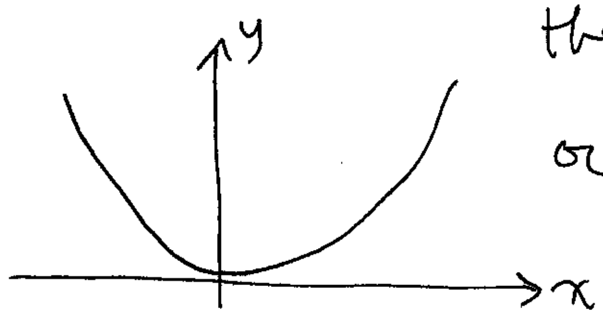
Math 1496 - Calc I

Consider $f(x) = x^2$ when $x > 0$ we see the graph "going up" or "increasing".



If $x < 0$ we see the graph going down or decreasing.

Def^{n} If a function $f(x)$ is increasing, then if $x_1 < x_2$ then $f(x_1) < f(x_2)$

If a function $f(x)$ is decreasing then if $x_1 < x_2$ then $f(x_1) > f(x_2)$

Consider the first example, show if $x > 0$ $f(x)$ is increasing. That is show if

$x_1 < x_2$ then $x_1^2 < x_2^2$

Now if $x_1 < x_2$ then $0 < x_2 - x_1$

Also since $x \geq 0$ then $x_1 + x_2 \geq 0$

so $0 < (x_2 - x_1)(x_2 + x_1)$

$\Rightarrow \quad 0 < x_2^2 - x_1^2$

$\Rightarrow \quad x_1^2 < x_2^2$ so yes it is increasing

similarly for decreasing if $x < 0$

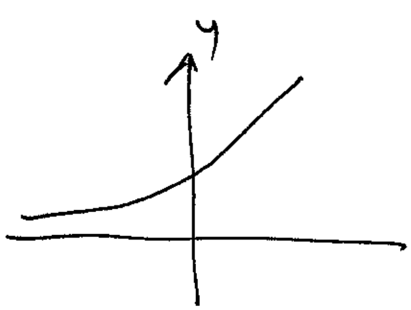
then $x_1 + x_2 < 0$ so if $x_2 - x_1 > 0$

$0 > (x_1 + x_2)(x_2 - x_1)$

$\Rightarrow \quad 0 > x_2^2 - x_1^2 \Rightarrow x_1^2 > x_2^2$

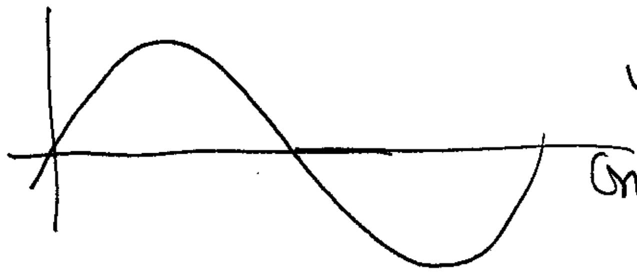
However, this def^n is really hard to use

Suppose we have $f(x) = e^x$



if $x_1 < x_2$ show $e^{x_1} < e^{x_2}$

or what about $f(x) = \sin x$



here we see on some intervals f is increasing and on others decreasing

So we have a better way.

$\underline{\underline{Th^m}}$ <u>Increasing/Decreasing</u>

If $f(x)$ is cont^s on $[a,b]$ and diff^{ble} (a,b)

(i) if $f'(x) > 0$ for all $x \in (a,b)$
f is increasing on (a,b)

(ii) if $f'(x) < 0$ for all $x \in (a,b)$
f is decreasing on (a,b)

(iii) if $f'(x) = 0$ for all $x \in (a,b)$
f is constant on (a,b)

Proof (i) Consider 2 pts x_1 & x_2 in (a,b) and that $x_1 < x_2$. If $f(x_1) < f(x_2)$

then $0 < f(x_2) - f(x_1)$

so $$\frac{f(x_2)-f(x_1)}{x_2-x_1} > 0$$

By MVT there is a c in (a,b) such that

$$f'(c) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$\Rightarrow$ $f'(c) > 0$ true for any c in (a,b) $\boxed{H}$

Ex 1 $f(x) = x^2$

$f'(x) = 2x$

$f'(x) > 0$ when $x > 0$

$f'(x) < 0$ when $x < 0$

} real easy to use

Ex 2 $f(x) = e^x$ $f'(x) = e^x$

$f'(x) > 0$ for all x so f is always increasing

Ex 3 Find values of x where the following is increasing & decreasing

$$y = x^3 - 6x^2 + 9x$$

Sol^n $y' = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-1)(x-3)$

So when is the >0 increasing and <0 decreasing

Sign chart - First find when $y' = 0$ so $x = 1, 3$

x		1		3	
$x-1$	$-$	0	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	0	$+$
$(x-1)(x-3)$	$+$	0	$-$	0	$+$
slope	/	—	\	—	/

increasing

$(-\infty, 1)$ $(3, \infty)$

decreasing

$(1, 3)$

So what happens at $x=1$, $x=3$?

when the derivative changes (1st deriv. test)

$f' > 0 \to f' < 0$ max

& $f' < 0 \to f' > 0$ min

so here we have a relative max at $x=1$

min at $x=3$

Ex 4 $\quad f(x) = \dfrac{x}{x^2-1}$

$$f'(x) = \frac{1(x^2-1) - 2x \cdot x}{(x^2-1)^2} = -\frac{(x^2+1)}{(x^2-1)^2}$$

so $f'(x) < 0$ as long as $x \neq \pm 1$

so decreasing $(-\infty,-1)$ $(-1,1)$ $(1,\infty)$

Ex 5 $\quad f(x) = \dfrac{x^2}{x^2-1}$

$$f'(x) = \frac{2x(x^2-1) - 2x \cdot x^2}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

so $f'(x)$ will change sign $\quad f' = 0$ $x = 0$ $\quad$ DNE $x = \pm 1$

x		-1		0		1	
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$-x$	$+$	$+$	$+$	0	$-$	$-$	$-$
$(x^2-1)^2$	$+$	0	$+$	$+$	$+$	0	$+$
$\dfrac{-2x}{(x^2-1)^2}$	$+$	∞	$+$	0	$-$	∞	$-$
slope	/	|	/	—	\	|	\

increasing $(\infty,-1)$ $(-1,0)$

decreasing $(0,1)$ $(1,\infty)$

max at $x = 0$