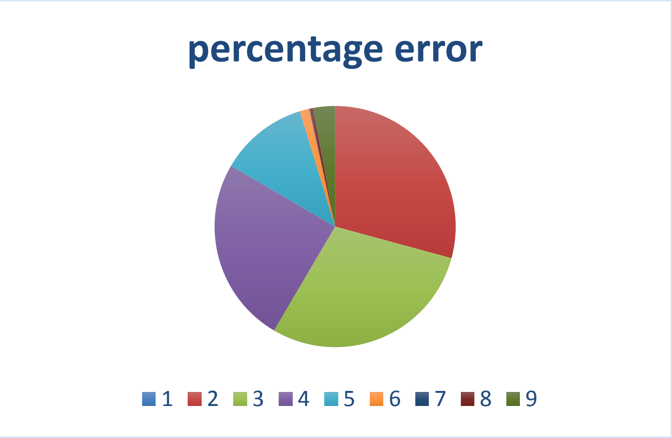

Figure 3 Pie chart to demonstrate the different errors in percentage

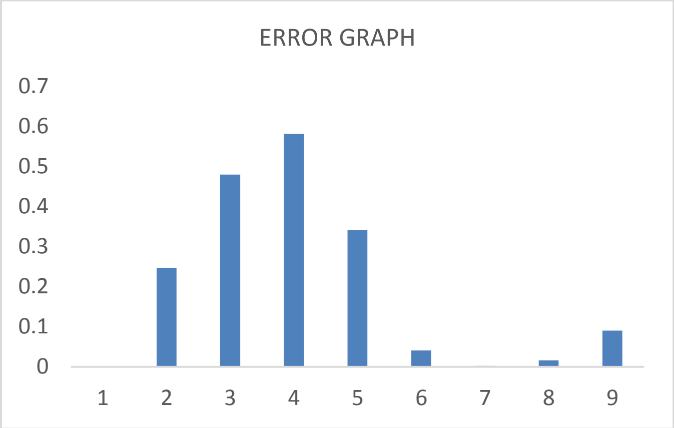

Figure 4 Error Analysis in Lagrange method

VI. CONCLUSION:

With the increase of digital data and other data it is found that data Analysis is one ways while finding a solution for an existing problem. In this paper, we have tried to present the merits, de-merits, and the areas that obtain such significant data by applying the afore mentioned methods, each of which presents a different characteristic in terms of accuracy, condition of use, time taken or the number of steps involved.

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