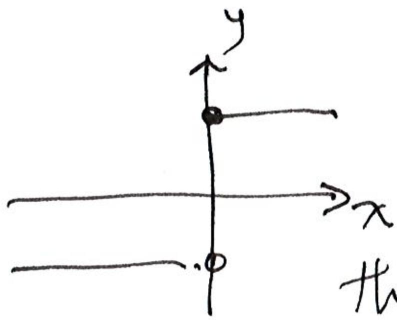
Math 1496 – Calc 1

One sided Limits

Consider $f(x) = \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases}$



As we approach $x = 0$ from the left and right the limits are different

From Left

$$\lim_{x \to 0^-} f(x) = -1$$

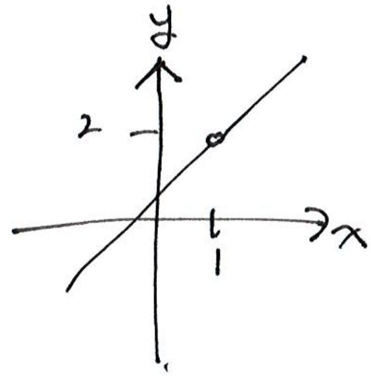
From Right

$$\lim_{x \to 0^+} f(x) = 1$$

As these are ~~different~~ different then

$$\lim_{x \to 0} f(x) \text{ DNE}$$

ex $\displaystyle \lim_{x \to 0} \frac{x^2 - 1}{x - 1}$



$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2$$

so the limit does exist

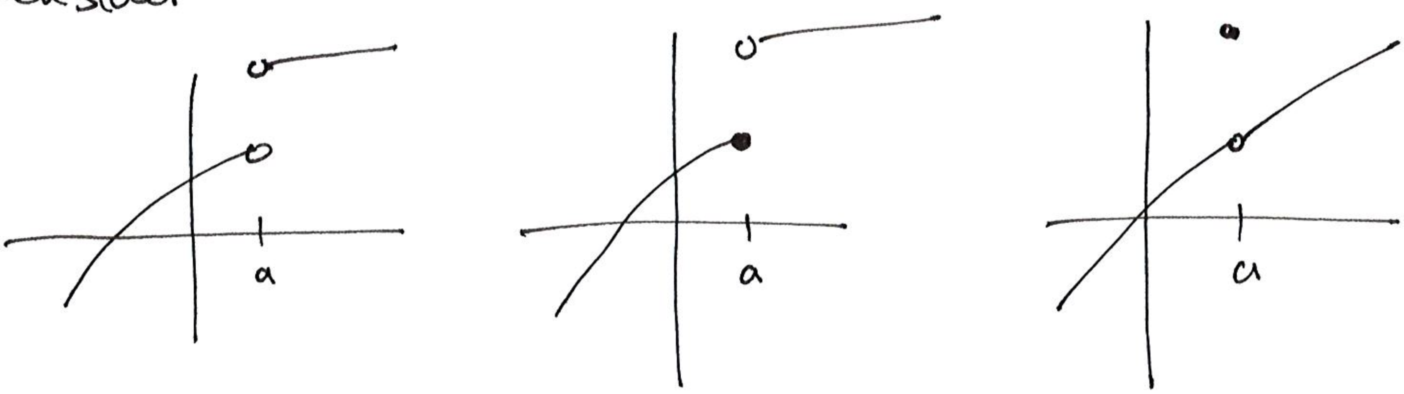
In general

if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$ (they both exist and $=$)

we say the limit exist

and $\lim_{x \to a} f(x) = L$

Consider



$\lim_{x \to a^-} f \neq \lim_{x \to a^+} f$

$f(a)$ not defined

$\lim_{x \to a^-} f \neq \lim_{x \to a^+} f$

$f(a)$ defined

$\lim_{x \to a^-} f = \lim_{x \to a^+} f$

so limit exist

but $f(a)$ ~~not~~ is defined but not limit

if (1) $\lim_{x \to a^-} f = \lim_{x \to a^+} f$ (limit exists)

(2) $f(a)$ defined

(3) $\lim_{x \to a} f = f(a)$ we say f is continuous at $x = a$

Ex $f(x) = \begin{cases} x & x \le 1 \\ x^2 & x > 1 \end{cases}$

(1) $\lim_{x \to 1^-} f = \lim_{x \to 1^-} x = 1$

$\lim_{x \to 1^+} f = \lim_{x \to 1^+} x^2 = 1$

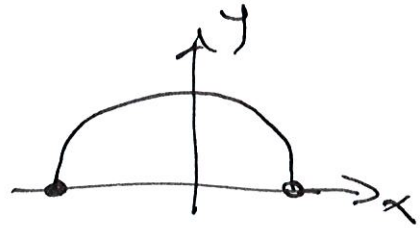
so limit exists so $\lim_{x \to 1} f = 1$

(2) $f(1) = (1) = 1$

(3) $\therefore \lim_{x \to 1} f(x) = f(1) \quad (= 1)$

f is continuous at $x = 1$

<u>Note</u>! $f(x) = \sqrt{1-x^2}$



$\lim_{x \to 1^-} \sqrt{1-x^2} = 0$

$f(1) = 1$ / same so f is cont^s from the left.

<u>Note</u>: f is not defined the right side of $x = 1$