Fall 2021 Math 4315/5315 - PDEs Home Work 1

1. Calculate the first order derivatives u_x and u_y for the following change of coordinates

$$
\begin{aligned}
&(i)\ \ r = 2x - y, \quad s = x + y,\\
&(ii)\ \ r = x\,e^{y}, \qquad s = x\,e^{-y},
\end{aligned}
$$

2. Using the chain rule $u_r = u_x x_r + u_y y_r$, find u_x and u_y in terms of r for the following:

$$
\begin{aligned}
&(i)\quad x = r, \quad y = -r, \quad u = 1\,,\\
&(ii)\quad x = r, \quad y = \ 1\,, \quad u = r^2,
\end{aligned}
$$

3. Solve the following first order ordinary differential equations

$$
(i) \qquad xy' = 3y + x^2 \quad (ii) \qquad xy' + y = x^2 y^2
$$

$$
(iii)\ \ \frac{dy}{dx} = \frac{y^2 - 3x^2 y}{x^3 - 2xy} \quad (iv)\ \ x^2 y' = x^2 y^2 + xy - 3
$$

4. Solve the following systems of ODEs

$$
\begin{aligned}
&(i) \quad \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}\\
&(ii) \quad \frac{dx}{y} = \frac{dy}{x} = \frac{dz}{z}\\
&(iii) \quad \frac{dx}{y} = \frac{dy}{x - z} = \frac{dz}{y}
\end{aligned}
$$

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5. Consider the following system of ODEs

$$
\frac{dx}{P(x,y,u)} = \frac{dy}{Q(x,y,u)} = \frac{du}{R(x,y,u)}.
$$

Show that for some real constants a, b and c

$$
\frac{dx}{P(x,y,u)} = \frac{dy}{Q(x,y,u)} = \frac{du}{R(x,y,u)} = \frac{d\,(ax + by + cu)}{aP + bQ + cR}.
$$

Due: Friday Sept. 3, 2021