

Math 3331 ODEs - Sample Final - Spring 2024

1. Solve the following first order ODEs

$$(i) \quad \frac{dy}{dx} = (\ln y - \ln x + 1) \frac{y}{x}$$

$$(ii) \quad x \frac{dy}{dx} + 2y = x^2 y^2$$

$$(iii) \quad \frac{dy}{dx} - y = 2e^x, \quad y(0) = 3$$

$$(iv) \quad \frac{dy}{dx} = \frac{1 - 2xy^2}{1 + 2x^2y}, \quad y(1) = 1$$

2. Solve the following second order ODEs

$$(i) \quad y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

$$(ii) \quad y'' + 2y' + 10y = 0, \quad y(0) = -1, \quad y'(0) = 4$$

$$(iii) \quad 4y'' - 4y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

3. Solve

$$(i) \quad (x^2 - 2x)y'' - (x^2 - 2)y' + 2(x - 1)y = 0, \quad y_1 = x^2,$$

$$(ii) \quad xy'' - (x + 1)y' + y = 0, \quad y_1 = e^x$$

4. Solve using any method (reduction of order, method of undetermined coefficients or variation of parameters)

$$(i) \quad y'' - 6y' + 9y = e^{3x},$$

$$(ii) \quad y'' - y' = 2x - 3x^2$$

5. Solve the following systems

$$(i) \quad \frac{d\bar{x}}{dt} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \bar{x},$$

$$(ii) \quad \frac{d\bar{x}}{dt} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \bar{x}, \quad \bar{x}(0) = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$(iii) \quad \frac{d\bar{x}}{dt} = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \bar{x}.$$

6. A large tank is filled with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at a faster rate of 10 gal/min. Find the number of pounds of salt in the tank at any time. Find the amount of salt in the tank at the moment when the tank is empty.

7. A population of rabbits satisfies the logistic eq. i.e. the population grows proportional to the amount of rabbits present and to the space available (which we will assume is 1000 rabbits). If the initial population of rabbits is 100 and it is observed that after 1 week there are 120 rabbits, how many rabbits will there be after 2 weeks? How long will it take for the populations to reach 90% of the limiting population? (This model also applies to spreading of rumors and epidemics)

8. A roast is removed from an oven and placed in a room where the temperature is 70°F . A meat thermometer indicated the internal temperature of the roast to be 160°F . Twenty minutes later the meat thermometer indicates 150°F . Assuming Newton's law of cooling, find the temperature of the roast at any given time