Math 1496 - Calc 1

Limits

Consider $\lim_{x \to 1} \frac{x^2-1}{x-1} = \text{"}\frac{0}{0}\text{"}$

One technique is factoring

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim_{x \to 1} x+1 = 2$$

Consider $\lim_{x \to \infty} \frac{2x^2+1}{x^2+3x} = \text{"}\frac{\infty}{\infty}\text{"}$

$$\lim_{x \to \infty} \frac{2+\frac{1}{x^2}}{1+\frac{3}{x}} = \frac{2+0}{1+0} = 2$$

How about $\lim_{x \to 1} \frac{\ln x}{x-1} = \text{"}\frac{0}{0}\text{"}$?

A problem we don't know how to do (until now)

L'Hôpital Rule

Suppose that $\lim_{x \to c} f(x) = 0$ $\lim_{x \to c} g(x) = c$

that f & g are differentiable in an open interval I containing $x = c$ & $g'(x) \neq 0$ I except at $x = c$ then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$ provided limits exists a $\pm\infty$

Also applies for "$\frac{\infty}{\infty}$"

Previous ex $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$ "$\frac{0}{0}$" so LH

$$\lim_{x \to 1} \frac{2x}{1} = 2 \checkmark$$

$$\lim_{x \to \infty} \frac{2x^2 + 1}{x^2 + 3x} = \text{"}\frac{\infty}{\infty}\text{"}$$ LH

$$\lim_{x \to \infty} \frac{4x}{2x + 3} = \text{"}\frac{\infty}{\infty}\text{"}$$ LH

$$\lim_{x \to \infty} \frac{4}{2} = 2 \checkmark$$

Now $\lim_{x \to 1} \frac{\ln x}{x-1} = "\frac{0}{0}"$ so LH

$$\lim_{x \to 1} \frac{\frac{1}{x}}{1} = 1$$

ex 2 $\lim_{x \to 0} \frac{2^x - 1}{x^2 - x} = "\frac{0}{0}"$ LH

$$\lim_{x \to 0} \frac{2^x \ln 2}{2x - 1} = -\ln 2$$

ex 3 $\lim_{x \to \infty} \frac{x^2}{e^x} = "\frac{\infty}{\infty}"$ LH

$\lim_{x \to \infty} \frac{2x}{e^x} = "\frac{\infty}{\infty}"$ LH again

$$\lim_{x \to \infty} \frac{2}{e^x} \to 0$$

ex 4 $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \text{"}\frac{\infty}{\infty}\text{"}$ LH

$$\lim_{x\to\infty} \frac{\frac{1}{2(x^2+1)^{1/2}} \cdot \cancel{2}x}{1} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

I think it's clear if we do LH again it will flip flop back to the original problem. so here we need to do something different.

$$\lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{1}{x^2})}}{x}$$

$$\lim_{x\to\infty} \frac{\cancel{x}\sqrt{1+\frac{1}{x^2}}}{\cancel{x}} = 1$$

There are forms call "indetermінate forms" which need special (more) work.

Ex $\lim_{x \to 0^+} x \ln x = $ " $0 \cdot -\infty$ "

Ex $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = $ " $\infty - \infty$ "

Ex $\lim_{x \to 0^+} x^x = $ " 0^0 "

Ex $\lim_{x \to 1} x^{\frac{1}{1-x}} = $ " 1^∞ "

Ex $\lim_{x \to \infty} (1+x)^{\frac{1}{x}} = \infty^0$

We consider these examples next class