Math 1496 – Calc I

Back in derivatives we saw

$$\frac{d}{dx}\ln|x| = \frac{1}{x} \quad \text{so} \quad \int \frac{dx}{x} = \ln|x| + c$$

In this lecture we explore this some more.

Ex 1 $\displaystyle\int \frac{dx}{2x-1}$ ~~dx~~

Here we will use a substitution.

Let $u = 2x-1$ so $du = 2\,dx$

$$\int \frac{\frac{du}{2}}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + c$$

so $\displaystyle\int \frac{dx}{2x-1} = \frac{1}{2}\ln|2x-1| + c$

In general

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + c$$

(we will use this in Calc 2)

$a \neq 0$

Ex 2 $\displaystyle\int \frac{x^3}{x^4+1}\,dx$

let $u = x^4+1$ so $du = 4x^3\,dx$

$$\int \frac{\frac{du}{4}}{\cancel{x^3}\,u} = \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|u| + c$$

$$\int \frac{x^3}{x^4+1}\,dx = \frac{1}{4}\ln|x^4+1| + c$$

don't really need | |

since $x^4+1 > 0$

Ex 3 $\displaystyle\int \frac{x\,dx}{(3x-1)^2}$

$u+1 = 3x$

let $u = 3x-1$ so $du = 3dx$ Now $x = \frac{u+1}{3}$

so $$\int \frac{\frac{u+1}{3}\cdot\frac{du}{3}}{u^2} = \frac{1}{9}\int \frac{u+1}{u^2}\,du = \frac{1}{9}\int\left(\frac{1}{u}+\frac{1}{u^2}\right)du$$

$$\frac{1}{9}\left(\ln|u| - \frac{1}{u}\right) + c = \frac{1}{9}\left(\ln|3x-1| - \frac{1}{3x-1}\right) + c$$

$$\int \frac{2x^2+7x-3}{x-2}\,dx$$

Here power num > power den

so 1st we need to divide poly's

$$\begin{array}{r|l} & 2x+11 \\ \hline x-2 & 2x^2+7x-3 \\ & 2x^2-4x \\ \hline & \quad 11x-3 \\ & \quad 11x-22 \\ \hline & \qquad 18 \end{array}$$

so $\displaystyle\int \frac{2x^2+7x-3}{x-2}\,dx = \int\left(2x+11+\frac{18}{x-2}\right)dx$

$= x^2+11x+18\ln|x-2|+c$

from 1st page.

Trig Integrals

we know

$$\int \sin x\,dx = -\cos x + c \qquad \int \cos x\,dx = \sin x + c$$

but what about

$$\int \tan x\,dx, \ \int \cot x\,dx, \ \int \sec x\,dx, \ \int \csc x\,dx?$$

$\int \tan x\,dx$

Here $\displaystyle\int \frac{\sin x}{\cos x}\,dx$ let $u = \cos x$

$du = -\sin x\,dx$

so $\displaystyle\int -\frac{du}{u}$

$-\ln|u| + c = -\ln|\cos x| + c$

so $\displaystyle\int \tan x\,dx = -\ln|\cos x| + c$

$\int \sec x \, dx$

$$\int \sec x \, dx = \int \sec x \frac{(\sec x + \tan x)}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx \quad (*)$$

let $u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x) dx$

so our integral $(*)$ becomes

$\int \frac{du}{u} = \ln|u| + c$ back sub

so $\int \sec x \, dx = \ln|\sec x + \tan x| + c$ //

Similarly

$\int \cot x \, dx = \ln|\sin x| + c$

$\int \csc x \, dx = -\ln|\csc x + \cot x| + c$