CHAPTER TWO
REPRESENTATION AND BEHAVIOR OF FUNCTIONS

2.0 Introduction

A major technological and mathematical advantage of the graphing calculator is that you can 'see' visualizations of relationships between numbers. This provides you with valuable information about the behavior of the data, and will allow you to further analyze the relationship between the data (numbers). For example, consider the following data relationship:

Table 2.0.1 (Data available in the PROFIT49 program)

Number of calculators sold (*N*)	0	500	1000	1500	2000	2500	3000	3500	4000
Profit made (*P*)	0	12500	25000	37500	50000	62500	75000	87500	100000

Reading this numeric representation of the relationship between the number of graphing calculators sold and the profit earned by the calculator company tells you that as more calculators are sold, the profit increases. You can also see that if they sell exactly 2,000 calculators, they will earn $50,000 in profit. This type of analysis of the relationship between the set of numbers N and the set of numbers P is developed throughout this text. This kind of information is useful, for example, to an accountant for this company, she may be asked to estimate the profit when 26,250 of the calculators are sold. Based on the numerical data above, she may be asked to estimate the calculator sales needed to bring in $1,000,000 in profit. To help answer these and other questions and to explain the answers to these questions, a visualization of this data can be useful. The visualizations (pictures) may not be similar to those studied in arithmetic. You may have seen graphs like in Figure 2.0.1.a. From now on, you will not use this type. You will use graphs like shown in Figure 2.0.1.b. If the number of calculators sold is represented by the appropriate numbers on a horizontal number line and the profit from the number of calculators sold is displayed <u>above the number</u> sold, you will see a visualization of the data.

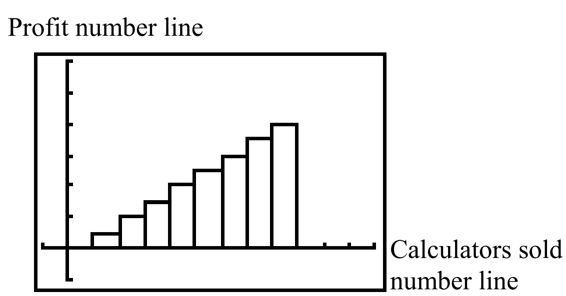


Figure 2.0.1.a

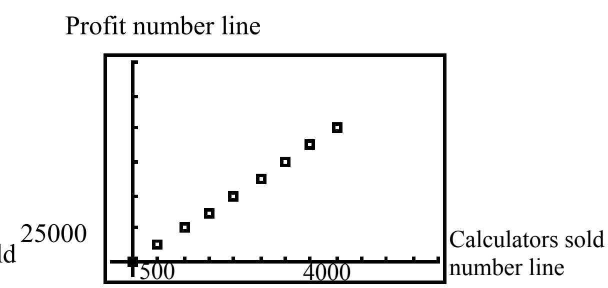


Figure 2.0.1.b

The visualization or **graphical representation** shows that the data is following a linear growth pattern. The data lie in a straight line, as do the tops of the bars. The linear growth pattern indicates a constant growth rate. It is this kind of a pattern that will help you analyze relationships at a higher mathematical level than just looking at the height of each bar as in Figure 2.0.1.a.

This same kind of analysis, although in much more detail, will be developed in this text. Not all data relationships are linear. As you progress through this text, you will analyze several types of relationships.

You will look at behaviors in data relationships such as data that sometimes increases and sometimes decreases, or has a maximum or minimum value(s). Knowing when data is positive, negative, or zero gives useful information to the person analyzing the data. Relationships can be quadratic, for example, many profit relationships are quadratic rather than linear. Relationships can be exponential, for example, the relationship between time and the number of humans on earth is exponential. (Quadratic and exponential relationships will be described later.) Throughout this text, you will investigate seven elementary relationships and several other combinations of relationships.

Have you ever wondered where symbols like $\frac{1}{2}x - 12$ or $-16t^2 + 48t + 100$ come from? Or, why do mathematicians study such things as $2|x-3|+5$ or $\sqrt{x+4} - 2$? Hopefully these and other questions will be answered in Chapter Two and in the remainder of the text. The process begins with the analysis of the data relationship in Table 2.0.2.

Consider the numeric relationship between time (t) and the average amount of garbage (g) generated each day by each person in the United States.

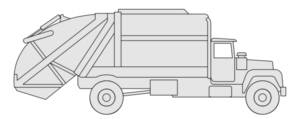


Table 2.0.2

t	1960	1970	1980	1988	1993	1998	2004	2006	2010
g	2.7	3.2	3.6	4.0	4.1	4.2	4.3	4.4	4.6

(g is in pounds: Data available in the program GARBG50)

It has been projected that in the year 2015, the average amount of daily garbage generated by each person in the United States will be 4.8 pounds. How is it possible to make such a prediction? Perhaps if the shape of the relationship is known, you can think of a way of answering the question. The four steps necessary to get the graphical representation of the relationship are shown again in Figure 2.0.2.

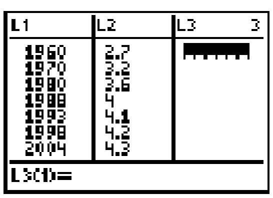


Figure 2.0.2.a

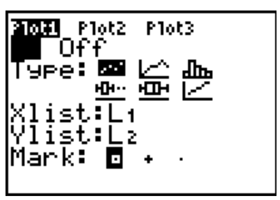


Figure 2.0.2.b

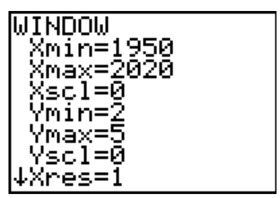


Figure 2.0.2.c

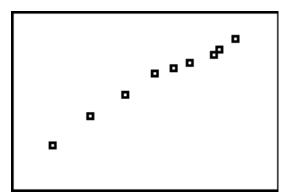
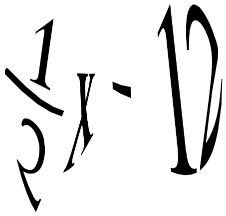


Figure 2.0.2.d

The graphical representation of the relationship suggests that the data has been nearly linear since 1960. If, in fact, it is a linear relationship, then a line drawn through the data points and extending the domain to include the year 2015 should reveal a data point at 4.8. Should you be curious if this method works, try it using a correctly scaled drawing and a good ruler. Of course, you can do the same thing on the calculator and not waste your time and paper. While this method is acceptable in the classroom, mathematicians, engineers, business people, and scientists may predict the garbage generation in the year 2015 by the method described below.

The numeric and graphic representations of the garbage problem have already helped to estimate what the per person per day garbage will be in the year 2015 by drawing a line. However, there is one more very important and useful idea that will allow you to not only predict for the year 2015, but for any year – within reason. There is one more way of representing the data relationship; it can be represented symbolically. Many data relationships can be represented symbolically. As this chapter unfolds, you will



discover that symbols like $\frac{1}{2}x - 12$ can be used to represent data relationships! To find a "good" symbolic representation requires that you first know the shape of the data relationship. For more complicated relationships, you must learn more about their behavior. That is, thus far, you know how to find the shape of a relationship. Chapters Two and Three will help you learn about other behaviors of relationships. This, in turn, will help you learn how to find the symbolic representation of a relationship.

One suggested symbolic representation of the garbage problem is $0.0371t - 69.9$ where t is time in calendar years and $0.0371t - 69.9$ is the amount of garbage created per person per day. The symbolic representation can also be used to obtain the numeric or graphic representations. For example, Table 2.0.2 is created by using the symbolic representation for the garbage with t being replaced with members of the domain of your choosing!

Table 2.0.3

t	1960	1970	1980	1988	1990	1993	1995	1998	2000	2010	2015
$0.0371t - 69.9$	2.8	3.2	3.5	3.8	3.9	4.0	4.1	4.2	4.3	4.6	4.8

The symbolic representation provides a very simple method for predicting any year you want to put in the problem domain.

You should not yet concern yourself with where the symbolic representation of the garbage problem comes from. You will learn where it comes from and how it is developed in Section 2.2. The point for now is that the relationship "can" be expressed in symbolic form. If data relationships are expressed in symbolic form, it may become easier to analyze certain features of the relationship.

This discussion may cause you to wonder if all of the "algebra-like" symbols are nothing but symbolic representations of data relationships. Of course it is possible; however, it is not always necessary or desirable to relate all symbolic representations to actual data relationships. To help you learn mathematics, it is desirable to develop new ideas in the context of a data relationship. After you have learned a mathematical idea, it may no longer be necessary to relate mathematical symbols to real-world data relationships. It is true that "algebra-like" symbols such as $3.5t + 80$, $-800|t - 30| + 24000$, $-16t^2 + 48t + 25$, and $10000 \cdot 2^{\frac{t}{20}}$ do have graphical representations that behave like the graphical representations of the data relationships presented in Section 2.1. The types of data relationships found in Section 2.1 are linear, absolute value, quadratic, and exponential.

$3.5t + 80$

Linear

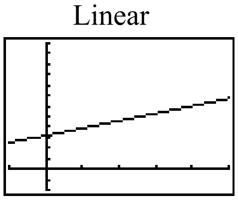


Figure 2.0.3a

$-800|t - 20| + 24000$

Absolute Value

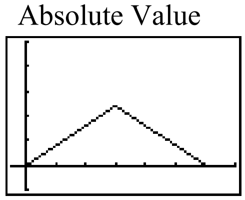


Figure 2.0.3b

$-16t^2 + 48t + 25$

Quadratic

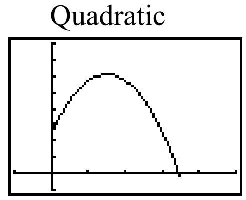


Figure 2.0.3c

$10000 \cdot 2^{\frac{t}{20}}$

Exponential

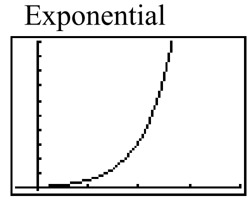
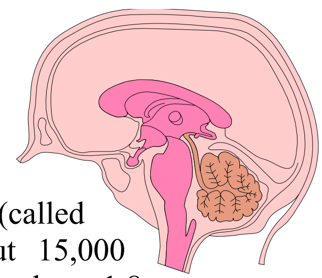


Figure 2.0.3d

These relationships will be studied numerically, graphically, and symbolically in Chapter Two.

The analysis of data relationships studied in Chapter Two will also introduce the concept of a variable and of a function. By symbolizing the real numbers in data relationships, you can make generalizations about the characteristics (behaviors) of relationships. Once some of the behaviors of relationships are known, you can solve problems encountered in the everyday world of business people, teachers, engineers, scientists, and others. As students, you will use the mathematics you learn to solve a variety of real-world problems often encountered by these professional people and others.

Chapter Two serves a two-fold purpose. You will learn about variables, behaviors of relationships, and three different ways of representing relationships - numeric, symbolic, and graphic. Second, Chapter Two gives you an opportunity to *experiment* with and learn how to use the graphing calculator to help you analyze and solve problems involving relationships. Using the three representations in conjunction with the calculator will enhance and enrich the teaching and learning of mathematics in this and all remaining chapters.



Basic Brain Function – Can Algebra be Simple?

As your brain is developing, from about the second month after conception to your second birthday, your brain is extremely busy creating connections (called synapses) among neurons. During this time, each neuron forms about 15,000 connections to other neurons – all on their own – without learning. This is about 1.8 million synapses (connections) per second for over two years of time. Why is this important and what does this have to do with algebra being simpler than you might think? The answer is that when you learn something (not just algebra), the brain stores what you learned in a network of connected neurons. It is the series of connections (synapses) that hold the memory of something you just learned. When the all the neurons in the neural circuit holding the memory fire (discharge electricity) from beginning to the end, you get the memory of what you are trying to remember. So, as a person under two years old, the circuits already exist. This is what makes learning extremely easy for a two year old, or three years old etc.

Now for the bad news: Which of these trillions of synapses (connections) remain, and which wither away, depends on whether they carry any traffic. That is, the traffic means learning and using what you learned. By one estimate, approximately 20 billion synapses are pruned every day between childhood and **early adolescence**. That is, after the age of two, the UNUSED connections start to wither – get cut off. With fewer connections available to process and hold a memory, the more difficult it becomes to learn. Adults have a more difficult time learning than do teenagers because most of the “free” circuits are gone.

Now for the good news: At around age 10-12, humans develop another burst of “free” synaptic growth. This growth is not as prolific as in the new brain, but because of the new neural circuit growth, learning again becomes simpler for teenagers. Adults are out of luck, because all the new unused neural connections (synapses) are cut by the time they become young adults. So they must CREATE new synaptic connections to hold and process anything new they learn. Woops, no free lunch for adult learners.

Teenagers, on the other hand, are offered a free lunch. But are you hungry?

2.1 Data Relationships Represented Numerically and Graphically

Related Data The study of related data extends throughout this text. In this section, you will study the geometric shapes of the data relationships. Yes, if you look at a picture (**graphical representation**) of the related data, many times you will see a very clear and definite shape to the data. This is not to say that all data has a clear and distinct shape; it doesn't. In addition to identifying shapes of data relationships, you will also be asked to identify if the relationship is increasing or decreasing. This behavior of the relationship becomes important in later studies. The data relationships you will analyze in this text come as pairs of numbers. If they are related, you will likely see a recognizable geometric shape. To get the graphical representation of the related data, the graphing calculator will be used. It will use the first number in the pair to determine the horizontal position of the data point, just like in Figure 2.1.3. The second number in the pair will determine how far above or below the horizontal number line to put a mark representing the graph of the pair of numbers. Below in Table 2.1.1 is the numeric representation of seven pairs of numbers. The first number in each pair is time and the second number is the percent of the Unites States Gross Domestic Product spent on health care.

Table 2.1.1

Time	1960	1963	1970	1975	1981	1984	1987	1989	1996	2000	2007
% *of GDP*	5.2	5.7	7.2	8.1	9.2	10.1	10.8	11.5	13.5	13.6	16

Source: OECD Health Data 2009. Data available in program GDPHCR53

Hopefully, you can see that the amount of money spent on health care in the United States is related to time. As time passes, money spent on health care increases. It does not fluctuate wildly. If the numbers were not related, you would see no pattern in the percent of GDP. While the data is useful in numeric form and will be further analyzed, the purpose here is to look at the graphical representation of the data and give a name (if possible) to the shape. The graphical representation can be obtained by using your graphing calculator. Below are screens from a TI-84 graphics calculator outlining the steps needed to graph the data relationship.

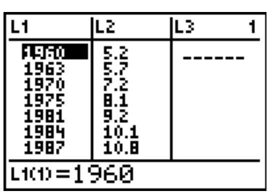


Figure 2.1.2a

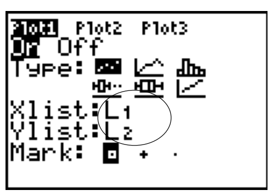


Figure 2.1.2b

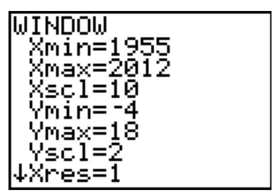


Figure 2.1.2c

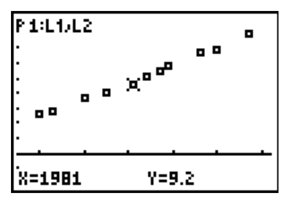


Figure 2.1.2d

The data is entered in L_1 and L_2 by running the TI-84 program GDPHCR53, or by entering it manually as shown in Figure 2.1.2a. The set of all numbers in L_1 is called the **domain** (Xlist) of the relationship and the set of all data in L_2 is called the **range** (Ylist) of the relationship. Figure 2.1.2b tells the calculator that the first number in each pair is stored in L_1 and the corresponding numbers are in L_2. This figure also shows that plot mode is on, the calculator is to plot points only, and to use the □ sign as the mark where each data point is to be plotted. Figure 2.1.2c shows the calculator window to be used. It tells the calculator to start time in 1955, stop in 2012, and put a tic mark every 10 years. It also tells the calculator to show any health care money from –4% to 18% and to put a tic mark every 2%. The final figure shows the graphical representation of the data. This is also shown in Figure 2.1.3.

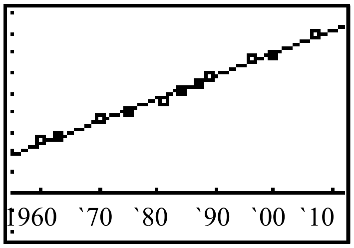


Figure 2.1.3

The issue here is simple. What is the shape of the data? Of course, it is nearly linear. The growth of money spent on health care as a percent of the Gross Domestic Product follows a linear growth pattern. The money spent on health care as a percentage of the Gross Domestic Product seems to depend on time. On many occasions, real data will not produce a "perfect" shape when it is graphed. The solid line drawn on top of the data is a good approximation to the actual data. Because the graphical representation of the data is like a straight line, the relationship is called **linear**.

As you think about linear data relationships, you quickly realize that the graph (being a straight line) either rises as time passes, falls as time moves forward, or is constant (a horizontal graph). In the example above, we know the GDP/health care is rising as time passes, so we say it is increasing. We will extend this idea for any data relationship to say that if the graph is rising as we trace to the right, it will be an increasing data relationship. If it falls as we read it from left to right, we will call it decreasing.

Below in Table 2.1.2 is data showing the relationship between the Centigrade (Celsius) temperature and the weight of one gallon of water in pounds. If you look closely at the weight of the water,

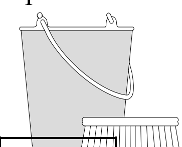


Table 2.1.2 (Data available in the program WATERW54)

T	0°	1°	2°	3°	4°	5°	6°	7°	8°
W	8.33461	8.33513	8.33551	8.33575	8.33585	8.33582	8.33565	8.33536	8.33495

Source: *CRC Handbook of Chemistry and Physics College Edition*. 1965-1966.

you will notice that as the temperature changes from 0° to 8°, the weight of the water increases part of the time and then it decreases in weight. Is the weight related to temperature? If so, how are they related? Is it a linear relationship? Can the data in Table 2.1.2 be used to predict the weight at 10°, or at –10° Centigrade? Again, these and other questions will be answered later in the text. For now, the question remains what is the shape of this data? To find out, proceed as in the first data set. The four screens below show the steps necessary to "see" the data on a TI-83 or 84. The set of numbers representing the temperature of the water (the domain) is stored in L_1. The corresponding set of numbers for the weight

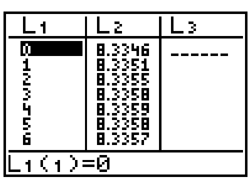


Figure 2.1.4a

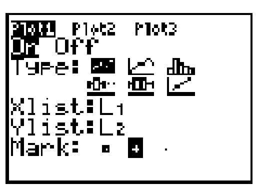


Figure 2.1.4b

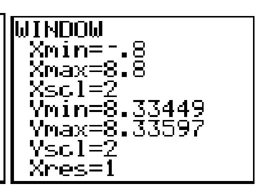


Figure 2.1.4c

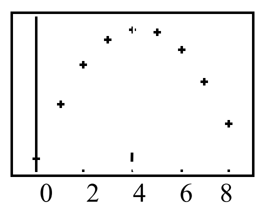


Figure 2.1.4d

of the water (the range) is stored in L_2. As with the first set of related data, Figure 2.1.4b shows that the data plotter is turned on, what type of graph to create, where the domain (Xlist) and range (Ylist) are stored, and what type of mark to use at each data point (+). The window shown in Figure 2.1.4c can be set manually or use ZOOM - ZoomStat on the TI calculators. You may recognize the shape as something like the path of a ball tossed lightly from one person to another. It is similar to the shape of water coming out of a drinking fountain. It looks like a parabola. The relationship between the temperature of a gallon of water and the weight of the water is parabolic in nature. Parabolic relationships are also called **quadratic relationships**. You will see why later in Chapter Three. The weight of the gallon of water depends on the temperature of the water in a quadratic-like fashion. Looking a Figure 2.1.4d or Table 2.1.2 we see that the weight (or the temperature-weight relationship) is increasing until the temperature is 4°, and then the relationship is decreasing when the temperature is above 4° Centigrade.

The third example is the relationship between the time and the national debt. As time passes, the national debt just keeps rising; that is, it appears that the national debt depends on (is related to) time. Table 2.1.3 shows the relationship between time and the national debt.

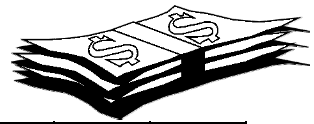


Table 2.1.3

Year	1940	1950	1960	1970	1980	1990	1994	2002	2006	2008	2010
National Debt	51	257	291	381	909	3113	4500	6000	8400	10024	13000

Data available in the program DEBT55: (Debt is in billions: as of 5/31/2010)

Is the national debt growing in a linear fashion? Is it following a quadratic relationship? Or does it have still another shape? To see the shape of the data, do as before and enter the data in the calculator's memory and then graph the data. As with the previous examples, the domain is stored in L_1 and the range in L_2. The graph type is data points connected and the mark to be used is the □ mark. Figure 2.1.5d shows the shape of the data.

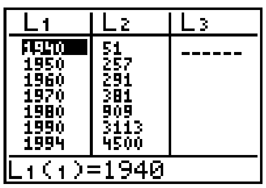


Figure 2.1.5a

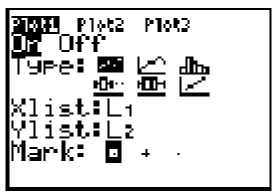


Figure 2.1.5b

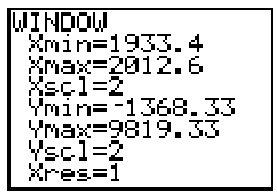


Figure 2.1.5c

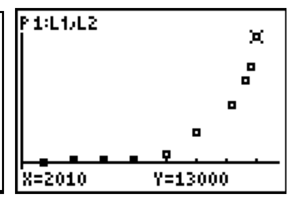


Figure 2.1.5d

Hopefully it is obvious that the data is not linear, not quadratic, but still another shape. This shape stays fairly flat for a while and then increases very sharply. This relationship is called **exponential**. Currently, human population is growing exponentially. Money invested in a compound interest account grows exponentially. Many bacteria grow in numbers (population) exponentially. In each of these examples, the population depends on the length of time spent growing. In the exponential relationship above, it is increasing. That is, the relationship between time and the USA national debt is increasing.

The few samples above demonstrate that real-world data relationships do behave according to very definite mathematical patterns. You will encounter several more shapes throughout the remainder of the text. The last example may convince you that not all data fits into a recognizable shape.

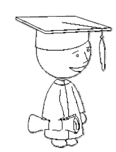


Table 2.1.4 shows the relationship between IQ and weight of 10 randomly selected students.

Table 2.1.4

Weight	110	115	118	127	135	138	142	150	155	170
IQ	120	115	130	103	95	137	120	112	128	116

Data is available in the program IQ55

Using the same methods as before, the steps used to graph the data are shown in Figure 2.1.6 with the numbers that make-up the domain stored in L_1 and the range is stored in L_2.

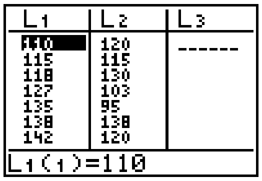


Figure 2.1.6a

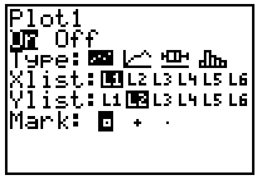


Figure 2.1.6b

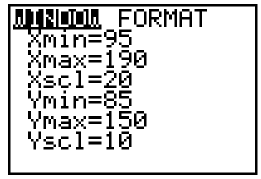


Figure 2.1.6c

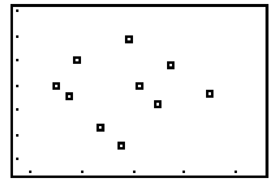


Figure 2.1.6d

As you can see in Figure 2.1.6d, the data represents no clear recognizable shape. The conclusion you may draw from this is that your IQ is not related to weight. That is, IQ does not depend on a person's weight. In looking at the visualization in Figure 2.1.6d, hopefully it is clear that the concept of increasing or decreasing is meaningless in this relationship.

Data relationships are found in your work life as well as your daily life. For example, the amount of gasoline in the gas tank of your car depends on the distance your car is driven. The time it takes you to get to work depends on how fast you drive. Your weekly gross wages may depend on the number of hours worked. The length of time it takes to paint a room depends on the surface area of the room. If you have a good understanding of mathematical relationships, you will have the power to control some of these relationships in your life.

2.1 STRENGTHENING NEURAL CIRCUITS

Please read the text before you do the exercises. Read the **next assigned section** after you finish the assignment below.

Priming the Brain

-4. Can your gross weekly wages be represented with mathematical symbols? Explain.

-3. If you throw a ball straight up, use English to describe how the height of the ball changes.

-2. What name might you give to the biggest height the ball reaches (see -3 above)?

-1. What is another word that describes the velocity (or speed) of the ball?

Neural Circuits Refreshed

1. What property is being demonstrated in the statement $4(3 + 5) = 4 \times 3 + 4 \times 5$?
2. Write a statement that shows the use of the associative property for multiplication.
3. Isolate the number 7 using the properties of equality. $7(4 - 3) + 4 = 5 + 6$. Do not simplify.
4. Evaluate (find the numerical value) $-4 + 5^2 + 3(2 - 7)$.
5. Evaluate $\sqrt{3^2 + 4^2}$.
6. Write 0.0000000001 in scientific notation.

Myelinating Neural Circuits

In Exercises 7 − 29, identify the shape of the data relationships, when the relationship is increasing and when it is decreasing. If a new shape occurs -- give it a name of your choice.

7. The percent of the U.S. population paid by the U.S. government as civilian employees measured over time. (Run GOVEMP57 Program for data in the calculator)

Year (*Time*)	1970	1975	1980	1985	1990	1995	2000	2005	2006	2007	2008
% Employed	3.81	3.35	3.01	2.80	2.72	2.36	2.10	1.91	1.87	1.85	1.88

Source: U.S. Office of Personnel Management. Accessed 6/1/2010: http://www.census.gov/compendia/statab/2010/tables/10s0484.pdf

8. The average cost (tuition and fees) of attending a public four-year college is increasing as time passes. Time is in academic years starting in the 1976 – 77 year. (Run TUITN57 Program)

Time	1976	1981	1986	1991	1996	2001	2006
Cost	617	909	1414	2107	2975	3766	5836

Source: The College Board

9. The annual poverty threshold for a family of four has been on the rise as indicated in the data below. (Run POVRTY57 Program)

Time	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2008
Income in $	3022	3223	3968	5500	8414	10989	13359	15569	17604	19971	22025

Source: U. S. Bureau of the Census

10. A ball is thrown straight upward with an initial speed of 32 feet per second from a point 20 feet above ground level. Below is the data that shows the height of the ball above the ground at *t* seconds. (Run BALL57 Program)

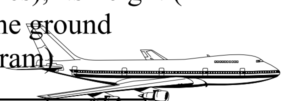
t	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.5
Height	20	25.76	30.24	33.44	35.36	36	35.36	33.44	30.24	25.76	20	12.96	4.64	0

11. Below is the data showing the number of inmates in state and federal prisons for recent years. (Run INMATE57 Program)

Time	1980	1982	1983	1984	1985	1986	1988	1989	1990	1991	1992	1997	2003	2008	2010
Inmates	330	414	437	462	503	545	632	713	773	824	884	1198	1470	1610	1612

(inmates are in thousands)

12. As a commercial airliner flies from Columbus to Pittsburgh (approximately 200 miles), its height (in feet) is related to the ground distance from Columbus. Below is the data showing the ground distance from Columbus and the related height of the plane. (Run FLIGHT57 Program)



Ground Distance	0	25	50	75	100	125	150	175	200
Height	0	4500	9000	13500	18000	13500	9000	4500	0

13. The amount of land being farmed is shown below as a percentage of the total land in the United States. The data shows that the amount increased for a while and recently has been decreasing. (Run FARMLN57 Program)

Time	1910	1930	1940	1950	1960	1970	1980	1990	2002	2007
% Farmed	38.8	43.6	46.8	51.1	49.5	47	44.8	42.7	41.4	40.8

14. If you invest $1000 at 6% compounded annually, the data below shows that the amount of money in your account depends on how long it is in the account. (time is in years)

Time	0	5	10	15	20	25	30	35	40	45	50
Amount	1000	1338	1791	2397	3207	4292	5744	7686	10286	13765	18420

Run the program COMPIN58 for data

15. The data below shows a checking account balance for the month of May 2010. The time is in the day of the month and the balance is rounded to the nearest dollar.

Time	1	3	5	7	9	11	13	15	17	19	21	23	25	27
Balance	3245	3202	958	885	623	623	623	623	623	1847	551	464	198	43

Source: author's checkbook (Run the program CHKBAL58 for data)

16. The data below shows that the number of hospitals in the United States has recently been decreasing; earlier, the number of hospitals was increasing. (Run the program HOSPIT58)

Time	1950	1955	1960	1965	1970	1975	1980	1985	1990	2000	2002	2003	2006
Number	6788	6956	6876	7123	7123	7156	6965	6872	6649	6116	5794	5764	5759

Source: American Hospital Association, *Hospital Statistics* (annual).

17. The human population of the earth has been steadily increasing as time passes. Below is the data showing the relationship between time and population. The population is in billions. For example, 2.8 means 2.8 times 1,000,000,000 or 2.8 billion. In 1950 the earth had 2,800,000,000 people. All numbers are rounded. (Run the program POPULT58 for the data)

Time	1750	1800	1850	1900	1950	1994	1999	2001	2006	2008	2010
Population	0.8	1.0	1.2	1.7	2.8	5.9	6	6.2	6.6	6.70	6.82

As of June 3, 2010

18. Below are the possible wages for a server working at the Blue Point Café in Duck, NC. He is paid a salary plus he earns an average of $3.50 in tips per person served. His wages depend on how many people he serves on during the week. (Run the program BLUEPT58 for the data)

Persons	10	15	20	25	30	35	40	45	50	55
Wages	75.00	92.50	110.00	127.50	145.00	162.50	180.00	197.50	215.00	232.50

19. This same server is responsible for filling the ice chest in the Blue Point Café. He kept the following records on the number of times per day he had to fill the chest. Time is expressed in the day of the month. (Run the program ICE58 for the data)

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number	6	6	8	4	12	9	3	4	6	5	8	11	6	5

20. Below is the data showing the number of people enrolled in Medicare from 1975 to 2008. The numbers enrolled are in millions. (Run the program MEDCRE58 for the data)

Time	1975	1980	1985	1990	1995	2000	2002	2006	2008
Number	25	28.5	31.1	34.2	37.5	39.6	40.5	43.3	45.3

21. Below is the volume of water as it changes with temperature changes. The original volume of water in this experiment was 1 milliliter at 4° C. (Run the program WATERV59 for the data)

T	−10	−5	0	2	4	6	10	20
V	1.00186	1.00070	1.00013	1.00003	1.00000	1.00003	1.00027	1.00177

Source: *CRC Handbook of Chemistry and Physics College Edition*. 1965-1966. The Chemical Rubber Company. Cleveland, OH

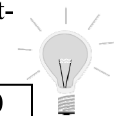
22. The internal revenue service has been collecting taxes at an average per-person per year rate as shown in the data below that has been rounded to the nearest dollar. (Run the program TAXPR59 for the data)

Year	1960	1965	1970	1975	1980	1985	1986	1987	1988	1989	1990	1991	2004	2007
Collected	508	589	955	1386	2276	3099	3233	3627	3792	4063	4222	4344	6874	8528

23. An author can write his book according to the schedule below. (Run the program WRITE59)

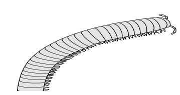
Time (in days)	1	2	3	4	5
Pages Written	2.5	5	7.5	10	12.5

24. Below is data showing how charges from the electric company depend on the number of kilowatt-hours of electricity used. (Run the program ELECT59)



kWh	200	400	600	800	1000	1200	1400	1600	1800	2000
Charge	23.82	38.14	52.47	66.79	81.11	95.43	109.75	124.08	138.4	152.72

25. Here is data showing the relationship between the number of E. coli bacteria (*N*) on an uncooked piece of hamburger and time (*t*) in minutes. (Run the program ECOLI59)



t	0	30	60	90	120	150	180
N	5000	14142	40000	113137	320000	905097	2560000

26. Below is the data for the relationship between the selling price (*s*) of a graphing calculator and the daily profit earned (*P*) from the sale of the calculators. (Run the program CALC59)

s	50	55	60	65	70	75	80	85	90
P	15890	17540	18690	19340	19490	19140	18290	16940	15090

27. Below is data for the relationship between time and the number of immigrants (in thousands) entering the United States for the last several decade. (30 means the decade of the 1930's)

Decade	30	40	50	60	70	80	90	100
Number	528	1035	2515	3322	4493	7338	9095	22000

Source: U.S. Bureau of Census. (Run the program IMMIG59)

28. The table below is data for the relationship between public education expenditures (in billions) in the United States and time? (Run the program EDUEXP59)

Time	1940	1950	1960	1970	1980	1990	2006
Expenses	3.3	8.9	23.9	68.5	165.6	377.5	900

Source: U.S Department of Education, National Center for Educational Statistics

29. Below is data for the relationship between blood velocity (v) in an artery (that has a radius of 0.5 *cm*) vs. the distance (d in *cm*) the blood is from the center of the artery. Velocity is in *cm*/sec. (Run the program BLOOD60 for the data)

d	0	0.1	0.2	0.3	0.4	0.49
v	25	24	21	16	9	0.99

30. Using the graphical representation of the number of prisoners in state and federal prisons you made in Exercise 11, predict how many prisoners there will be in 2015. Explain how you arrived at your answer.

31. Find or make-up one example of a relationship that is linear. Identify the domain and range and describe why you think the relationship is linear.

From Mathematics to English

32. After reading this section, make a list of questions you want to ask your instructor.

33 Continue in your daily journal and make an entry. In addition to your normal entry on thoughts about the mathematics in this section, list at least two positive comments about what you have learned about this topic.

34. In paragraph format, summarize the material in this section of the text in your daily journal.

35. Describe how your classroom instructor made this topic more understandable and clear.

36. After reading the text and listening to your instructor, what do you not understand about this topic?

37. In this section, you have used data relationships in numeric and graphic forms. Express your opinion as to whether these relationships could be represented in symbolic form.

From English to Mathematics

In the next three exercises, write the English statement as a mathematical statement. For example, "five times the difference of x and 4", would be $5(x - 4)$.

38. \$3.50 plus x

39. x squared minus 3, times 43

40. the length x times the width $(x - 1)$

Developing the Pre-Frontal Lobes

41. Create (make-up) a data relationship with 6 data points that is linear and increasing.

42. Create (make-up) a data relationship with 6 data points that is linear and decreasing.

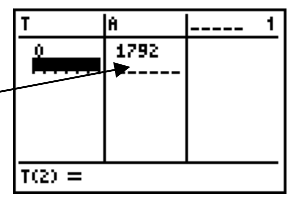
2.2 Data Relationships Represented Symbolically

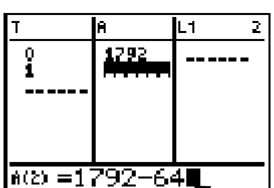
Thus far in Chapter Two, you have analyzed many data relationships using the numeric and graphic representations. In Section 2.2 we will learn how to represent data relationships in symbolic form. In Section 2.0 we suggested that $0.0371t - 69.9$ can represent the garbage problem. What do we mean? Simply stated when t is replaced with any year, the symbols $0.0371t - 69.9$ approximate the amount of garbage generated per person per day. So in 2004 ($t = 2004$), the symbols $0.0371t - 69.9$ become 4.25. The actual data for 2004 is 4.3. Or for example in 2006, the symbols $0.0371t - 69.9$ become 4.49, and the actual data is 4.4. The symbolic representation $0.0371t - 69.9$, nearly produces the "real" data. The symbols $0.0371t - 69.9$ appear to "model" the actual data relationship.

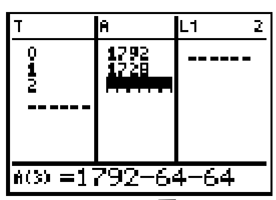
From Numeric to Symbolic Many times mathematical relationships are available in numeric form or are described using words. It is extremely important that you be able to convert this information to symbolic mathematical form. Below is a method that is based on your ability to recognize a pattern, as the data relationship is developed from a situation. We use pattern recognition because your brain is excellent at generalizing patterns.

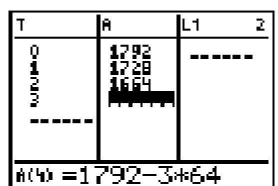
House Painter Measurements on a potential customer's house shows a surface area of 1792 square feet. Let t be time in hours and A the area that remains to be painted. Since you paint at an average rate of 64 square feet per hour, and the initial conditions are known, we can develop the algebraic model.

The initial conditions are known. At time 0, the area remaining to be painted is 1792 square feet.



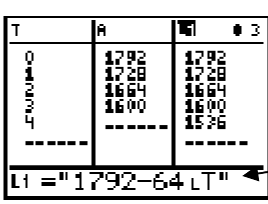






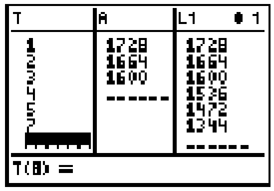
Since the rate is known, you also know that each hour worked is another 64 square feet less to paint. The mathematical process is subtraction of 64. Thus, we can create a numeric representation from the verbal description.

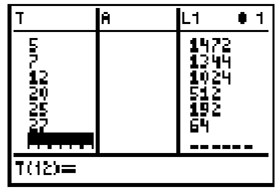
But repeated subtraction of 64 can best be accomplished by subtracting the product of 64 and the number of times it has been subtracted – as shown in the edit line.



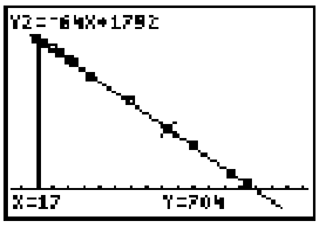
From a pattern of arithmetic operations, you can generalize to algebraic symbols such as $1792 - 64T$.
(LT represents all numbers stored in list T.)

The symbols 1792 − 64LT represent the house painter data and so they are called the symbolic representation of the situation. That is, as we saw above, the symbols automatically created the area, as we entered new values for time, below, the symbols create the corresponding areas.





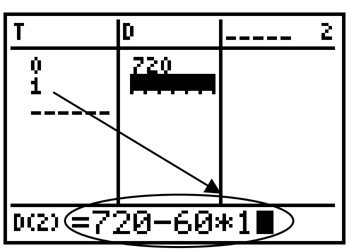
Typically, you may not find common or standard use of the symbols such as 1792 − 64LT to represent a situation. In mathematics, we normally us an "*x*" to represent a list of number such as is used above. So, we will likely use 1792 − 64*x*. When we graph the data relationship and the model 1792 − 64*x*, or −64*x* + 1792, we see the graph of the data points are on the graph of the model.

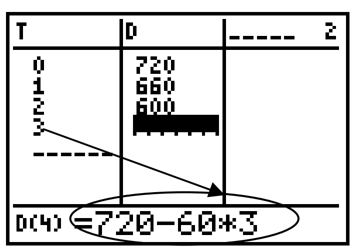


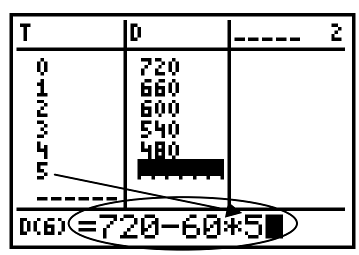
On the figure to the left, we see the graphical representation of the data relationship between time and the area remaining to be painted, and also displayed is the graphical representation of the symbols Y2 = −64X + 1792 with the data point (17, 704) displayed through the use of trace on the calculator. Through trace, we can see three representations of the house painter situation on the screen at the same time – with only one data point displayed at a time. It is important that you visualize all three representations simultaneously because the brain will automatically create neural associations of the neural circuits that process the three representations individually. This means you will more likely be able to recall the algebra involved.

When using a graphing calculator to graph the symbolic representation you will note the use of another symbol, Y2. It or another comparable symbol is used to represent the symbolic form of the situation −64*x* + 1792.

Driving Home On the 720-mile return trip home from Kitty Hawk, North Carolina to Columbus, Ohio, Professor Ed drove at an average rate of 60 mph throughout the night. Create a mathematical model that will find the distance *D* left to travel after *T* hours driven.

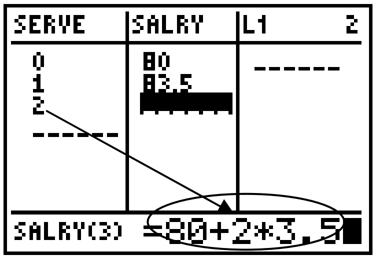


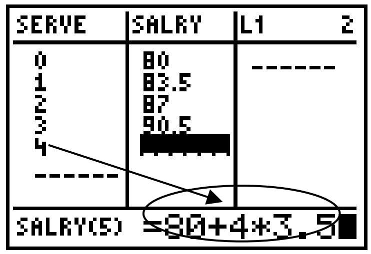


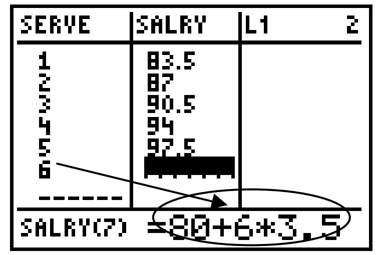


After each hour traveled, the distance left to travel changes (decreases) by 60 miles. The pattern shown above is very useful in developing the symbols 720 – 60*T*, where *T* is the time traveled. Since 720 – 60*T* represents the distance remaining in the trip, we might use *D* = 720 – 60*T* as the symbolic representation of the data relationship. However, if we want the graphical representation displayed on the graphing calculator, we will simply enter 720 – 60*x* in Y1 or Y2, etc. So the technology may force you into writing the symbolic representation as Y1 = 720 – 60*x*.

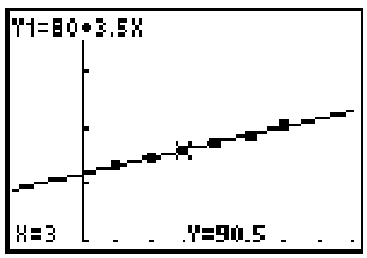
The Blue Point Café At many restaurants, servers are paid a set weekly salary plus tips. At the Blue Point Café in Duck, North Carolina, servers are paid $80 per week plus tips. A serious job seeker calls the local Restaurant Association and discovers that servers average $3.50 in tips per person served for the dinner meal.







The pattern established in the series of screens above suggests that the relationship is 80 + 3.5*SERVE*. Perhaps a more common way of writing the symbols is 80 + 3.5*S*, or even 3.5*S* + 80. Using a more common notation, we may even write *SALRY* = 3.5*SERVE* + 80. Like before, if we choose to find the graphical representation of the relationship using technology, we would likely use $y = 80 + 3.5x$, or Y1 = 80 + 3.5*x*.



TB Bacteria Below is the data relationship between time and the population of tuberculosis bacteria growing under unrestricted conditions. This is something like what might happen in a Petri dish in a medical laboratory. Time is in hours and the bacteria population is in thousands. This data is available in the program TBBACT63.

Tuberculosis Data

t	0	6	12	18	24	30	36	42	48
B	5	10	20	40	80	160	320	640	1280

The initial conditions are shown and the bacteria population is doubling every 6 hours.

T	*B*	Thoughts
0	5	At time 0-hours, the number of bacteria is obvious – the initial condition.
6	10	6 hours later, the number of bacteria is what? Sure, it is 5 doubled, or 10. How do you calculate it? OK, 5 × 2, but what is the relationship to “6 hours later?” It will help to think of 5 × 2 as 5×2^1? But what is the relationship to “6 hours later?” What arithmetic operation on 6 will yield 1?
12	20	6 hours later, the number of bacteria is what? Sure, it is doubled, or 20. How do you calculate it? OK, 10 × 2, but where does the 10 come from? OK, 5×2. So we now have 5×2×2, or 5×2^2. But what is the relationship to “12 hours?” What arithmetic operation on 12 will yield 2?

18	40	6 hours later, the number of bacteria is what? Sure, it is doubled, or 40. How do you calculate it? OK, 20 × 2, but where does the 20 come from? OK, 10×2. But where did the 10 come from? OK, 5×2. So we now have 5×2×2×2, or 5×2^3. But what is the relationship to “18 hours?” What arithmetic operation on 18 will yield 3?
24	80	6 hours later, the number of bacteria is what? Sure, it is doubled, or 80. How do you calculate it? OK, 40 × 2, but where does the 40 come from? OK, 20×2. But where did the 20 come from? OK, 10×2. But where did the 10 come from? OK 5×2. So we now have 5×2×2×2×2, or 5×2^4. But what is the relationship to “24 hours?” What arithmetic operation on 24 will yield 4?
…	…	Hopefully, at this point we see the pattern that the exponent on 2 is the number of hours divided by 6 – the doubling rate (they double every 6 hours).
T	*B*	So it looks like the model for this relationship is the initial condition times 2 (doubling) raised to the exponent of (time/6).

Finally, the generalized symbolic form is $5 \times 2^{\frac{T}{6}}$, or $B = 5\times2^{\frac{T}{6}}$

In every data relationship, there are two sets of numbers involved. One set contains those numbers that depend on input numbers selected by you. For example, in the tuberculosis bacteria situation above, time is determined by you, and it is independent of anything else. Once you select a time, you find that this time value determines the number of bacteria in the dish. All of the values you select are called the domain of the relationship, and the variable that represents these numbers is called the independent variable. All of the values determined from the input numbers you select are called the range of the relationship, and the variable that represents these numbers is called the dependent variable.

The data relationships used in Section 2.1 all had domains that were determined by the problem or the available information. A real-world problem determines its own domain. That is, the garbage problem presented in Section 2.0 has the domain [1960, 2010] because the data only exists for those years. However, when the symbolic representation is used to describe the garbage situation, there is no mathematical reason why the domain of $0.0371t - 69.9$ cannot be $(-\infty, \infty)$. Thus, you will quite frequently use a different domain when the symbolic representation is used.

A **variable** is a symbol that represents any element in the **domain** of a relationship. The *largest* set of real numbers that the variable in a relationship *can* represent is the **normal domain** of the relationship. The set of real numbers used for the variable when applied to a problem is the **problem domain**.

Since the domain is a set of real numbers, interval notation may be the best method of describing the domain.

Example 1: If the variable *x* represents all real numbers except 4, how can it be written?

Solution: It is sometimes written using set notation: $x \in \{\text{all real numbers}, x \neq 4\}$. The symbol $\in$ means belongs to, or is a member of. Another method is interval notation, $x \in (-\infty, 4) \cup (4, \infty)$.

⁂

Example 2: If the variable z represents all real numbers, describe this domain?

Solution: One method is to use set notation with English, $z \in \{\text{all real numbers}\}$, or use interval notation $z \in (-\infty, \infty)$.

⁂

The domain <u>can</u> be selected from the set of real numbers. If the domain of a relationship contains just one real number, the variable in the relationship is called a **constant**. For example, the symbol π is used to represent the one number, 3.14159...; thus, π is a constant. There are other symbols you will encounter that also represent only one real number; they are all constants.

π

Recalling the data relationships discussed in Section 2.1, you may have noted that the ranges of data relationships were also real numbers. Throughout most all of this text you will find that the range must be a set of real numbers.

> When a relationship is represented symbolically, the set of real numbers the symbols represent is called the **range** of the relationship.

Since the range must be a set of real numbers, you must check the symbolic from of the relationship for arithmetic operations that can create non-real numbers. You should recall two occurrences in your previous mathematical education that have given rise to non-real numbers, division by zero and the square root of a negative real number. Thus, if the symbolic representation of a relationship contains division or square root, you must exclude the values from the normal or problem domain that cause the relationship to be non-real.

The symbolic representation of relationships may contain a variety of different arithmetic operations. For example, the garbage problem $(0.0371t - 69.9)$ contains multiplication and subtraction. Many symbolic representations of relationships are terms and polynomials.

> A **term**, or **monomial**, is a <u>product</u> of real numbers and/or variables.

Examples of terms: $5x^2$, $-3x$, $14.3x^0$.

Recall that $x^0 = 1$, when x is not zero.

Also recall that $x^2 = x \cdot x$.

> A **polynomial** is a sum and/or a difference of terms.

For example, $3x + 7$ and $-3.6x^5 - 17x^3$ are polynomials. The following representations are not polynomials: $\frac{2}{x}$, $\sqrt{x} - 15$, and $\frac{x-5}{x^2 - 3x + 4}$.

Function Defined Finally, you may have noticed that all of the data relationships in this text have had exactly one element of the range (represented by the dependent variable) for each element of the domain (represented by the independent variable). Each relationship has *not* been like the following examples. Table 2.2.1 shows the relationship between the numbers (*N*) and letters (*L*) on your cell phone. For mathematical reasons think of the letters of the alphabet as integers 1 through 26.

Table 2.2.1

N	2	3	4	5	6	7	8	9
L	1,2,3	4,5,6	7,8,9	10,11,12	13,14,15	16,17,18,19	20,21,22	23,24,25,26

In this relationship, each number in the domain is related to three (or four) numbers from the range. Another example of this type of relationship comes from a family of five seated at a restaurant table designed for four as shown in Table 2.2.2.

Table 2.2.2

Seat	1	2	3	4
Person	1	2	3	4,5

In this example, the number 4 from the domain is related to both 4 and 5 from the range. Both of these examples demonstrate an important and basic difference between relationships. Nearly all relationships you will study will be of the first type -- where each element of the domain is related to only one element in the range. For the purpose of clarification, each type of relationship will be given a name. Instead of using the word relationship, mathematicians use the word **mathematical relation**, or just **relation**, to describe both types of relationships. The first type is also given the name function.

A **function** is a relation (relationship) where every real number in the domain is related (connected) to exactly one real number in the range.

Example 3: Is the relation $-2\sqrt{x+3}$ a function?

Solution: Yes, because any number from the domain will cause $-2\sqrt{x+3}$ to be exactly one real number.

⁂

Example 4: Is the relation below a function?

Table 2.2.3

t	–2	–1	0	1	2
$3t \pm 6$	–12, 0	–9, 3	–6, 6	–3, 9	–12, 0

10
5

Solution: No. For a number like –1 from the domain, there are two related numbers from the range, –9 and 3.

⁂

Example 5: Is the relation in Figure 2.2.1 a function?

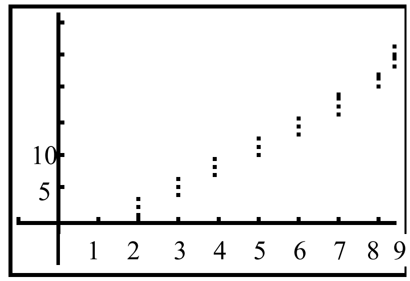


Figure 2.2.1

Solution: No. For example, the number 7 from the domain is related to four numbers from the range, 16, 17, 18, and 19. (Note: this is the graphical representation of the phone relationship.)

⁂

<u>Domain</u> The variable in a function must have a domain that causes the function to be a real number. Suppose a data relationship has a symbolic representation of $\frac{x^2 - x - 2}{x - 2}$; the domain of the function $\frac{x^2 - x - 2}{x - 2}$ cannot contain the number 2. That is, if x has a value of 2, the function is undefined, not a real number. The domain is $(-\infty, 2) \cup (2, \infty)$.

Table 2.2.4

x	. . .	0	1	2	3	4	. . .
$\frac{x^2 - x - 2}{x - 2}$	. . .	1	2	und.	4	5	. . .

The function $\sqrt{x+2}$ does not represent a real number if the domain contains numbers smaller than –2. The square root of a negative number is not a real number, and if x is less than −2, the function is the square root of a negative number. Therefore, the normal domain must be real numbers −2 and larger, or $[-2, \infty)$.

Table 2.2.5

x	. . .	–4	–3	–2	–1	0	1	2.5	. . .
$\sqrt{x+2}$	. . .	not real	not real	0	1	1.41	1.73	2.12	. . .

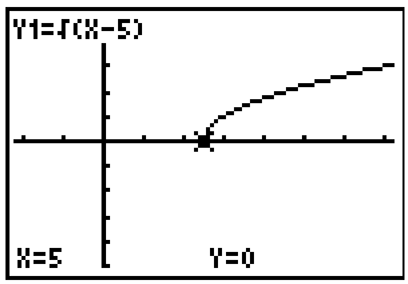
Example 6: Suppose the function $0.00153\sqrt{80000 - m}$ models the thickness of a brake pad (in inches) on a car driven m miles, what is the problem domain? What is the normal domain?

Solution: If m is larger than 80000, the function becomes a non-real number because $80000 - m$ is negative. The number of miles (m) driven cannot be less than 0. This means that the problem domain must be [0, 80000]. When the function is considered out of the context of the problem, it is mathematically acceptable for m to be negative (try it). Thus, the normal domain is $(-\infty, 80000]$.

⁂

Example 7: Find the normal domain of the function described by $\sqrt{x-5}$.

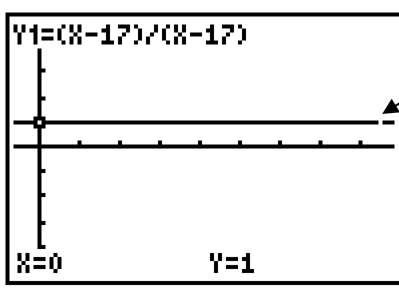
Solution: The square roots of negative numbers are not real numbers, thus the domain must be numbers that cause $(x - 5)$ to be on the interval $[0, \infty)$. If $x \in [5, \infty)$, then $x - 5$ is greater than or equal to zero. Thus the normal domain is all real numbers larger than or equal to 5. In interval notation, the normal domain is $[5, \infty)$. The graph below suggests that the function does not represent a real number to the left of 5 because there is no graph.



⁂

Example 8: Find the normal domain of the function described by $\dfrac{x-17}{x-17}$.

Solution: A real number divided by zero is not a real number; thus, the domain cannot contain 17. The normal domain is all real numbers except 17. In interval notation this is $(-\infty, 17) \cup (17, \infty)$.



Notice the hole in the graph when x is 17.

⁂

Example 9: Find the normal domain of the function described by $\dfrac{2}{(x-1)(x+2)}$.

Solution: Division by zero is undefined. Thus the quotient is not a real number, if x has a value of 1, $(x - 1)$ is zero. The function contains division by $(x - 1)$. So $x \neq 1$. If x has a value of -2, $(x + 2)$ is zero. The function contains division by $(x + 2)$. So $x \neq -2$. The normal domain is all real numbers except 1 and -2. Interval notation is a little messy; it is $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

⁂

Example 10: Find the normal domains of $\dfrac{1}{x+1}$, $\dfrac{1}{x+2}$, and $\dfrac{-5}{x+3}$.

Solution: The only operation that can cause the functions to not be real numbers is division by 0. Thus to avoid division by zero, eliminate -1 from the domain of $\dfrac{1}{x+1}$, eliminate -2 from the domain of $\dfrac{1}{x+2}$, and -3 from the domain of $\dfrac{-5}{x+3}$. The respective normal domains are $(-\infty, -1) \cup (-1, \infty)$, $(-\infty, -2) \cup (-2, \infty)$, and $(-\infty, -3) \cup (-3, \infty)$.

⁂

Example 11: Find the normal domain of $\sqrt{x-1}$, $\sqrt{x-2}$, and $\sqrt{x-3}$.

Solution: The operation that causes non-real numbers is the square root of negative numbers. To avoid the square root of negative numbers, $x - 1$ must be zero or positive. This can only happen if x is 1 or larger. Thus the normal domain of $\sqrt{x-1}$ is $[1, \infty)$. By the same reasoning, the normal domain of $\sqrt{x-2}$ is $[2, \infty)$, and the normal domain of $\sqrt{x-3}$ is $[3, \infty)$.

⁂

2.2 STRENGTHENING NEURAL CIRCUITS

Many times you can work the exercises by algebraic, graphic, or numeric methods. You can learn about mathematics no matter which method you use. Part of the learning process is for you to decide which method is best for you to use. Please read the text before you do the exercises. Read the **next assigned section** after you finish the assignment below.

Priming the Brain

-4. If you throw a ball straight up, does it have a maximum height?

-3. If you throw a ball straight up, does it have a zero height?

-2. Do you think the linear data below is increasing or decreasing? Why?

Time	1	2	3	4	5
Salary	\$6.50	\$13	\$19.50	\$26	\$34.50

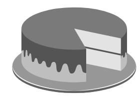
-1. When you get a drink at a drinking fountain, what do you think is the shape of the water path?

Neural Circuits Refreshed

1. Fifty-four percent of the current contribution to global warming comes from carbon dioxide. It is 392 parts per million of the current atmosphere (as of 5/2010). Write this number (392/1000000) in scientific notation.

2. In the United States, 14×10^9 pounds of trash is dumped into the ocean each and every hour. Write this number in standard notation.

3. Assuming that every person, on average, eats 1.5 pounds of food per day, how much food is consumed by all 6.9×10^9 people on earth in one year? (2010 data)



4. Use interval notation to describe all numbers greater than 4.02.

5. Describe the set of numbers $[-4, 3)$.

6. Is the function below linear, quadratic, exponential, or none of these? This data is available in the program MEDINS70.

Yearly Medical Charges	50	80	130	200	225	290	350	480	600
After Insurance Charge	50	80	130	200	205	218	230	256	280

Myelinating Neural Circuits

Find the symbolic representation of the relationships in Exercises 7 – 12.

7. A telemarketer makes $100 per week plus $1.00 per call made. Express salary in terms of calls made.

8. A lawn spreader (A device that puts fertilizer on your lawn.) is loaded with 50 pounds of fertilizer and is set to release approximately 0.01 pounds per square foot of lawn. Express fertilizer left in the spreader in terms of the area covered.

9. The electric company charges residential customers $9.50 a month as a meter fee plus they charge $0.11 per kilowatt hour of electricity used in the monthly billing cycle. Express monthly charges in terms of electricity used.

10. A 4 meg file can be down-loaded from the Internet at the rate of 0.25 meg per second. Express the amount remaining to be down-loaded in terms of time (in seconds).

11. 4000 quarters are dropped on the table. Approximately ½ of them are heads for each toss, and the heads are removed after each toss. Express the number of heads up in terms of the number of tosses x.

12. Control mice had a mean tumor volume doubling rate of 9.0 days. The size of the tumor upon first observation was 4 *ml*. With time in days, express the volume of the tumor in terms of time passed (in days). The data is available in the program MOUSEP70.

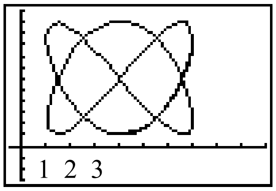
time	0	9	18	27	36	45
population	4	8	16	32	64	128

Are the relations in Exercises 13 – 18 functions?

13.

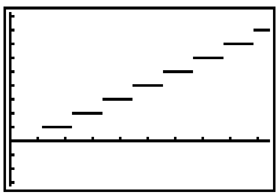
D	1	2	3	4	5	6	7	9
R	12	19	6	32	47	2	26	12

14.



15. $\sqrt{-x}$

16.



17. x^3+1

18.

D	1	2	3	4	5	4	3	2	1
R	1	3	5	7	9	8	6	4	2

Specify the normal domain of the functions in Exercises 19 – 50. START HERE

19. $\frac{1}{2}x-5$

20. $\frac{x-5}{3}$

21. $\frac{3}{4}x^2-\frac{5}{8}x+\frac{2}{3}$

22. $\frac{37}{x+5}$

23. $\frac{-58}{x+5}$

24. $\frac{9}{x+5}$

25. $\frac{(x-2)(x+3)(x-17)}{x+5}$

26. $\frac{x+5}{x+5}$

27. If the denominator of a fraction function is $x + n$, and the numerator represents a real number, what is the domain?

28. $\sqrt{x+1}$

29. $\sqrt{x+2}$

30. $\sqrt{x+3}$

31. $\sqrt{x+4}$

32. $\sqrt{x+19}$

33. What is the domain of $\sqrt{x+n}$, where n is on the interval $[0, \infty)$?

34. $\sqrt{x-5}$

35. $\sqrt{x-5}+8$

36. $-4\sqrt{x-5}+6$

37. $-4\sqrt{x-5}+6x$

38. $\sqrt{x-5}+3x^2$

39. What is the domain of $\sqrt{x-n}$, where n is on the interval $[0, \infty)$?

40. $\dfrac{x+1}{x-3}$

41. $\dfrac{x+1}{(x-3)(x+2)}$

42. $\dfrac{x+1}{(x-3)(x+2)(x+4)}$

43. $\dfrac{x+1}{(x-3)(x+2)(x+4)(x-7)}$

44. What is the domain of the function $\dfrac{2x^2-5x+17}{(x-n)}$, where n is on the interval $[0, \infty)$?

45. $\sqrt{2-x}$

46. $\sqrt{x+4}-\sqrt{x-2}$

47. $\dfrac{1}{\sqrt{-x}}$

48. $-\dfrac{\sqrt{x+4}}{x-2}$

49. $\dfrac{-4}{\sqrt{x+3}}$

50. $\dfrac{\frac{1}{x}}{\sqrt{x+5}}$

Identify a problem domain that makes sense with respect to the situation described in Exercises 51 – 58.

51. $0.055x$ — 5.5% sales tax function, with x representing the selling price

52. $0.27t + 0.50$ — long distance telephone charge of \$0.27 per minute and a \$0.50 connect charge, with t representing time in minutes

53. $3.80m + 4.25$ — taxi fare at \$3.80 per mile plus a \$4.25 user charge, with m representing the distance traveled in miles

54. $0x$ — financial return on investing x dollars in the state lottery

55. $-0.105\,|d-60| + 6.2$ — the height of an airplane above ground level on a trip from Columbus to Cincinnati, with d representing the ground distance from Columbus

56. $-16t^2 + 45t$ — the height of a rock thrown upward with t being time in seconds.

57. $\dfrac{R}{1-R}$ — the rate of markup based on the cost price, where R is the rate of markup based on the selling price

58. $\dfrac{100}{1+R}$ — the cost price of an item selling for \$100, where R is the markup rate based on the cost price

In Exercises 59 – 64, create any function that has a domain of:

59. $(-\infty, 6) \cup (6, \infty)$

60. $(-\infty, -4) \cup (-4, 9) \cup (9, \infty)$

61. $(-\infty, 3]$

62. $(-\infty, -2]$

63. $[4, \infty)$

64. $(4, \infty)$

In the next three exercises, find the symbolic form of the given relationship.

65. Elizabeth the nurse set the drip rate on a 1,000 *ml* IV bottle at 2 *ml* per minute. Find the symbolic form of the *TIME-IV* relationship. (Where *IV* is the amount left in the bottle at time *TIME*.)

66. The North Carolina Utility Commission sent a letter to all North Carolina Power customers with the following proposed usage charges, where k is the number of *kWh* used and E_c is the monthly electric bill. The data is available in the program ELECT73.

k	200	500	700	1000	1500	2000	3000
E_c	21.37	42.69	56.90	78.21	113.74	149.26	220.31

67. Farmer Tom bought a 100 bushel grain dryer and it came with the following "moisture chart" for corn initially at 24% moisture. Find the symbolic form of the *time-moisture* relationship. The data is available in the program CORN73.

Drying Time	0	15	30	45	60	75	90	105
% Moisture	24	22.8	21.6	20.4	19.2	18	16.8	15.6

From Mathematics to English

68. After reading this section, make a list of questions you want to ask your instructor.

69. Continue in your daily journal and make an entry. In addition to your normal entry on thoughts about the mathematics in this section, list at least two positive comments about what you have learned about this topic.

70. In paragraph format, summarize the material in this section of the text in your daily journal.

71. Describe how your classroom instructor made this topic more understandable and clear.

72. After reading the text and listening to your instructor, what do you not understand about this topic?

73. Explain why the normal domain of $\sqrt{x^2}$ is all real numbers.

74. Why can the domain of $x^2 + x + 1$ contain $\sqrt{7}$?

75. Can the domain of $x - 1$ contain 0.3333...? Why?

76. Describe the normal domain of the function $\dfrac{1}{(x-1)(x+1)(x-2)(x-3)(x+2)(x-5)(x+7)}$.

77. What is a function?

78. Is the relationship between the length of time a copier is running and the number of copies made a function relationship? Explain.

79. If the symbolic representation of the relationship between time and the number of hotel rooms cleaned by the housekeeping staff is $3.6t$, what are the differences between the normal domain and the problem domain? (t is time in hours.)

80. Why does the normal domain of $\dfrac{-7}{\sqrt{x+3}}$ not contain −3?

From English to Mathematics
In the next three exercises, write the English statement as a mathematical statement. For example, "seven times the sum of x and 4", would be $7(x + 4)$.

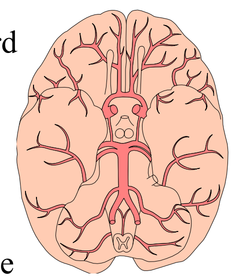
81. seventeen plus x

82. the difference of x minus 3, times 43

83. the length x times the width $(x + 4)$

Developing the Pre-Frontal Lobes
84. In a function, for every x there must be only one related value of the function. List the symbolic representations of two functions where not only is this true but it is also true that for every value of the function there is only one corresponding value for x.

Basic Brain Function – What's with the Prefrontal Lobes Stuff?



Directly behind your forehead are the prefrontal lobes of your brain. The word lobes is used because there are two regions – one on left, and one on the right. These regions contain the decision-making circuits (connected neurons), as well as most all of the high level functioning activity in your brain. The prefrontal lobes are the last circuits of your brain to develop. Actually, they are not finished developing until you are about 21 to 23 years of age. The implication is that people under this age sometimes do not make the best decisions. As you might be thinking, even adults do not always make the best decisions, or cannot always think clearly. You are right. But why, when they are over 23?

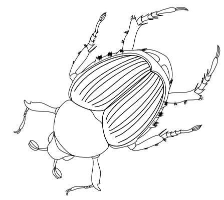
There is a common saying in life, "use it or lose it." Or saying it another way: mental activity increases the production of molecules that are thought to enhance mental activity. This applies directly to brain function – especially thinking and decision-making. It may not be obvious, but the items under the heading **Developing the Pre-Frontal Lobes** can sometimes be a little more challenging. Actually there are a variety of explorations, concept quizzes, investigations, or modeling projects (found in the ancillary workbook) that can be challenging. Even many of the other exercises in this textbook can be challenging. Why are they in the textbook if they are "hard?" Because the only way to have, and keep properly functioning prefrontal lobes is to do challenging stuff – like the exercises in this textbook and in the ancillary activity workbook. As it turns out, if you do not do challenging mathematics, or anything that is challenging, it is akin to having a lobotomy. When an adult has a lobotomy (for unrelated medical reasons), they can no longer think critically nor can they make complicated decisions. So we are back to why some (many) adults cannot think clearly and are poor at decision-making. We are also back to why you need to do challenging stuff – like algebra.

Well, all this information is something to think about.

6.1 Solving Equations Containing the Linear Function

The Beetles Are Coming

Suppose that you raise red raspberry plants and you notice that on June 28 there are 17 Japanese Beetles on your small set of plants. The next day you see 4 more and the day after 4 more, etc. The function that models the number of beetles on your raspberry plants is $17 + 4x$, where x is time in days starting at 0 on June 28. This model may also be written as $4x + 17$.



You should recognize the function $y = dx + e$ as a linear function with a rate of change of d and initial condition of e. Solving the equation $dx + e = f$ is just another way of asking the question: When does the graph of $dx + e$ have (equal) a value of f? Another way of looking at solving equations is to find when (the value of x) the numeric representation of the function is f. That is, consider the point (–3, 5) (This means 3 days before June 28 there were 5 beetles on your raspberry plants.) on the graphical representation of the function $4x + 17$ or as a data pair in the numeric representation of $4x + 17$, as shown below in Figures 6.1.1 and 6.1.2.

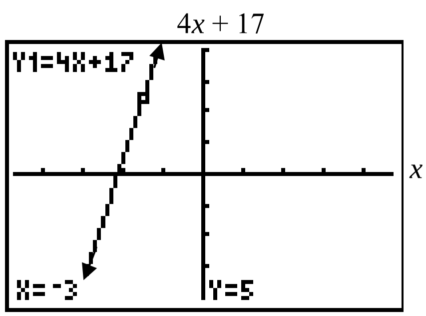


Figure 6.1.1

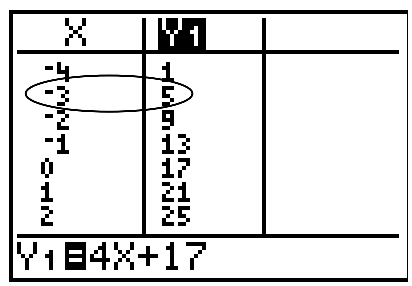


Figure 6.1.2

This pair of numbers describes the solution to the equation $4x + 17 = 5$, where –3 is the solution. That is, the value of –3 causes the functions to be equal (in this case, a value of 5).

Of course an equation containing the linear function may be considerably more complicated, such as an equation that has a linear function on both sides of the equal sign, like $-3(x + 5) - 9 = 4x + 4$. The idea behind solving the equation remains the same; find a value for x that makes the linear function on the left have the same value as the linear function on the right? If both functions are graphed and the intersection of the graphs is on the calculator screen, you have found the solution. Why? Because at the point of intersection, the x value causes both function to be the same (equal).

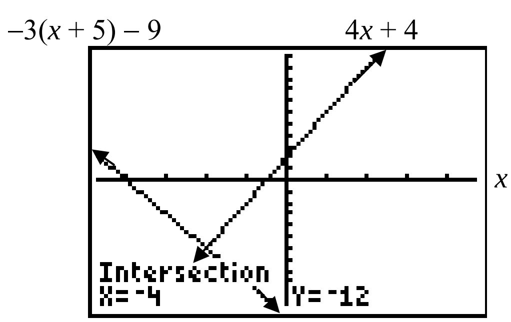


Figure 6.1.3

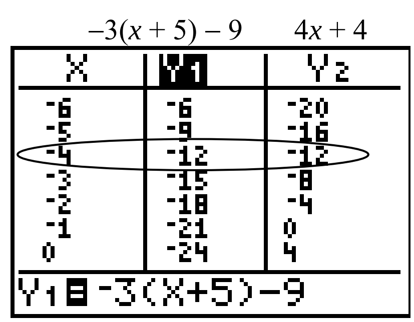


Figure 6.1.4

Thus, you may now see a method for solving equations. If the equation is simple, you may want to try to trace to the solution on a decimal window. If the equation is a little more complicated, you may want to try to find the intersection with the intersection finder on your calculator. Also demonstrated above is the numerical method -- finding the numeric representation that shows a value for x that makes both functions the same. Still another technology-based method will be developed later and a method using algebra will end the possible methods for solving an equation. It will not suffice for you to learn one method. You will find that any one method will either not solve all types of equations or may be very difficult to use on some types of equations.

The Numerical Method

The objective in solving equations is to find all values for the variable that make the equation a true statement. That is, in solving the equation $2x + 5 = 13$, you know $2x + 5$ can have an infinite number of values because the linear function $y = 2x + 5$ has a range of all real numbers. Since the goal is to make a true statement, look for a value for x that yields the element 13 from the range. The numerical approach to solving equations is to make a table and look for a value of x that causes the function $2x + 5$ to be 13. Below the table shows the exact solution to be 4 - a value for x that causes the function to be 13.

Table 6.1.1

x	1	2	3	**4**	5	6	7	8	9
$2x + 5$	7	9	11	**13**	15	17	19	21	23

π -6 $\sqrt{3}$ 0 $\frac{1}{2}$

What if the table does not show the solution? This is possible for a couple of reasons. First, what if the solution to the above equation were 4.5 instead of 4? You would have observed that the function is below 13 when x is 4 and above 13 when x is 5; therefore, you conclude that the solution must be between 4 and 5 and then you must make a table for the function between 4 and 5 with steps in x of 0.1. This may give the answer. The other case where the solution is not in your table is when the solution is smaller than your first number for x or larger than your biggest number for x. Make a new table where the function values are closer to 13 and hope to find the solution in the new table. This whole process may seem long; however, a graphing calculator can make the table for you as fast as you can enter the function. The beauty of the table is that once it is set, you can scroll forward or backward until you find the value of x that causes the function to be 13. If you can only come close to 13, you can set a new table with Δx of 0.1 or smaller and try again for the value of x that causes the function to be 13.

Trying a second equation may help you understand the numerical method. Below is the solution to the equation $-2(x + 5) + 7 = -32$. Start with a table where *xmin* is a reasonable guess. For example, start the table with -5 (*xmin*) and let Δx be 1.

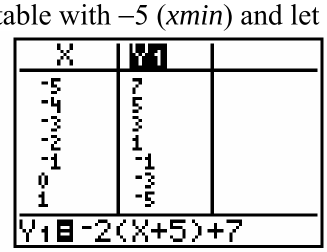
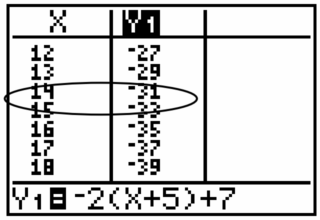


Figure 6.1.5

For values of x from -5 to 1, the function is not -32. You can see that if you scroll down, the function gets closer to -32. Figure 6.1.6 shows the results of scrolling down until numbers near -32 are on the table.

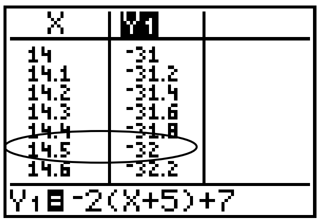


X	Y1	
12	-27	
13	-29	
14	-31	
15	-33	
16	-35	
17	-37	
18	-39	

Y1 = -2(X+5)+7

Figure 6.1.6

You can see that this table has x values that cause the function to jump over the function value of –32. Refine this table by using an *xmin* of 14 and Δx of 0.1 This may show a better solution than the approximate value of 14 or 15.



X	Y1	
14	-31	
14.1	-31.2	
14.2	-31.4	
14.3	-31.6	
14.4	-31.8	
14.5	-32	
14.6	-32.2	

Y1 = -2(X+5)+7

Figure 6.1.7

Figure 6.1.7 shows that if x is 14.5, the function is exactly –32. The equation has been solved by the numerical method.

The Trace Method

A small search party is looking for a missing camper in a 150 acre wooded area. They can cover 2 acres in 6 minutes. Therefore the function that models the amount of area remaining to be search at time t is $A = -\frac{2}{6}t + 150$. How much time will it take until 80 acres are left to search? To find out, solve the equation $80 = -\frac{2}{6}t + 150$. Using the trace method requires that you graph the function $A = -\frac{2}{6}t + 150$. Use trace until the value of A is 80. If you can't get an exact value of 80 for A from the trace cursor, either use the approximate value or change to a decimal window. Figure 6.1.8 shows an approximate solution of about 208.5 minutes and Figure 6.1.9 shows the trace cursor on the exact solution of 210 minutes.

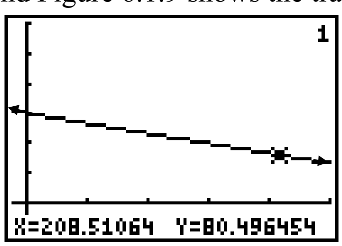


Figure 6.1.8

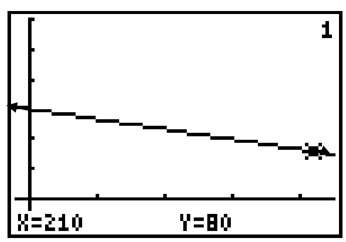
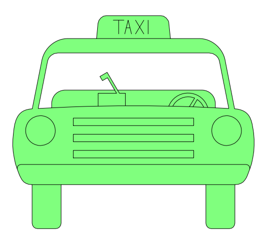


Figure 6.1.9

The Intersection Method

The Taxi Fare



Suppose that taxi company A has a fare schedule of $1.20 per mile plus a $3 usage fee. Taxi company B charges $0.90 per mile plus a $5 usage fee. The model of the taxi fare for company A is $F_A = 1.2m + 3$ and for company B it is $F_B = 0.9m + 5$. How far can you travel so that the fares are equal? The answer to this question can be found by solving the equation $1.2m + 3 = 0.9m + 5$. What you really want to know is when F_A equals F_B. Remember, this means you must find a value for m that causes function F_A to equal function F_B.

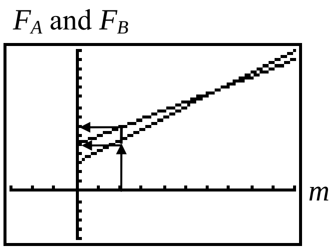


Figure 6.1.10a

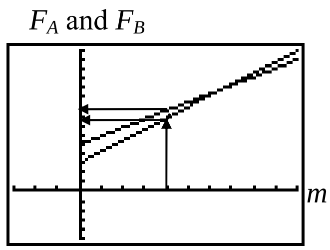


Figure 6.1.10b

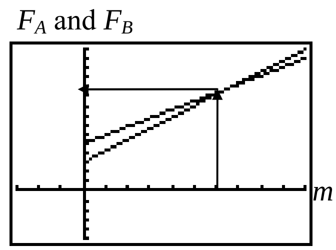


Figure 6.1.10c

Please notice in Figure 6.1.10a that at 2 miles, F_A (2) = 5.4 and F_B (2) = 6.8. So, at two miles, the fares are not equal. Then in Figure 6.1.10b, at 4 miles driven, F_A (4) = 7.8 and F_B (4) = 8.6. So, at four miles, the fares are not equal. But at 6.67 miles driven, F_A (6.67) = 11 and F_B (6.67) = 11. That is, the only distance traveled (6.67 miles) that gives the same taxi fare ($11.00) is at the intersection of the two fare models. Thus, at the intersection of the two graphs, F_A equals F_B and the *m*-coordinate at that point is the value of *m* that causes F_A to equal F_B. Figure 6.1.11a shows you that if you drive approximately 6.67 miles the taxi fare is $11 for both companies. The solution to the equation is approximately 6.67 miles, or

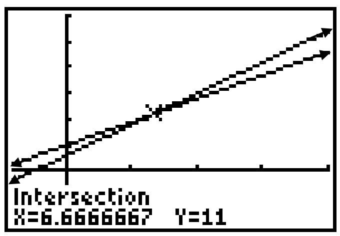


Figure 6.1.11a

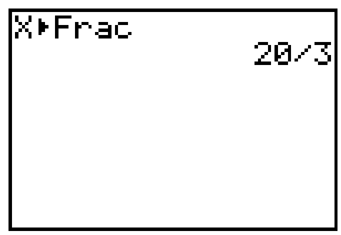


Figure 6.1.11b

as shown in Figure 6.1.11b, the exact solution is $\frac{20}{3}$ or $6\frac{2}{3}$ miles.

The Zeros Method

The last function-based method combines what you learned in Chapters Two, Three, and Four about finding the zeros of a function and what you learned about manipulating the symbols in an equation in Chapter One.

As a sample of how to solve with the zeros method, let's solve the equation used above; that is, solve the equation $1.2m + 3 = 0.9m + 5$. To use the zeros method, use the properties of equality from Chapter One to transform the above equation so zero is on one side of the equation and all symbols on the other.

$1.2m + 3 = 0.9m + 5$	subtract 0.9*m* and 5 from both sides
$1.2m + 3 - 0.9m - 5 = 0$	no need to simplify unless you want to

Recall that using the subtraction property of equality does not change the solution to the equation, thus solving the equation $1.2m + 3 - 0.9m - 5 = 0$ is the same as solving the equation $1.2m + 3 = 0.9m + 5$. The left side of the transformed equation is the function $1.2m + 3 - 0.9m - 5$, and the question you are being asked to answer is when is the function $1.2m + 3 - 0.9m - 5$ equal to 0? This is not new! You learned how to find zeros with the "zero finder" on the calculator. Your calculator may use the word "root" instead of "zero". This is a name mathematicians use for the solution to any equation. Since the **root** of the equation is the same as the "zero" of the function on the left, you are using the zeros method for solving the equation by finding the zero of the related function.

The roots of an equation are the zeros of the transformed function.

Figure 6.1.12 shows the approximate zero of the function 1.2*m* + 3 – 0.9*m* – 5 and Figure 6.1.13 shows the exact zero.

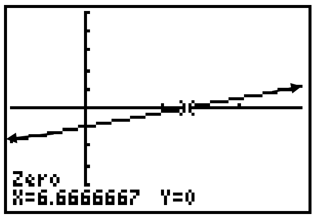


Figure 6.1.12

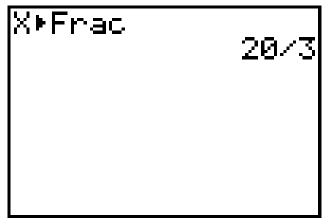


Figure 6.1.13

A nice feature of this method is that you always know where to find the real solution to the equation - it is always on the *x*-axis.

The Symbolic Method

The Electric Company

If the North Carolina Power Company charges $0.07 per *kWh* (kilowatt hour) of electricity used plus a monthly service fee of $9.50 and the Ohio Power Company charges $0.08 per *kWh* used plus a $3.75 monthly service charge, what number of *kWh* used will cause the monthly electric bill from North Carolina to be the same as from Ohio? The mathematical model of the Carolina Power electric bill is $E_C = 0.07k + 9.5$, and the Ohio Power model is $E_O = 0.08k + 3.75$. Since you want to know what value of *k* makes the electric bill from North Carolina Power (0.07*k* + 9.5) equal the electric bill from Ohio Power (0.08*k* + 3.75), solve the equation $0.07k + 9.5 = 0.08k + 3.75$. The equation can be solved by using the properties of equalities studied in Chapter 1. As a quick reminder they are listed below.

Addition Property:	If $a = b$ then $a + c = b + c$
Subtraction Property:	If $a = b$ then $a - c = b - c$
Multiplication Property:	If $a = b$ then $ac = bc$ where $c \neq 0$
Division Property:	If $a = b$ then $\frac{a}{c} = \frac{b}{c}$ where $c \neq 0$

The equations

$$a = b$$

$$a + c = b + c$$

$$a - c = b - c$$

$$ac = bc$$

$$\frac{a}{c} = \frac{b}{c}$$ are all equivalent equations ($c \neq 0$).

If a number is added to, subtracted from, or a non-zero number is multiplied by, or divided into both sides of an equation, the original equation and the **transformed** equation have the same solutions. These basic properties of equality allow for a symbolic method for solving equations.

The algorithm used in solving equations with algebra is to isolate the variable on one side of the equation. This can be accomplished by using the properties of equality. With the variable by itself on one side of the equation, the solution is on the other side of the equal sign. When the algebraic method works, it can give the *exact* solution to the equation, or it can be approximate as shown below. (An **algorithm** is a set of steps used to accomplish a goal.)

One option to solving the equation is:

$0.07k + 9.5 = 0.08k + 3.75$	subtract $0.08k$ from both sides
$0.07k + 9.5 - 0.08k = 0.08k + 3.75 - 0.08k$	simplify
$-0.01k + 9.5 = 3.75$	subtract 9.5 from both sides
$-0.01k + 9.5 - 9.5 = 3.75 - 9.5$	simplify
$-0.01k = -5.75$	divide both sides by –0.01
$\frac{-0.01k}{-0.01} = \frac{-5.75}{-0.01}$	simplify
$k = 575$	

If a customer in North Carolina and one in Ohio use 575 *kWh* of electricity, their electric bills will be identical.

You should try each of these methods and select two or three methods to use on a regular basis. You must not assume you can always use just one method because any one of these methods may not solve all equations you encounter.

Example 1 Solve the equation $3(x - 6) - 5(x + 7) = -23$ numerically.

Solution: Make a table and look for a value of x that causes the function $3(x - 6) - 5(x + 7)$ to have a value of –23. The written page cannot contain a dynamic table as you may make on your calculator; instead, you will only see a static table as shown below.

Table 6.1.2

x	–10	–9	–8	–7	–6	–5	–4	–3	–2
$3(x-6)-5(x+7)$	–33	–35	–37	–39	–41	–43	–45	–47	–49

After looking at the behavior of the function, it appears that the solution is less than –10. Make another table.

Table 6.1.3

x	–20	–19	–18	–17	–16	**–15**	–14	–13	–12
$3(x-6)-5(x+7)$	–13	–15	–17	–19	–21	**–23**	–25	–27	–29

The solution is –15; it is a value for x that causes the function to be –23.

⁂

Example 2: Solve the same equation $3(x-6)-5(x+7)=-23$ with the trace method.

Solution: Graph the function $y=3(x-6)-5(x+7)$ and trace to a point where it has a value of –23.

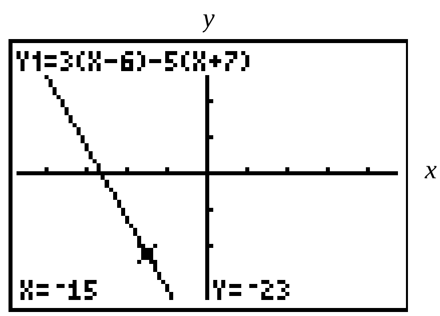


Figure 6.1.13

The trace cursor is on the point where the function is –23. The solution to the equation $3(x-6)-5(x+7)=-23$ is –15.

⁂

Error

You may want to think of error as the difference between your answer and the exact answer. Further, since you may want to interpret error as a zero or positive number, find the absolute value of the difference between your solution and the exact solution.

> **Error:** The absolute value of the difference between the exact root (R) and your approximation of the root (r).
> $$error = |R - r|$$
> where R is the exact root of an equation and r is your approximation of R.

Since the calculator does not always give exact values for the solution (root) to an equation, as a general rule, you must express all roots with an error less than or equal to 0.01. The exceptions are 1) if the solution (root) is less than 0.01, or 2) if the solution is a very large number. In these two cases, you may approximate the root to whatever makes sense in the equation being solved.

Example 3: Solve the equation $\frac{1}{4}x - 12 = -20$ using the zeros method.

Solution: Add 20 to both sides of the equation to get the transformed equation $\frac{1}{4}x - 12 + 20 = 0$. Find the zero of the function $f(x) = \frac{1}{4}x - 12 + 20$ with the zero finder on the calculator. The exact solution is –32.

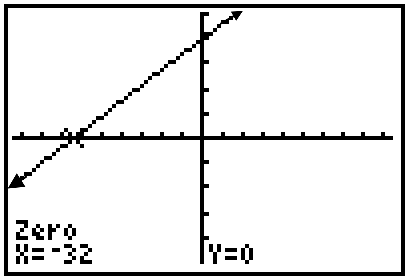


Figure 6.1.14

⁂

Example 4: Solve $3.5x + 2.1 = -5.9$ using the analytical (algebraic) method.

Solution:

$3.5x + 2.1 = -5.9$	subtract 2.1 from both sides
$3.5x + 2.1 - 2.1 = -5.9 - 2.1$	simplify
$3.5x = -8$	divide both sides by 3.5
$\frac{3.5x}{3.5} = \frac{-8}{3.5}$	simplify
$x = -2.29$	

–2.29 is an approximate solution and $\frac{-16}{7}$ is the exact solution. The solution is also called the root of the equation.

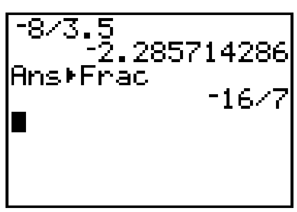


Figure 6.1.15

⁂

Example 5: Solve $5.3 - \sqrt{7}x = 12.6$ with an error ≤ 0.01. Use the zeros method.

Solution: Subtract 12.6 from both sides of the equation and then graph the related linear function $y = 5.3 - \sqrt{7}x - 12.6$. In entering the number $\sqrt{7}$ times x on the calculator, you may want to put a times sign between them or parentheses around one of the factors to avoid the difference between $\sqrt{7x}$ and the correct form $\sqrt{7} \cdot x$. Find the zero with the zero finder.

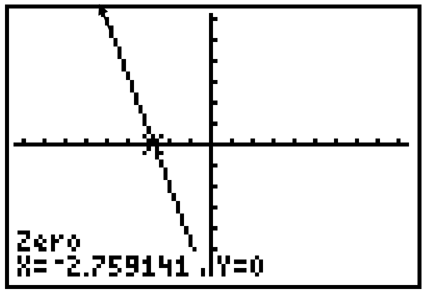


Figure 6.1.16

The solution to the equation $5.3 - \sqrt{7}x = 12.6$ is –2.76, where the error ≤ 0.01

⁂

Example 6: Find the exact solution to $5.3 - \sqrt{7}\,x = 12.6$.

Solution:

From $5.3 - \sqrt{7}\,x = 12.6$	subtract 5.3 from both sides
$5.3 - 5.3 - \sqrt{7}\,x = 12.6 - 5.3$	simplify
$-\sqrt{7}\,x = 7.3$	divide both sides by $-\sqrt{7}$
$\dfrac{-\sqrt{7}x}{-\sqrt{7}} = \dfrac{7.3}{-\sqrt{7}}$	simplify
$x = \dfrac{-7.3}{\sqrt{7}}$	the exact solution is $\dfrac{-7.3}{\sqrt{7}}$.

⁂

Example 7: Solve $3x - 5 = x + 2$ with an error ≤ 0.01 using the numerical method.

Solution: Subtract x and 2 from both sides of the equation to yield the equivalent equation $2x - 7 = 0$. Find the zero of the related function $y = 2x - 7$. The zero is shown below in the second table.

Table 6.1.4

x	–1	0	1	2	**3**	**4**	5	6
$y = 2x - 7$	–9	–7	–5	–3	–1	1	3	5

The data above shows that there is a zero between 3 and 4. At 3 the function is negative and at 4 it is positive; some place between 3 and 4 it must be 0. Make a new table between 3 and 4 using Δx of 0.1.

Table 6.1.5

x	3	3.1	3.2	3.3	3.4	**3.5**	3.6	3.7
$y = 2x - 7$	–1	–0.8	–0.6	–0.4	–0.2	**0**	0.2	0.4

The value of x that causes the function to be 0 is 3.5. The solution to the equation $3x - 5 = x + 2$ is exactly 3.5. (The error is 0.)

⁂

Example 8: Solve the equation $5(x - 2) = -3(2x + 3) - \pi$ by the intersection method.

Solution: The intersection method works because the only value of x that causes the function $y = 5(x - 2)$ and the function $y = 3(2x + 3) - \pi$ to be equal is the x-coordinate of the intersection of the graphs. The intersection finder is used to locate the intersection.

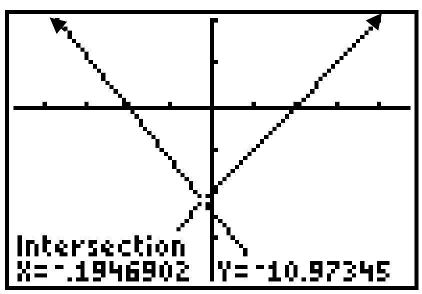


Figure 6.1.17

If the calculator values are rounded to the hundredths place, the error will be less than 0.01. The solution to the equation $5(x - 2) = -3(2x + 3) - \pi$ is –0.19.

⁂

6.1 STRENGTHING NEURAL CIRCUITS

Many times you can work the exercises by algebraic, numeric, or graphical methods. You can learn about mathematics no matter which method you use. Part of the learning process is for you to decide which method is best for you to use. Please read the text carefully before you do the exercises. Read the **next assigned section** after you finish the assignment below. All numeric answers must have an error ≤ 0.01.

Priming the Brain

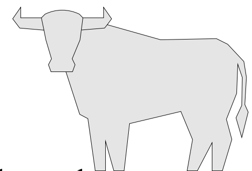
-4. Every point on the graph of $y = x + 1$ can be symbolized as (x, y) or $(x, x + 1)$. Describe all points below the graph of $y = x + 1$.

-3. If $|x - 1| = 2$, does this imply that $x - 1 = -2$?

-2. Describe a relationship where as one variable increases, the related variable increases.

-1. What happens to the thickness of a sheet of paper every time you fold the paper in half?

Neural Circuits Refreshed



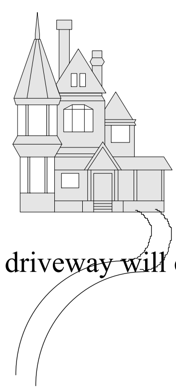
For every fast-food hamburger from Central American cattle, about 55 square feet of rain forest is destroyed. *Earth Right - Every Citizen's Guide*, H. Patricia Haynes, p. 217, 1990.

If n is the number of hamburgers sold and A is the area of rain forest destroyed to make grazing land for the cattle used to make the hamburgers, answer the first three questions.

1. Express the rain forest area destroyed as a function of the number of hamburgers sold.
2. How many acres of rain forest must be used to produce the beef for 10,000 hamburgers?
3. How many hamburgers are sold for every 10,000 square feet of rain forest destroyed?
4. What linear function contains the points $(-2, 5)$ and $(-3, -7)$?
5. What is the linear function whose graph has a slope of $-\frac{2}{3}$ and contains the point $(0, -2)$?
6. What are the slope and y-intercept of the graph of $y = \frac{1}{4}x - 7$?

Myelinating Neural Circuits

Solve the following equations by any method of your choice.



7. If it costs \$2,400 to pave the driveway to your home with asphalt and each year it costs \$325 to re-seal the driveway with blacktop sealer, the function that models the total cost of the driveway is $C = 325t + 2400$. Find when the driveway will cost \$3,000. Find when it will cost \$3,500. Find when it will cost \$4,200.

8. If the cost of using concrete on the same driveway in Exercise 7 is \$3,650 and it costs \$0 to maintain the driveway year after year, the function that models the total cost of the concrete driveway is $C = 0t + 3650$. In what year does the cost of the asphalt driveway equal the cost of the concrete driveway?

9. The Cleanit Car Company (CCC) pays each member of its towel drying crew a weekly salary of $50 plus $.75 per car. The crew, being nearly graduated from college, realizes that with special care of each car they can average $.45 per car in tips. How many cars must be washed at the Cleanit Car Company for a towel dryer to gross $200 per week? The function that models gross salary is $S(c) = 0.75c + 0.45c + 50$.

10. The Better Cleanit Car Company (BCCC) pays each member of its towel drying crew a weekly salary of $80 plus $.55 per car. This crew also has nearly graduated from college and realizes that with special care of each car they can average $.71 per car in tips. How many cars must be washed at the Better Cleanit Car Company for a towel dryer to gross $200 per week? The function that models gross salary is $S(c) = 0.55c + 0.71c + 80$.

11. What number of cars must be washed for the gross salary of a CCC employee to equal the salary of a BCCC employee?

12. Archeologists hired college students to simulate the building of a Hopewell Indian burial mound. Each of 150 students was given a 2 cubic foot basket in which to transport soil. One student could increase the volume of the mound by 2 cubic feet per 20 minutes. How long did it take for all 150 students to build a mound with a volume of 50,000 cubic feet? How long would it take if only 35 students worked? How long would it take for all 150 students to build a 140,000 cubic foot mound?

13. At a New Years Eve party, the caterer supplied a bowl with approximately 1000 chips. During the party, guest #1 ate the chips at a rate of 22 chips per minute (cpm), guest #2 at 13 cpm, guest #3 at 26 cpm, and the caterer added chips to the bowl at an average rate of 47 cpm. Are the number of chips increasing or decreasing? When are there 200 chips left in the bowl? When is the bowl empty? If guest #2 doubles her eating rate, when will the bowl be empty?

14. To prepare for a mathematics conference, hotel workers must take 5000 chairs from the storage room and set them up for the opening session. David sets up 10 chairs every 3 minutes. Diana sets up 12 chairs every 4 minutes, and Debby sets up 7 chairs every 2 minutes. How long does it take to be half finished? How long does it take to have no chairs left to set up?

15. The ABC Painting Company is doing a job cost estimation for a house with 4200 square feet surface area. Painter *A* paints and prepares at a rate of 125 square feet per hour. Painter *B* paints and prepares at a rate of 65 square feet per hour. Painter *C* works at 23 square feet per hour. (Painter *C* is a trainee and is painter *B*'s son.) How long will it take to paint 75% of the house? How long will it take so that there is 0 square feet left to paint? If the ABC Painting Company charges $10.50 per person per painting hour, how much should they estimate as the labor cost for the job?

16. Downloading software from a web site to your computer transfers at a rate of 44K per second. The software being downloaded is 391K is size. How long will it take to download the software? If you log in at a faster rate of 54K per second, how long will it take? What download rate would cause the software to download in 1 minute?

17. $5.7x + 2.9x - 1.4 = \sqrt{6}$

18. $\sqrt{3}x + 2 = 1.5$

19. $\pi x = 2$

20. $3.2(1.5x + 7.2) - 1.5(x - 5.1) = 7(x + 3) - 5$

21. $\sqrt{2}(x - \sqrt{5}) = x + \frac{1}{2}$

22. $-(-[x + 4] - 13) = 2x - 5$

23. $0.002x + 15 = 300$

24. $500x = 5000$

25. $\sqrt{7}(x - \pi) = 2.5(x + \sqrt{2.9})$

26. $2(x + 1) = 2x + 1$

27. $65000(x - 5) = 42000x$

28. $(\frac{1}{4})x + \frac{1}{2} = (\frac{3}{4})x$

29. $0.80 = \dfrac{416 + x}{580}$

30 List any linear equation (other than $x = 5$) that has a solution of 5.

31. List any linear equation (other than $x = -5$) that has a solution of –5.

32. List any linear equation (other than $x = \pi$) that has a solution of π.

From Mathematics to English

33. After reading this section, make a list of questions that you want to ask your instructor.

34. Continue in your daily journal and make an entry. In addition to your normal entry on thoughts about the mathematics in this section, list at least two positive comments about what you have learned about this topic.

35. In paragraph format, summarize the material in this section of the text in your daily journal.

36. Describe how your classroom instructor made this topic more understandable and clear.

37. After reading the text and listening to your instructor, what do you not understand about this topic?

38. What is the most important idea presented in this section? Why?

39. Which method would you use to solve the equation $0.002x - 1.05 = 3.001$? Why?

40. Describe how to solve the equation $2x - 5 = 4.1$ with an error less than 0.01, using the graphical method.

41. Describe how to solve the equation $2x - 5 = 4.1$ with an error less than 0.01, using the numeric method.

42. Which method would you use to solve the equation $3x = -15$? Why?

43. Describe the relationship between solving an equation and finding the zeros of a function.

44 Discuss why you know some linear equations have no solution and other linear equations have an infinite number of solutions. Give examples of each type.

45. Using the analytic (algebraic) method for solving equations, describe the steps you would use to solve the equation $-3(x + 5) = 17.6$. Do not actually solve.

46. Using the graphical method for solving equations, describe the steps you would use to solve the equation $-3(x+5)=17.6$. Do not actually solve.

47. What is an equation?

48. What is the “solution” to an equation?

49. Why should you check a solution to an equation?

50. If you memorize a method for solving an equation, do you understand what a solution to an equation is? Explain.

51. Can every equation be solved using technology? Explain.

52. Can every equation be solved using algebra or other mathematics? Explain.

From English To Mathematics

In the next three exercises, convert the English statement to an equivalent mathematical statement.

53. three times a number *x* is fifty-two

54. the length is three times the width

55. the product of seven and a number *x* is four minus the number *x*

Developing the Pre-Frontal Lobes

56. Using interval notation (except for zeros), describe when each of the linear functions has a value less than zero, zero, and greater than zero. Approximate numbers are acceptable as long as the error is ≤ 0.01

		$f(x) < 0$	$f(x) = 0$	$f(x) > 0$
a.	$f(x) = 4x + 7$			
b.	$f(x) = -3x + 2$			
c.	$f(x) = 0.06x - 9$			
d.	$f(x) = -53x + 80$			
e.	$f(x) = (\frac{1}{4})x + 2$			
f.	$f(x) = -(\frac{3}{4})x + 5$			
g.	$f(x) = \pi x + 1$			

57. Solve the following linear equations by a method of your choice. If you use the algebraic method, show your work; if you use the graphical method, show the graph used to yield the solution.

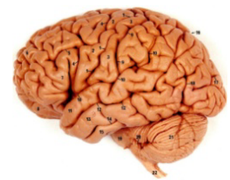
a. $-2[x - 5(3x + 4) - 6] + 6[-4 + 7(9x - 1) - 2(x - 6)] = 0$

b. $0.003x - 4.932(x + 79) = -23\{5x - 2[-7.4x + (-5)] - 32.69\}$

c. $-420x - 250(32x - 57) - [-49x - 21(17x + 89) - 430x] = 425(84x - 7)$

d. $0 = 0.24x - 0.11(-88 + 14x) - 7\{2 - 5[3 - 5(2x - 1) - 9.3] + 12.5\}$

Basic Brain Function – Connections Revisited



The central theme of this textbook is the mathematical idea of "function." The reason is that every concept or procedure in the textbook can be connected through function representation (numeric, graphic, or symbolic) or function behaviors (increasing/decreasing, maximum/minimum, zero/positive/negative, rate of change, and domain/range). Why is this important? Because of the way the brain stores and recalls something learned – memories.

When your brain is presented with something new, it AUTOMATICALLY tries to connect the new stuff to what you already know. The problem in an algebra course like this one is that there may be nothing in your brain related to the algebra you are learning. But this does not stop your brain from connecting to something – anything. And here is the problem for teaching and learning. What we really want to happen is for your brain to connect to previously learned algebra and to simple real-world contexts (like an I. V. drip or a ball tossed straight up) that have mathematical properties that are related to the topic at hand. Otherwise your brain may connect the current algebraic concept/procedure to a wide variety of totally unrelated ideas already in your brain. If teachers knew what your brain likely connects an algebraic concept to, that information could be used to teach other mathematical ideas. Anything you learn is stored in your brain as clusters of related (connected) ideas. Therefore, this textbook connects every new idea or procedure with previously learned ideas or to contextual real-world situation to help your brain store and retrieve memories. Why is this important when you know people who could learn without the teacher and textbook connecting everything?

It turns out that ALL recall of something learned is processed in the brain through a series of connections. If every neuron in the networks storing a concept or procedure fires (discharges electricity to the next neuron in the circuit) it is likely that that thought will reach consciousness – you will recall it. Further, to be able to recall better, you need lots of connections to the thing you need to recall. There are many connections to every mathematical concept presented in this text. As for people who can recall without the connections being made in the textbook or by the teacher, he/she has very likely learned how to make good connections without thinking about it. But not many people are like this.