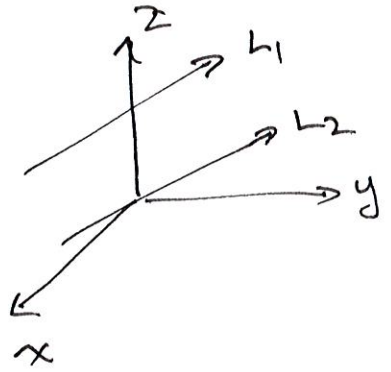
Math 1497 Calc II

Lines & Planes - Intersection

2 Lines



2 lines parallel

$x = 1+t$ $\quad x = 4-s$

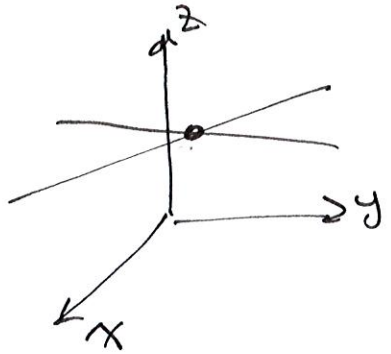
$y = 2-t$ $\quad y = -3+s$

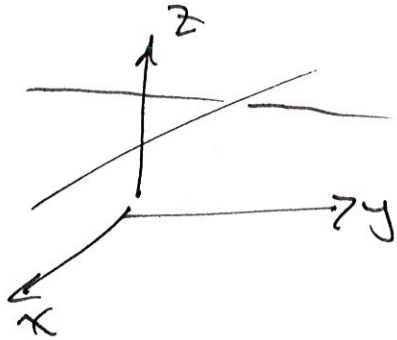
$z = 3+2t$ $\quad z = 7-2s$

L1: $\vec{u} = \langle 1, -1, 2 \rangle$ $\quad$ L2: $\vec{v} = \langle -1, 1, -2 \rangle$

$\vec{u} = -\vec{v}$ so the point in the same direction

If not parallel they could intersect a be skewed





Ex $x = 1+t$ $\quad y = 4-s$

$y = 2-t$ $\quad y = -3+2s$

$z = 3+2t$ $\quad z = 2-s$

equate 2 and solve for t & s and consider if the remaining are the same

for example, equate x & y

$1+t = 4-s$ ← $t = 3-s$

$2-t = -3+2s$ $\quad 2-3+s = -3+2s$

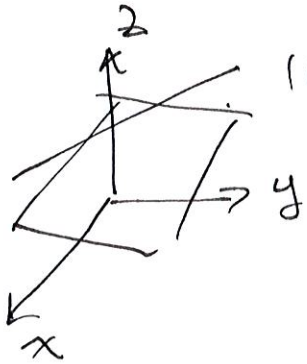
$s = 2 \quad t = 1$

Now consider z:

$L_1 \; z = 3+2 = 5 \qquad L_2 = 7-2 = 5$ ✓

So they intersect

A Line & a Plane



(i) line doesn't intersect plane
line is parallel to a vector in the plane.

(ii) line actually lies on the plane

(iii) line intersects plane at a point

Ex $x+2y+3z = 6$ $\quad \vec{n} = \langle 1, 2, 3 \rangle$

$\vec{u} = \langle 1, -2, 1 \rangle$

$x = 1+t$

$y = 1-2t$ $\quad \vec{n} \cdot \vec{u} = 1-4+3 = 0$

$z = 1+t$ $\quad$ so $\vec{n} \perp \vec{u}$ so parallel to plane

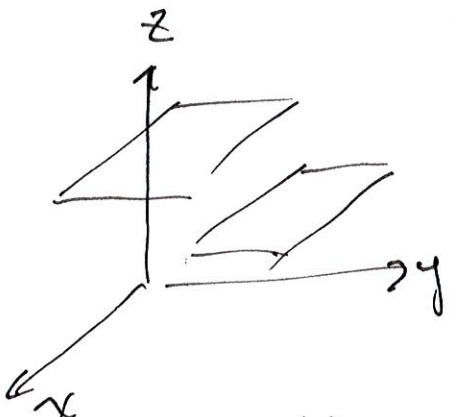
substitute line into eq^n of plane so...

$$(1+t) + 2(1-2t) + 3(1+t) \stackrel{?}{=} 6$$

$$1+t + 2-4t + 3+3t \stackrel{?}{=} 6$$

$$6 \stackrel{?}{=} 6 \checkmark \quad \text{so line lies on plane.}$$

2 Planes



if the planes are parallel

then $\vec{n}_1 \parallel \vec{n}_2$

or $\vec{n}_1 \times \vec{n}_2 = \vec{0}$

otherwise they will intersect

ex

$P_1: x+2y+3z=6$

$P_2: 4x+y+z=3$

$z = 3-4x-y$

sub into P_1

$$x+2y+3(3-4x-y) = 6$$

$$x+2y+9-12x-3y = 6$$

$$-11x+y = 3$$

let $x=t$, $y = 3-11t$

so $z = 3-4t-3+11t = 7t$