Math 3331 - ODE

Model's

1. Population Growth

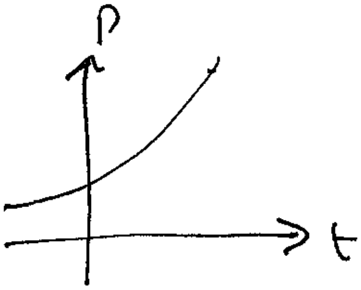
Let $P = P(t)$ be the size of a population & $\frac{dP}{dt}$ the rate of change.

One model is

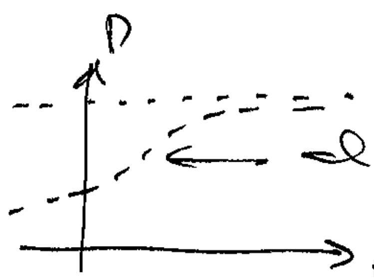
$$\frac{\frac{dP}{dt}}{P} = k$$ — larger populations change faster than smaller ones (k const > 0)

Sol^n (to be shown later)

$$P(t) = P_0 e^{kt}, \quad P_0 \text{ - initial population}$$



← suggest given enough time population grows unbounded.



← expect this — so new model!

suppose there's a max, say 1 Million people
b/c land, food etc

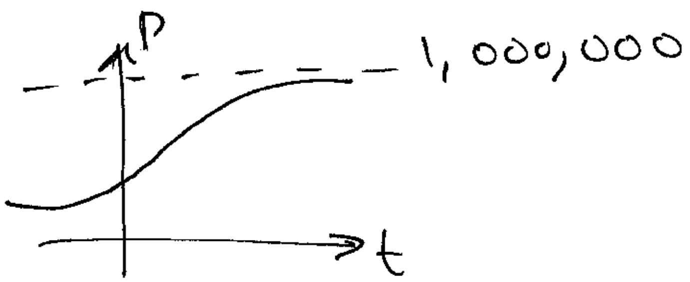
so P - population size
1,000,000 - P - available spots

<u>New Model</u>

$\frac{dP}{dt} \propto P$ & $\frac{dP}{dt} \propto 1,000,000 - P$

model $\frac{dP}{dt} = kP(1,000,000 - P)$ $P(0) = P_0$

Does this new form give a sol^n like?



<u>Cooling</u>

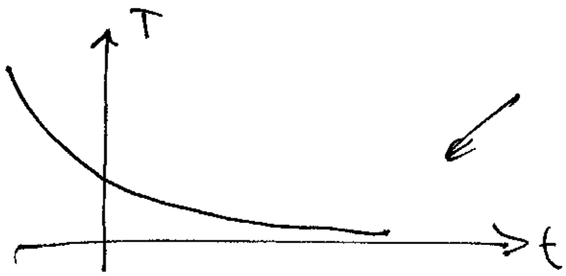
Suppose we have a cake in the oven (400°F)
and we bring it into a room at 70°F.
so what happens?. The cake cools to
room temp.

Model I

Let $T = T(t)$ temp of cold

$$\frac{dT}{dt} = kT \qquad (k < 0 \text{ cooling})$$

$$T = T_0 e^{-kt}$$



predicts $T \to 0$ as $t \to \infty$

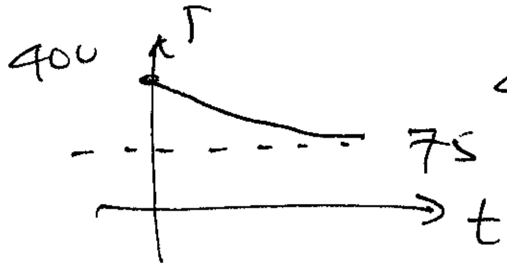
what about room temp?

New Model

$$\frac{dT}{dt} = k(T - T_\infty)$$

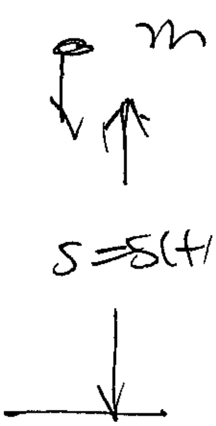
T_∞ – ~~75~~ 70°F room temp

$T(0) = 400$



will this be the temp curve?

3. Falling Bodies



$$F = ma = m\frac{d^2s}{dt^2} = -mg$$

$$\frac{d^2s}{dt^2} = -g$$

v_0 - initial velocity

$$s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

s_0 - init. position

$$v(t) = -gt + v_0$$

If we're high enough $v(t)$ will be very large. Prob. - no air resistance

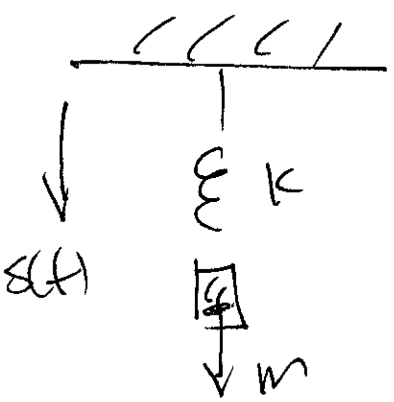
Model $\frac{dv}{dt} = a$ so $\frac{dv}{dt} = -g$

$$\frac{dv}{dt} = -g + kv, \quad \frac{dv}{dt} = -g + kv^2$$

If we solve these the will

$$\lim_{t\to\infty} v(t) \to v_{max}$$ terminal velocity

4. Spring-Mass System



$$F = ma = m\frac{d^2 s}{dt^2}$$

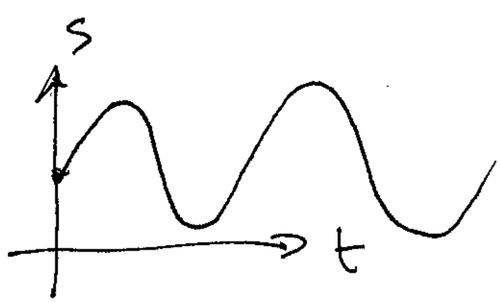
restoring Force - Hooke's Law

$$F = -kx$$

so $m\frac{d^2 s}{dt^2} = -kx$ or $\frac{d^2 s}{dt^2} + w^2 \cancel{s} = 0$

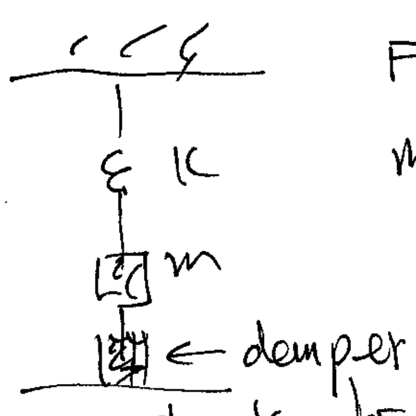
when $w^2 = \frac{k}{m}$

Soln $s(t) = c_1 \sin wt + c_2 \cos wt$ (will show later)



oscillitory

not good if this is your car.



λ - constant

$F = F\downarrow - F\uparrow$

$m\frac{d^2 s}{dt^2} = -kx - \lambda \frac{ds}{dt}$ depends on velocity

will this predict