Technical discussion on risk mathematics

Q.8-02. Why is the measurement of risk problematic?

The difficulty with the concept of measuring risk is one of knowledge; risk is probabilistic in nature and its measurement hinges on the underlying probability distribution. The two great challenges to measuring risk are to catalog all possible outcomes and to know the probability of each outcome. The list of all outcomes and the probability assigned to each is referred to as the probability distribution of the random variable of interest. Any statement about the expected outcome of a process or the likelihood of a specified set of outcomes presumes that the underlying distribution is known.

Unfortunately, the underlying distribution is not something that can be observed in any but the most trivial cases. It is only possible to estimate the underlying probability distribution based upon historic data and this can be done only through certain assumptions about the stability of the random process and the observability of the outcomes. Many different models are likely to fit the same set of observations, but these different models may have significantly different consequences, particularly in the odds on "rare" events.[1]

Q 8-02.01. What should be done if certainty is not obtainable?

Because one cannot know with certainty the shape of the probability distribution that generates our risks, misspecification of that distribution can carry dangerous consequences.

It is important to exercise caution in drawing conclusions. Do not assume that everything can be modeled by the Normal (or Gaussian or bell-curve) distribution and that no alternative is needed. Assuming a Normal distribution may make the mathematics easy and allows the use of readily tabulated values for formulae, but it may also result in catastrophic events presumed to be extremely rare - those once-in-a-century events – occurring with distressing frequency. Begin by testing the data against several candidate probability distributions and be conservative in rejecting any. Consider blending forecasts from several models to derive a composite forecast.

[1] This situation highlights what is popularly referred to as "the black swan" problem. When it is impossible to identify which of several underlying stochastic processes is generating observations, selection of one of the candidate models may understate the probability of catastrophic rare events if the wrong selection is made. In this case, all statistical efforts to ameliorate the anticipated hazard may be inadequate and will result in a loss greater than any hedged against.

Q 8-02.02. What are the problems in measuring rare events?

Extreme values (that is, values that are more than three standard deviations from the mean) fall in probability very quickly under the Normal distribution. This is not so for all possible distributions. Small data samples are unlikely to reveal the odds of an extreme event and it is exactly this information that is needed to be sure of the underlying probability distribution.

Rare or low probability events require a long data series even to indicate their existence. For example, to get at least a 50-50 chance of seeing just one event that has a 1% chance of occurring requires at least 69 observations; for a 0.01% chance, 6,932 observations; for a 0.0001% chance, 693,147 observations. The last example would be equivalent to almost 2,000 years of daily observations in order to find even one example of a one-in-a-million event. Since it is unlikely that such a rich dataset is available, the probability models are likely to be truncated. At this point, assumptions about the shape of the distribution in the absence of complete data can be very flawed.

Q 8-02.03. What are some ways risk is represented?

This discussion about risk has focused on its measurement or representation. There are several ways that risk might be summarized. It can be described by words or concepts, but it may be difficult to achieve a common understanding among all involved parties as to the meanings. It also may be difficult to incorporate textual descriptions into a system that measures risk in any automated way. Risk also may be calculated as a single value or set of values that represents some relevant metrics. That these are numeric values, however, is not enough to justify their use. The metrics must sufficiently encapsulate those aspects of the situation that no additional useful information would be added by other statistics. For example, if an event's probability were known to follow the Normal distribution, the mean and the variance of the process would be sufficient metrics or information.

Given the metrics that might be used to measure the outcome, one must determine a decision rule based on those metrics. A decision rule uses data to prescribe some action or decision. Decision rules are common in daily life: one looks at the probability of rain to decide whether to carry an umbrella for the day.

Risk in this statistical sense is the potential cost that may be incurred when one uses a decision rule, based upon data, to choose a course before events are known. After the outcomes are known, it is too late to change plans. A decision has been made and the consequences of choosing the course taken must be borne. Statistical risk may be measured in various ways: expected loss, minimax loss, and quadratic loss are examples. The differences among the calculations reflect differences in interpretation of risk and in responses to risk. The approach used to calculate risk affects the risk mitigation strategy, i.e., the decision rule, and the shape of the portfolio held to be adequately immunized from risk

Q 8-02.04. What are implications of using expected loss to measure risk?

Using *expected loss*, the average of all positive and negative outcomes multiplied by their probabilities, may induce an unwarranted sense of safety when catastrophic losses are given very low probability of occurrence.

There are two problems with this calculation. The resulting value may describe a state that is nothing like any of the outcomes. For example, in a world where only two outcomes may occur – catastrophic loss and unimaginable benefit, each about equally likely – the calculation may suggest that the world would be basically unchanged. In fact, under either outcome the world will be very changed from the current state. The second problem is that the probabilities of the extreme outcomes can significantly affect the resulting calculation. As described above, however, it is in the estimation of extreme probabilities that many approaches are weak.

Q 8-02.05. What are implications of using minimax loss to measure risk?

Minimax strategies are the solutions to many problems in game theory. A decision rule based on the minimax criterion would look at the worst loss that would result given each possible decision and choose the decision that had the least painful worst case. In many situations, this leads to highly conservative strategies to avoid as much risk as possible regardless of likelihood of outcome.

Q 8-02.06. What are implications of using quadratic loss to measure risk?

Decisions based on minimizing quadratic loss are the most common in portfolio risk management. A quadratic loss function imposes an increasing weight (with the square of the distance) on the difference between the target and the actual outcome. These decision rules usually use the statistical variance of the outcome as a measure of loss. Although the mathematical properties are easy to work with, quadratic loss is symmetric. A loss the same distance from either side of the target point is given the same weight. This may not mirror the true costs when they are not symmetric.