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Business Mathematics – Vol 1

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Content

1. Set Theory

Sets - Definition

A Group | well – defined (unique / distinct / condition - based) objects

Representation:

Rule/Set-Builder / Selection / Property / Algebra Method	Defines the rule	A:$\{x: x\ is\ a\ vowel\ in\ English\ Alphabet\}$ A is a set of all x, such that x is a vowel in English Alphabet" A- Set of Vowels
Listing / Tabular / Roster Method	Lists the elements	A = $\{a, e, i, o, u\}$

Types / Terms

Sl. No	Types/Terms	Notation	Examples / Remarks
1	Elements / Member / Object -	"ϵ" ϵ - Epsilon symbol Read as " belongs to"	Let A = $\{a, e, i, o, u\}$ Then, a ϵ A [Read as: "a belongs to A"] Also b∉A [Read as: "b does not belongs to A"] A = {A}Type equation here. Here A(element) ϵ A(set)
2	Cardinal Number - The number of elements in a set n is Positive(+ve) or Zero(0), But not negative(–ve)	'n'	A = $\{a, e, i, o, u\}$ n(A) = 5 Read as "n of A = 5"
3	n=0: Empty / void / null set	{ } , ϕ	A – A month having 32 days A – Number of stupid students in my class
4	n=1: singleton set		A = {1}
5	n – finite Number, Finite set	defines an end	A = {a,e,i,o,u}

<table>
<tr><td>6</td><td>n – Infinite Number,

Infinite set</td><td>defines NO end</td><td>N - – set of natural numbers</td></tr>
<tr><td>7</td><td>Subset - A set contained in a set

Number of subsets = 2^n</td><td>$A_i \subseteq$ A</td><td rowspan="4">Consider A= {1,2,3} ⟹n(A) =3

Vary n ≤ 3

n=o: ϕ

n=1: A_1 ={1}, A_2 = {2}, A_3 ={3}

n=2, A_4 = {1,2}, A_5 = {2,3},

A_6 = {1,3}

n=3, A_7 = {1,2,3}

Here,

Subsets - ϕ, A_1, A_2, A_3, ….. A_6, A_7

Number of subsets, $2^n = 2^3 = 8$

Proper subsets –ϕ, A_1, A_2 ….A_6

Number of proper subset= 2^n-1 = 7

Note: A_7 =A itself, hence not a proper subset

Superset – A

P(A) = {ϕ, A_1 , A_2, , A_4, A_5, A_6 &A_7 }

Therefore, n(P(A)) = 2^n</td></tr>
<tr><td>8</td><td>Proper Subset - A set properly contained in a set

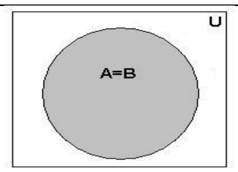
Note: ϕ has no proper set

Number of Proper subsets = $2^n - 1$

Not Equal and Equivalent to Superset</td><td>$A_i \subset$A</td></tr>
<tr><td>9</td><td>Superset: A set contains a set</td><td>$A_i \subseteq$ A (A - super set)</td></tr>
<tr><td>10</td><td>Power set: Set of all subsets

Note: Power Set is never a null set</td><td></td></tr>
<tr><td>11</td><td>Universal Set ∪ - The whole set (i.e. The set which contains all)</td><td>The whole set</td><td>A - Set of English alphabet</td></tr>
<tr><td>12</td><td>Complimentary Set - - The set which has elements not in A</td><td>A'/$\bar{A}$/A^c</td><td>U - Set of English alphabet

A – Set of vowels

A' – Set of Consonants</td></tr>
<tr><td>13</td><td>Equal sets : The two sets containing same elements</td><td>
</td><td rowspan="2">Let A= {1,2,3}, n(A) =3,

B = {2,4,5}, n(B) = 3, C={1,2,3}, n(c) = 3

Here,

A and C are Equal Sets

A and B, B and C are</td></tr>
<tr><td>14</td><td>Equivalent Sets: The two sets containing same number of Elements</td><td>Equal set ⟹ ($correct$) Equivalent set?

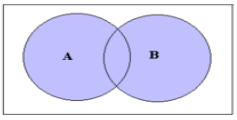
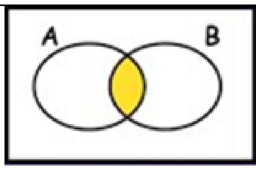
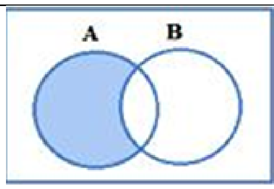
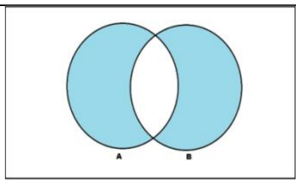
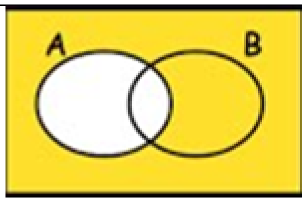
Equivalent set</td></tr>
</table>

		⟹ (wrong) Equal Set?	Equivalent Sets
15	Disjoint set: There exists no common elements in A and B	A ∩ B = ϕ	Let A = {1,2,3}, B= {a,c,d} Then, A ∩ B = ϕ and n(A ∩ B) =0
16	Ordered / Product set: A$\times B$ ={a$\times b: a \in A \& b \in B$}	Read as "A cross B"	Let A = {1,2}, B= {a, c, d} A $\times B$ = {(1,a),(1,c),(1,d),(2,a),(2,c),(2,d)} n(A) = 2 and n(B) =3 n(A×B) = n(A) ×n(B) = = 2×3 = 6

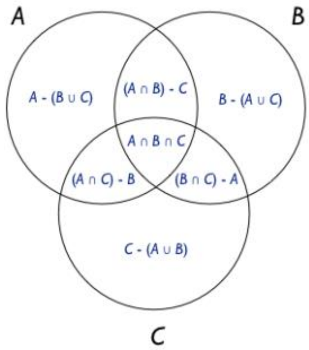
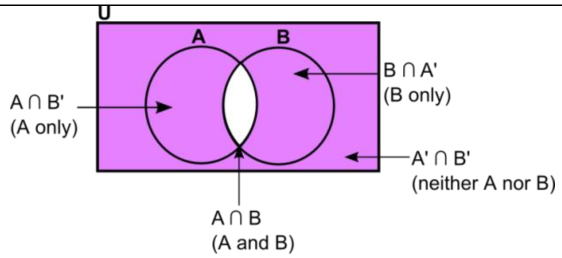
Points to Ponder:

∅ and {} - null set {∅} = singletion set with null set as the only element ∅ is a subset of a Power set.	What is the power set of a null set? For a null set, n(∅) =0 p (∅) = {∅} - Singleton Set

Operations on Sets / (& - X/or -+)

Sl. No	Operations	Definitions	Venn Diagram	Example **A ={1,2}** **B = {2,3}**
1	Union A∪B / Either A or B	A∪B = $\{x: x \in A \text{ or } x \in A\}$		A∪B = {1,2,3}
2	Intersection A∩B Both A & B	A∩B = $\{x: x \in A \,\&\, x \in B\}$		A∩B = {2}
3	Difference A-B: only A, but not B B –A: only B, but not	A – B = $\{x: x \in A \,\&\, x \notin B\}$		A – B = {1} B – A = {3}
4	Symmetric Difference AΔB = (A-B)∪(B-A) ⟹ only A, but not B or only B, but not A	A△B = $\{x: x \,\&\, A - B \text{ or } x. B - A\}$		A△B = {1,3}
5	Complement Set A′	A′ = $\{x: x \notin A\}$		**Let S ={1,2,3,4}** A′ = {3,4}

Points to Ponder:

1	$A - B \neq B - A$	
2	A – B in terms of other operations	$A - B = A - (A \cap B)$ / $(A \cup B) - B$ / $A \cap B'$ / $(A \Delta B) - (B - A)$ / $U - (A - B)'$
3	A – B with 3 sets	"only A" implies only A, but not B and C [Notation: $(A - B) \cap (A - C)$] "only A, but not B" implies the set may contain elements in C also [Notation: $A - B$] Neither A nor B $\Longrightarrow (A \cup B)' = A' \cap B' = U - (A \cup B)$
Venn Diagram with different operations		
4		

Addition Theorem on Sets

On 2 Sets	**On 3 Sets**
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$	$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
Points to Ponder	
If $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$	$A \cap B \cap C = \phi \Longrightarrow A \cap B = \phi, B \cap C = \phi, A \cap C = \phi$, Hence, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

Other Laws

De morgans law	$(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$
Commutative (2 sets 1 operation)	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative: (3sets 1 operation)	$(A \cup B) \cup C = A \cup (B \cup C)$

	$(A \cap B) \cap C$ = A∩ $(B \cap C)$
Distributive: 3sets 2 operation	$(A \cup B) \cap$ C = $(A \cap C) \cup (B \cap C)$ $(A \cap B) \cup C$ = $(A \cup C) \cap (B \cup C)$
Identity	A ∪A = A A∩ A = A

Simple Problems

1. Which of the following statements are correct /incorrect?
   (a) 3 ∈ (1, 3, 5); (b) 3 ∉ (1, 5); (c) (3) ⊆ (1, 3, 5); (d) (3) ∈ (1, 3, 5)

2. Fill up the blanks by appropriate symbol ∈, ∉, , ⊆, ⊂, =

(i) 3 : …..(3, 4) ∪ (4, 5, 6) (ii) (6) ….. (5, 6) ∩ (6, 7, 8)	(iii) {3, 4, 5}…{2, 3, 4}∪{3,4,5} (iv) (a, b)…..(a)	(v) 4….(3, 5) ∪ (5, 6, 7) (vi) (1, 2, 2, 3) ….(3, 2, 1)

3. A∪A is equal to
   a. A
   b. E
   c. ∅
   d. None of these

4. A∩A is equal to
   a. ∅
   b. A
   c. E
   d. None of these

5. (A∪B)′ is equal to
   a. (A∩ B)′
   b. A∪B′
   c. A′ ∩ B′
   d. None of these

6. (A∩ B)′ is equal to
   a. (A′∪ B)′
   b. A′∪B′
   c. A′∩B′
   d. None of these

7. A∪E is equal to (E is a superset of A)
   a. A

b. E

c. ∅

d. None of these

8. A∩E is equal to

a. A

b. E

c. ∅

d. None of these

9. E∪E is equal to

a. E

b. ∅

c. 2E

d. None of these

10. A∩E′ is equal to

a. E

b. ∅

c. A

d. None of these

11. A∩ ∅ is equal to

a. A

b. E

c. ∅

d. None these

12. A∪A′ is equal to

a. E

b. ∅

c. A

d. None of these

13. The set $\{ 2^x : x\ is\ a\ positive\ rational\ number\}$

a. An infinite set

b. A null set

c. A finite set,

d. None of these

14. $\{1-(-1)^x\}$ for all integral x is the set

a. {0}

b. {2}

c. {0,2}

d. None of these

15. E is a set of positive even number
and O is a set of positive odd
numbers, then E∪O is a
   a. Set of whole numbers
   b. N
   c. A set of rational number
   d. None of these
16. If R is the set of positive rational number and E is the
set of real numbers then
   a. R⊆E
   b. R⊂E
   c. E⊂R
   d. None of these

17. If N is the set of natural numbers
and I is the set of positive integers, then
   a. N= 1
   b. N⊂1
   c. N⊆I
   d. None of these

18. If I is the set of isosceles triangles
and E is the set of
equilateral triangles, then
   a. I⊂E
   b. E⊂I
   c. E=I
   d. None of these

19. If R is the set of isosceles right angled triangles and I is set of isosceles triangles, then
   a. R=I
   b. R⊃I
   c. R⊂1
   d. None of these

20. $\left\{\frac{n(n+1)}{2} : n \text{ } is \text{ } a \text{ } positive \text{ } integer\right\}$is
   a. A finite set
   b. An infinite set
   c. Is an empty set
   d. None of these

Word Problems

Question 1: A survey shows that 40% of the Indians like grapes, whereas 68% like bananas. What percentage of the Indians like both grapes and bananas?

Question 2: In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like

a. Maths only b. Science only c. Either Maths or Science d. Neither Maths nor Science

Question 3: In a class of 50 students appearing for an examination of ICWA, from a centre, 20 failed in Accounts, 21 failed in Mathematics and 27 failed in Costing, 10 failed both in Accounts and Costing, 13 failed both in Mathematics and Costing and 7 failed both in Accounts and Mathematics. If 4 failed in all the three, find the number of (i) Failures in Accounts only (ii) Students who passed in all the three subjects.

Sahasri Singar Academy – Business Mathematics

Chapter 2 Permutations and Combinations

Introduction

Arrangement – The sequence / order of things is taken on account

Grouping – Sequence/order is not necessary

Why Permutations & Combinations?

Illustration

The manager of a large bank has a difficult task of filling two important positions from a group of five equally qualified employees. Since none of them has had actual experience, he decides to allow each of them to work for one month in each of the positions before he makes the decision. How long can the bank operate before the positions are filled by permanent appointments?

Answer to above – cited situation requires an efficient counting of the possible ways in which the desired outcomes can be obtained. A listing of all possible outcomes may be desirable, but is likely to be very tedious and subject to errors of duplication or omission. We need to devise certain techniques which will help us to cope with such problems. The techniques of permutation and combination will help in tackling problems such as above.

Permutations: A group of persons want themselves to be photographed. They approach the photographer and request him to take as many different photographs as possible with persons standing in different positions amongst themselves. The photographer wants to calculate how many films does he need to exhaust all possibilities? How can he calculate the number?

Combinations: There are situations in which order is not important. For example, consider selection of 5 clerks from 20 applicants.

Fundamental Principles of counting

Rule	Explanation	Example
Multiplication Rule AND⇒m×n multiply	If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways then total number of ways of doing both things simultaneously = m×n.	If one can go to school by 5 different buses and then come back by 4 different buses then total number of ways of going to and coming back from school = 5×4 =20
Addition Rule OR ⇒ Add	If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in (m + n) ways.	If one wants to go school by bus where there are 5 buses or to by auto where there are 4 autos, then total number of ways of going school = 5 + 4 =9.

Factorial notation

Definition – The Factorial n, represents the product of all integers from 1 to n both inclusive

Notation – n!

n! = n$\times (n-1) \times (n-2) \times \ldots\ldots \times 3 \times 2 \times 1$

Justification: 0! = 1

1. As per definition

2. Default case - In the case where no arrangement is needed, there is an arrangement in default, hence,

3. On reverse working

0! = 1 1! = 1 2! =2×1 3! = 3×2×1 4! = 4×3×2× 1	$\frac{1!}{1}$ = 1 $\frac{2!}{2}$ = 2 $\frac{3!}{3}$ = 2! $\frac{4!}{4}$ = 3!

4. Apply r =n in the formula for $^nP_r = \frac{n!}{(n-r)!}$

, we get ^nP_n = n!/(n-n)! = n!/0!

We know that ^nP_n = n! Therefore, by applying this we drive, 0! = n!/n! =1

Permutations and Combinations - on the whole

	Permutations	**Combinations**
Meaning	Arrangement	Grouping
Definition	The ways of arranging or selecting smaller or equal number of persons or objects from a group of persons or collection of objects with due regard being paid to the order of arrangement or selection	The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important
Formulae	$^nP_r = \frac{n!}{(n-r)!}$	$^nC_r = \frac{n!}{r!(n-r)!}$ = $\frac{nPr}{r!}$ ➔ nP_n =n!/nC_n
Repetitions	$\frac{n!}{P!\,q!\,r!}$	No such in combinations
Circular Case	1. Circular / Ring - (n-1) ! 2. circular + no difference in order (clockwise and anti-clockwise) $\frac{(n-1)!}{2}$	No such in combinations

Restricted Case		
Should occur	$(n-p)^P{}_{(r-p)} \times r^P{}_p$	$(n-p)^C{}_{(r-p)}$
Should not occur	$(n-p)^P{}_r$	$(n-p)^C{}_r$
Cases at different r values		
r = 0	$\frac{n!}{[n-0]!} = \frac{n!}{n!} = 1$	$\frac{n!}{(n-0)0!} = 1$ $(Note: nP_r = nC_r)$
r = 1	$\frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$	$\frac{n!}{(n-1)!1!}$ = n $(Note: nP_r = nC_r)$
r = n-1	$\frac{n!}{[n-(n-1)]!} = \frac{n!}{1}$	$\frac{n!}{(n-n+1)(n-1)!}$ $\frac{n! \times n \times (n-1)!}{1(n-1)!}$ = n
r = n	$\frac{n!}{(n-n)!} = \frac{n!}{0!}$ = n!	$\frac{n!}{(n-n)n!} = 1$
Remarks	At, r = n and r= n-1, $nP_n = nP_{n-1}$	$nC_r = nC_{n-r}$ $(Complimentary\ Combinations)$ If we form a group of r things out of n different things, (n-r) remaining things which are not included in this group form another group of rejected things.

Points to ponder	
Relation between Permutations and Combination $(n+\ 1)C_r = nC_r + nC_{r-1}$ and $nP_r = n-1P_r + (n-1P_{r-1})$	
Grouping & Order = Arranging $nC_r . rP_r\ = nP_r$ $\Rightarrow nC_r\ = \frac{nP_r}{rP_r} = \frac{n!/(n-r)!}{r!/(r-1)!} = \frac{n!}{r!(n-r)!}$ (since (r-r)! = 0! = 1)	Prove that ${}^{n}C_{r} = {}^{n-2}C_{r-2} + 2\ {}^{n-2}C_{r-1} + {}^{n-2}C_{r}$ RHS = ${}^{n-2}C_{r-2} + 2\ {}^{n-2}C_{r-1} + {}^{n-2}C_{r}$ = ${}^{n-2}C_{r-2} + {}^{n-2}C_{r-1} + {}^{n-2}C_{r-1} + {}^{n-2}C_{r}$ (Refer Note 1 and 2) = ${}^{n-1}C_{r-1} + {}^{n-1}C_{r}$ (Refer Note 3) = (n-1) + 1C_r = ${}^{n}C_{r}$ = L.H.S
Note	
${}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}$ Take n = n-2 & r = r-1 $\therefore {}^{n-2}C_{(r-1)-1} + {}^{n-2}C_{r-1} = {}^{n-2+1}C_{r-1}$ ${}^{n-2}C_{r-2} + {}^{n-2}C_{r-1} = {}^{n-1}C_{r-1}$	Take n = n-2 & r = r $\therefore {}^{n-2}C_{r-1} + {}^{n-2}C_{r} = {}^{(n-2)+1}C_{r} = {}^{n-1}C_{r}$ Take n = n-1 & r = r $\therefore {}^{n-1}C_{r-1} + {}^{n-1}C_{r} = {}^{(n-1)+1}C_{r} = {}^{n}C_{r}$

Theorem: The number of permutations of n things chosen r at a time is given by

${}^{n}P_{r} = n(n-1)(n-2)\ ...\ (n-r+1)$, where the product has exactly r factors.

Results

Number of permutations of n different things taken all n things at a time is given by ${}^nP_n = n(n-1)(n-2)\ldots(n-n+1)$ $= n(n-1)(n-2)\ldots .2.1 = n!$	${}^nP_r = n.(n-1)((n-2)\ldots\ldots.(n-r+1)$ $= n.(n-1)((n-2)\ldots\ldots.(n-r+1) \times \frac{(n-r)(n-r-1)2.1}{1.2\ldots(n-r-1)(n-r)}$ $= n!/(n-r)!$ Thus, ${}^nP_r = \frac{n!}{(n-r)!}$

Standard Results

I. Permutations when some of the things are alike, taken all at a time, the number of ways p in which n things may be arranged among themselves, taking them all at a time, when n_1 of the things are exactly alike of one kind, n_2 of the things are exactly alike of another kind, n_3 of the things are different is given by, **P** = $\frac{\boldsymbol{n}!}{\boldsymbol{n_1}!\boldsymbol{n_2}!\boldsymbol{n_3}!}$

II. Permutations when each thing may be repeated once, twice ,…… up to r times in any arrangement, $\boldsymbol{n^r}$

III. Combinations of n different things taking some or all of n things at a time.

$\boldsymbol{\sum_{r=1}^{n} nC_r = 2^n - 1}$ **(that is one or more) [No or more =** $\boldsymbol{\sum_{r=0}^{n} nC_r = 2^n}$**]**

IV. Combinations of n things taken some or all at a time when n_1 of the things are alike of one kind, n_2 of the things are alike of another kind n_3 of the things are alike of a third kind.

$\boldsymbol{\{(n_1+1)(n_2+1)(n_3+1)\ldots\ldots) - 1\}}$

V. The notion of Independence in Combinations

Result: The combinations of selecting r_1 things from a set having n_1 objects and r_2 things from a set having n_2 objects where combination of r_1 things, r_2 things are independent is given by ${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$

Note: This result can be extended to more than two sets of objects by a similar reasoning.

$n_1C_{r_1} \times n_2C_{r_2}$

Simple Problems

Permutations

Sl. No	Question	Solution
1	Prove that "CALCUTTA" is twice of "AMERICA" in respect of number of arrangements of letters.	For CALCUTTA, $\frac{8!}{2!2!2!} = \frac{8\times7!}{2\times2\times2} = 7!$ For AMERICA, $\frac{7!}{2!} = \frac{7!}{2!}$ $\therefore$ CALCUTTA = 2 × AMERICA $7! = 2\times\frac{7!}{2!}$ ➔ $7! = 7!$
2	Four travellers arrive in a town where there are six hotels. In how many ways can they take their quarters each at a different hotel?	6 hotels & 4 travellers $6P_4 = \frac{6!}{2!} = \frac{720}{2} = 360s$
3	In how many ways can 8 mangoes of different sizes be distributed amongst 8 boys of different ages so that the largest one is always given to the youngest boy?	$nP_n = 7P_r = 7! \times 1!$
4	How many different odd numbers of 4 digits can be formed with the digits 1, 2, 3, 4, 5, 6, 7 ; the digits in any number being all different?	$4P_1 \times 6P_3 = 4\times\frac{6!}{3!} = 4\times\frac{720}{6}$ = 480 ($4P_1$ is the ways for fixing the odd number in the ones place)
5	How many number lying between 1000 and 2000 can be formed from the digits 1, 2, 4, 7, 8, 9 ; each digit not occurring more than once in the number?	$1P_1 \times 5P_3 = 1\times\frac{5!}{2!} = 1\times\frac{120}{2}$ = 60 ($4P_1$ is the ways for fixing the odd number in the ones place)
6	In how many ways can the colours of a rainbow (VIBGYOR) be arranged, so that the red and the blue colours are i. always together. ii. always separated.	Total: 7! Together: 6! × 2! =720 × 2 = 1440 Seperated = Total - Together = 7! – (6! × 2!) = 5040 – 1440 = 3600
7	In how many ways can 7 papers be arranged so that the best and the worst papers never come together?	7! – (6! × 2!) = 5040 – 1440 = 3600 (similar to Question 6)
8	Show that the number of ways in which 16 different books can be arranged on a shelf so that two particular books shall not be together is 14 (15) !. The value of $\sum_{r=1}^{10} r.$ ^rP_r is? **Note:** $(r+1)!$ - r! = $(r+1)r! - r!$ = r! $(r+1-1)$ = r×r! = r× rP_r	$\sum_{r=1}^{10} r \times rP_r = \sum_{r=1}^{10} r \times r!$ $= \sum_{r=1}^{10}(r+1)! - r! = (2! - 1!) + (3! - 2!) + \ldots\ldots + (10! - 9!) + (11! - 10!)$

		= 11! – 1!
9	i. In how many ways can 5 boys form a ring? ii. In how many ways 5 different beads be strung on a necklace?	i. $(n-1)!$ = $(5-1)!$ = 4! = 24 ii. $\frac{(n-1)!}{2}$ = $\frac{(5-1)!}{2}$ = $\frac{24}{2}$ = 12
10	There are 20 stations on a railway line. How many different kinds of single first-class tickets must be printed so as to enable a passenger to go from one station to another?	$20P_2$ = 19×20 = 380
11	If the letters word 'DAUGHTER' are to be arranged so that vowels occupy the odd places, then number of different words are?	No. of vowels = 3(A, U,E) No. of Odd places = 4 Places Hence, $4P_3$ ways for vowels 5 Consonants in remaining 5 places, i.e. $5P_5$ ∴ $4P_3 \times 5P_5$ = 2880 ways
12	The total number of ways in which six '+' and four '-' signs can be arranged in a line such that no two '- ' signs occur together is?	_ + _ + _ + _ + _ + _ $\frac{6! \times 7P_4}{6! \times 4!} = \frac{6! \times \frac{7!}{4!}}{6! \times 4!}$

Combination

No	Question	Solution
1	In an examination paper, 10 questions are set. In how many different ways can you choose 6 questions to answer. If however no. 1 is made compulsory in how many ways can you select to answer 6 questions in all?	$10C_6$ = 7 $9C_5 \times 1C_1$ = 126
2	Out of 5 ladies and 3 gentlemen, a committee of 6 is to be selected. In how many ways can this be done : (i) when there are 4 ladies and (ii) when there is a majority of ladies?	When there are 4 ladies, $5C_4 \times 3C_2$ = 5×3 = 15 When there are majority of ladies, $(5C_5 \times 3C_1)$+$(5C_4 \times 3C_2)$ =(1×3) + (15) =18(3+15)
3	Out of 16 men, in how many ways a group of 7 men may be selected so that (i) particular 4 men will not come and (ii) particular 4 men will always come?	(i) $12C_7$ (ii) $12C_3$
4	In a meeting after every one had shaken hands with everyone else, it was found that 66 handshakens were exchanged. How many members were present at the meeting?	66 = nC_2
5	A man has 3 friends. In how many ways can be invite one or more of them to dinner?	$2^n - 1 = 7$

6	n point are in space, no three of which are collinear. If the number of straight lines and triangles with the given points only as the vertices, obtained by joining them are equal, find the value of n.	$^nC_3 = {^nC_2}$ $\frac{n\times(n-1)\times(n-2)}{3\times2\times1} = \frac{n\times(n-1)}{2\times1}$ n -2 = 3 ➔ n = 5
7	How many different triangles can be formed by joining the angular points of a decagon? Find also the number of diagonals of the decagon?	$^{10}C_3$ → no.of Triangles $^{10}C_2$ -10 → no .of diagonals
	Note: In general, n points. $^nC_3$ → no.of Triangles and $^nC_2$ – n → no.of Diagonal	

Comprehensive problem

Question 1: Find the number of ways of selecting 4 letters from the word "EXAMINATION".

Answer:

There are 11 letters in the word of which A, I, N are repeated twice. Thus we have 11 letters of 8 different kinds (A,A), (I,I), (N,N), E, X, M, T, O. The group of four selected letters may take any of the following forms:	a. Two alike and other two alike, $^3C_2 = 3$ b. Two alike and other two different, $^3C_1\times{^7C_2}=3\times21 = 63$ c. All four different, $^8C_4 = \frac{8\times7\times6\times5}{1\times2\times3\times4} =70$ Hence, the required number of ways = 3 + 63 +70 = 136 ways

Question 2: The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is?

Answer: Select 2 lines from Set 1 (containing 4 parallel lines), $^4C_2 = 6$

Select 2 lines from Set 2 (containing 3 parallel lines), $^3C_2 = 3$

Hence, 6 × 3 = 18 parallelograms can be formed

Question 3: A boat's crew consist of 8 men, 3 of whom can row only on one side and 2 only on the other. The number of ways in which the crew can be arranged is __________

Answer:

Out of 8 persons, 5 are fixed and two persons are to be choosed from the remaining 3.

Hence, $3C_2$

The 4 persons in both the sides could be arranged in 4! × 4!

Therefore, $3C_2$ × 4! × 4! = $3C_2 \times (4!)^2$ = 1728

Question 4: If all the permutations of the letters of the word 'CHALK' are written in a dictionary the rank of this word will be__________

Answer:

1. A _ _ _ _ = 1 × 4! = 24	3. C H A K L = 1 × 1 × 1 × 1 × 1 = 1
2. C A _ _ _ = 1 × 1 × 3! = 6	4. **C H A L K** = 1 × 1 × 1 × 1 × 1 = 1

Hence, 24 + 6 + 1 +1 = 32, 32^{nd} rank

Question 5: A family of 4 brothers and three sisters is to be arranged for a photograph in one row. In how many ways can they be seated if, 1. All the sisters sit together, 2. No two sisters sit together?

Answer:

1. Consider the sisters as one unit and each brother as one unit. 4 brothers and 3 sisters make 5 units which can be arranged in 5! Ways. Again 3 sisters may be arranged amongst themselves in 3! Ways

Therefore, total number of ways in which all the sisters sit together = 5!×3! = 720 ways.

2. In this case, each sister must sit on each side of the brothers.

4 brothers may be arranged among themselves in 4! Ways. For each of these arrangements 3 sisters can sit in the 5 places in ${}^{5}P_{3}$ ways.

Thus the total number of ways = ${}^{5}P_{3}$ ×4! = 60×24 = 1,440

Question 6: The Supreme Court has given a 6 to 3 decision upholding a lower court; the number of ways it can give a majority decision reversing the lower court is

a. 256 b. 276 c. 245 d.115

Answer:

It can by 5 votes for or 6,7,8 and the whole 9 to get majority so ,

9C9+9C8+9C7+9C6+9C5 =1+9+36+84+126 =256

Question 7: Five bulbs of which three are defective are to be tried in two bulb points in a dark room. Number of trials the room shall be lighted is

a. 6 b. 8 c. 5 d. 7

Answer:

The number of ways in choosing 2 bulbs to fit in the bulb points are C(5,2) = 10

The number of ways it will not light, due to choosing of defective bulbs are C(3,2) = 3

Hence, 10-3 = 7 ways