Masses and Mixings of Quarks and Leptons

I. Generalities

- We assume that gravity is characterized by two fundamental scales:

  Hubble: $H^2 \equiv \frac{\Lambda}{3}$ $\quad H^{-1} \sim 10^{28}$ cm.

  Planck: $G_{Newton} = \frac{1}{M_{pl}^2}$ $\quad M_{pl}^{-1} \sim 10^{33}$ cm.

- Other scales (even in "pure gravity") emerge:

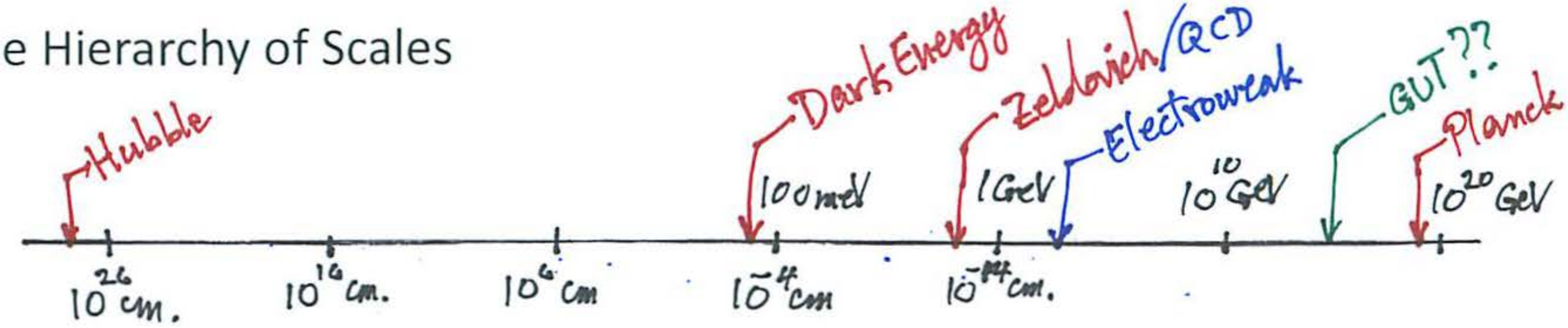
  Dark-energy scale:

  $$H^2 = \frac{8\pi}{3M_{pl}^2}\rho_{DE} \equiv \frac{8\pi}{3M_{pl}^4}\mu_{DE}^4 \qquad \frac{1}{\mu_{DE}} \sim 8\times10^{-3}\,\text{cm}.$$

  Zeldovich scale:

  $$\Lambda_Z^3 \sim \Lambda_{QCD}^3 \sim HM_{pl}^2 \qquad \Lambda_Z^{-1} \sim 10^{-12}\text{-}10^{-13}\,\text{cm}.$$

- The Hierarchy of Scales



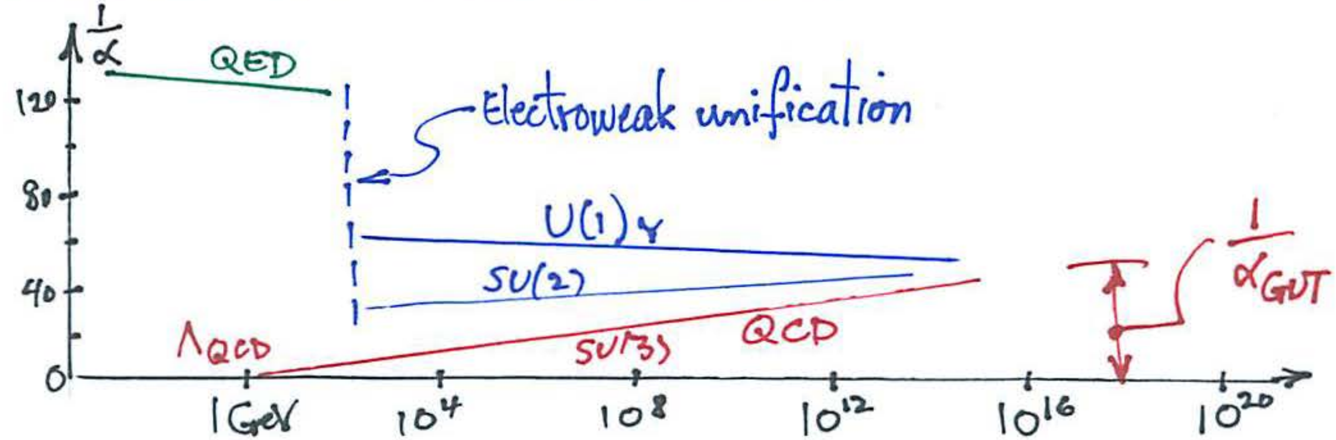
- Definition of a "theory of everything":

  Effective field theory is valid over the entire range.
  The theory encompasses all known interactions.
  The theory is concise.

A really good theory of everything will have

  very few input parameters.
  a minimum of fundamental scales.

- The most important non-gravitational “new scale is QCD:
- Dimensional transmutation



- The value of Λ_{QCD} is controlled by α_{GUT}
- Why is $\alpha_{GUT} \sim 1/30$ (or $1/40$ or whatever)?
- Can α_{GUT} be influenced by the presence of the Zeldovich gravitational scale Λ_Z ?

- The "QCD vacuum" couples to the "Zeldovich vacuum":

  It has the same energy.

  It is in the same place.

  The two vacuua are coupled dynamically.

- A hypothesis:

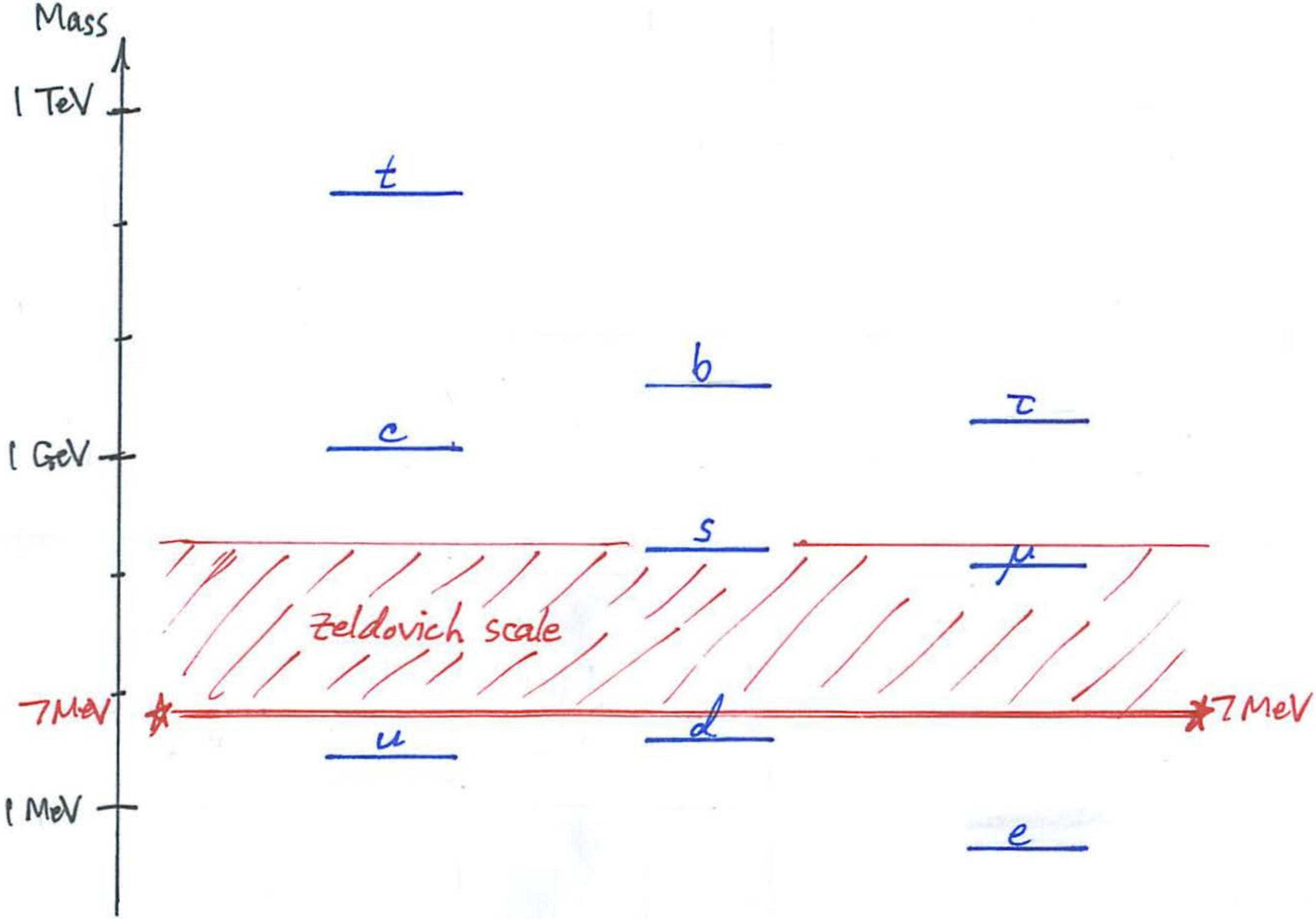
  Stability criteria drives Λ_{QCD} toward Λ_{Z}.

  Λ_{Z} is "input."

  Λ_{QCD} is "output."

Now look at other scales the same way.

Masses of Quarks and Leptons



Masses of Quarks and Leptons

The masses center about the Zeldovich (and/or QCD) scale.

Since leptons are involved, this suggests that it is the Zeldovic scale that controls the origin of quark as well as lepton masses.

This seems to imply that the “flavor problem” (origin of masses and mixings) largely is resolved at the (remarkably low!) Zeldovich scale.

I try to implement this idea with a “dark sector” of SU(5) singlet scalar bosons with flavor structure. There will be more details regarding this in the next section.

The Higgs Sector

- The fundamental dimensional parameter of the standard-model electroweak sector is

$$\langle \phi \rangle \equiv v = 242 \text{ GeV}$$

- There exists the curious relation

$$v \cong m_{top} \sqrt{2}$$

- If the origin of the top quark mass can be traced to the (outer limits of the) Zeldovich scale, perhaps the electrweak vev v has a common origin.

- This idea is toxic. BUT

- If so, the standard model phenomenology as we know it would require introduction of no scales other than what the gravity theory provides, namely the Hubble scale and the Planck scale.

The Bottom Line:

- Maybe a really good theory of everything is thinkable!
- We should never give up.

II. The Dark Sector

- General properties of the dark sector:

  Low mass scalar fields (mass <<< m_{top}).

  Nontrivial flavor indices.

  The fields are probably pseudo-Goldstone bosons (think pions).

  The fields probably interact strongly amongst themselves.

- The role of the dark sector:

  Implementing the masses and mixings of the quarks and leptons.

  Accounting for the existence of dark matter.

- If the idea were to work, the main implication would be that the "flavor problem" largely resolves itself at the Zeldovich scale.

What is flavor?

- Chiral quarks and leptons are flavor triplets:

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \quad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

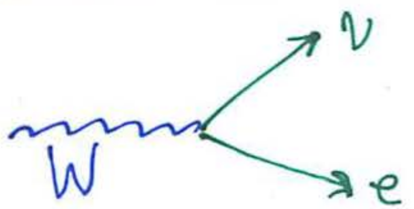
- The photon, the W , the Z , and the gluons are flavor singlets.
- The flavor group is maximally SU(3) X SU(3) .
- It is broken by nonvanishing masses and mixings.
- The Higgs sector is flavor nontrivial.
- We will assume that the vector subgroup controls the description at the low energies of interest:

$$SU(3)_L \times SU(3)_R \Rightarrow SU(3)_V \quad (\Rightarrow SO(3)\ ??)$$

Flavor currents

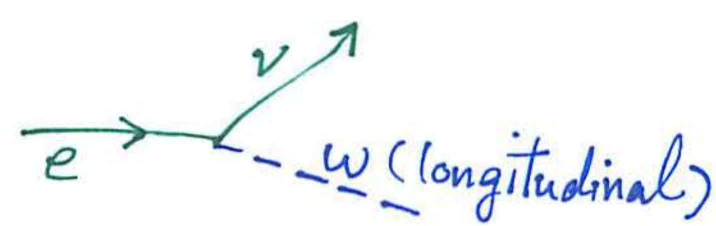
- The diagonal vector currents of quarks and leptons are conserved at tree level, and are broken only by Higgs effects.
- For example, turn off the weak and electromagnetic gauge couplings to see the origin of flavor violation. In that limit, e.g., the electron becomes unstable:

BEFORE GAUGELESS LIMIT



W eats goldstone boson and gets massive.

AFTER GAUGLESS LIMIT ($m_W \to 0$)



W vomits goldstone boson as $m_W \to 0$. ($g_{weak} \to 0$)

- We will attempt to use Occam's razor to construct toy models of the dark sector.
- This is work in progress. Not much has been done (or accomplished!) yet.
- Introduce a single self-conjugate Higgs triplet.

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \vec{\phi}^{\dagger}$$

- Couple it off-diagonally to the quarks and leptons via the divergence of the vector current:

$$\mathcal{L} \overset{?}{\ni} \bar{\Psi}_i \gamma_\mu \Psi_j (\partial_\mu \phi_k) \epsilon^{ijk} + h.c.$$

- After an integration by parts, one might have an effective interaction of Yukawa form:

$$\mathcal{L} \overset{?}{\sim} (m_i - m_j)\, \overline{\psi}_i \psi_j \phi_k \epsilon^{ijk} + h.c$$

- We anticipate that these scalar bosons $\vec{\phi}$ are described by a scale-dependent Higgs potential V (ϕ , μ). (Note that the above coupling is suggestive of this interpretation.)

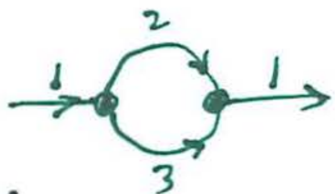
- We assume that a prominent component of this potential is a coupling term which is cubic in the Higgs fields. In the extreme limit of only including this term, one would have

$$\mathcal{L} \sim \frac{1}{2} \sum_{i=1}^{3} (\partial_\mu \phi_i)^2 - F(\mu)\, \phi_1 \phi_2 \phi_3$$

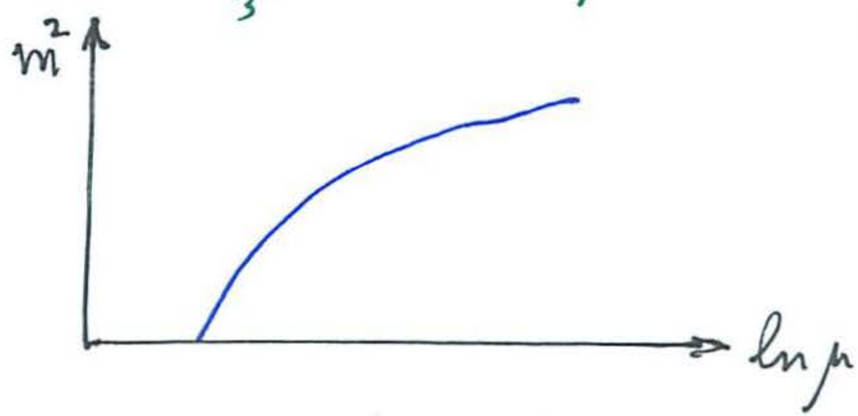
- This is known in the literature as the ABC model (Per Kraus and David Griffiths, Am. J. Phys 60,1013 (1992)).

- Kraus and Griffiths only do perturbation theory; we need to do more.

- But they identify a running mass emergent from the only divergent Feynman diagram in the theory:



$$m^2(\mu) \sim -F^2\left(\ln \frac{\Lambda^2}{\mu^2} + \text{const.}\right)$$



- But the φ^3 theory is unstable—something Kraus and Griffiths ignore.
- It is easily stabilized by adding in a quartic term to the cubic potential. There are various ways of doing this; e.g.

$$\mathcal{L} \sim (\varphi_1^2+\varphi_2^2+\varphi_3^2)^2 \quad \text{AND/OR} \quad \mathcal{L} \sim (\varphi_1^4+\varphi_2^4+\varphi_3^4) \quad \text{AND/OR} \quad \mathcal{L} \sim (\varphi_1+\varphi_2+\varphi_3)^4$$

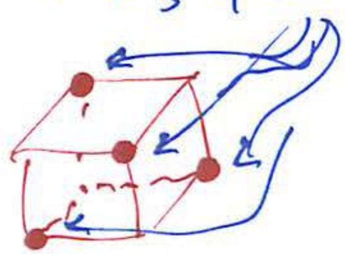
- The simplest is to include only the most symmetric contribution. Write

$$V(\varphi) = (\varphi_1^2+\varphi_2^2+\varphi_3^2)^2 - 12F\varphi_1\varphi_2\varphi_3$$

- Then minimize:

Choose $\varphi_1 = \varphi_2 = \varphi_3 \equiv \varphi$ Then $V = 9\varphi^4 - 12F\varphi^3$

4 Minima in the $\varphi_1\varphi_2\varphi_3$ space:



Minimum occurs at $\boxed{\langle\varphi\rangle = F}$

(Reverse signs of two φ's to get the others.)

- Now compute oscillations about the minimum. This is accomplished by use of tribimaximal mixing:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

- At the minimum, we have

$$X = \sqrt{3}\, F$$

$$Y = Z = 0$$

(We have rotated to a frame with axis along the body diagonal)

- There is a bit of algebra needed to get the Higgs potential V (X, Y, Z):

$$V = (X^2+Y^2+Z^2)^2 - 12F\left[\frac{X^3}{3\sqrt{3}} - \frac{3XZ^2}{6\sqrt{3}} - \frac{XY^2}{2\sqrt{3}} - \frac{ZY^2}{\sqrt{6}} + \frac{Z^3}{3\sqrt{6}}\right]$$

- Now expand to quadratic order in Y and Z :

$$V \cong X^4 - \frac{4FX^3}{\sqrt{3}} + \cdots$$

- Only the mass associated with oscillations in the (radial) X direction is nonvanishing:

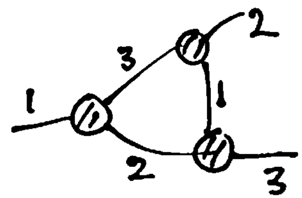
Write $X = \sqrt{3}F + h$

Then $V(X) \cong -3F^4 + 6F^2h^2$

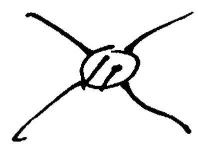
Therefore $m_h \approx 2\sqrt{3}F$

- There is much more to do:

  Induced mass terms (with scale dependence)
  Scale-dependent vertex parts:



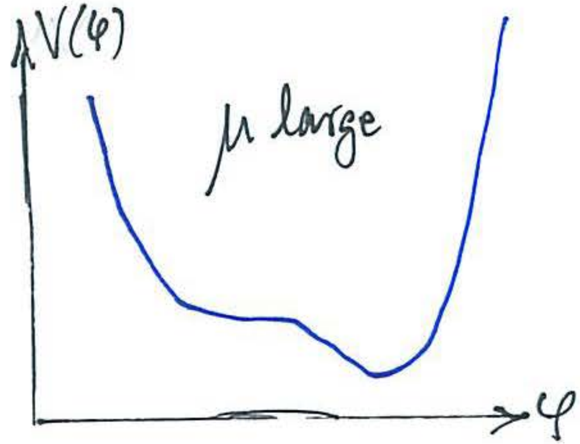
  Vacuum stabilization via quartic terms.

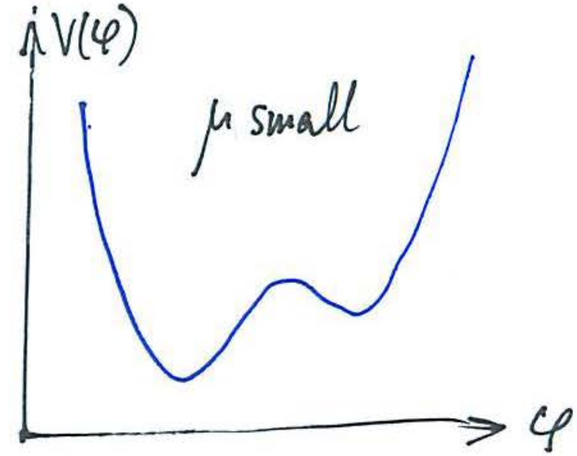


- It is an interesting theory!

- Our desideratum: a strong enough scale dependence to create a phase transition at 7 MeV.

- A possible schematic:





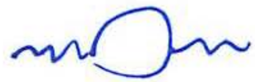
- There are many detailed options even within this simple model.

- A close analogue is finite temperature field theory ("compactification of a Euclidean time variable on a torus"). As sketched below, we will try to do the same thing, but with the space degrees of freedom.

- Perhaps the ABC parameters "lock" to the Zeldovich scale in a way similar to that conjectured above for the QCD scale.

What is Meant by Scale?

- Consider the running coupling constant in QED. It is related to the photon propagator;



$$D(q^2) \cong \frac{e_0^2}{q^2\left[1 - \frac{e_0^2}{12\pi^2} \ln \frac{\Lambda^2}{q^2}\right]} \equiv \frac{e^2(q^2)}{q^2}$$

- The scale-dependence a la Wilson is induced by looking at the effect on the renormalized coupling e_0^2 as one varies the UV cutoff with fixed bare coupling e_0^2 .

- Such scale dependence and the actual q^2 dependence of the propagator carry essentially the same information.

- I am thinking of looking at the theory at fixed bare coupling, but as a function of varying the infrared cutoff. In QED, this point of view again carries essentially the same information as varying q^2 .

What is meant by "infrared cutoff"?

- Quantize the theory in a small box of size $\sim\mu^{-1}$ with periodic boundary conditions.

- Consider μ as the renomalization-group scaling variable.

- For QCD, when $\mu >> \Lambda_{QCD}$ the theory is perturbative and the description is in terms of quarks and gluons.

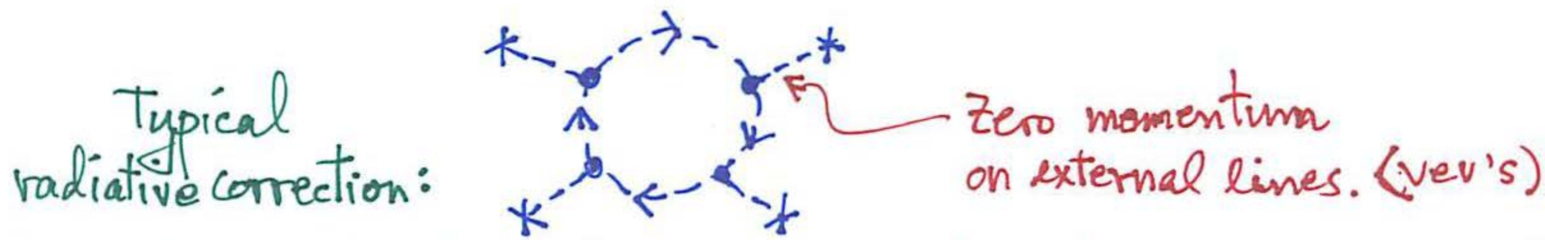
- For QCD, when $\mu << \Lambda_{QCD}$, the theory is non-perturbative and is described approximately by the chiral effective Lagrangian (Gasser-Leutwyler) whose degrees of freedom are pions and nucleons.

- Thus this description contains a confinement/deconfinement "phase transition" (loosely speaking) as a function of μ .

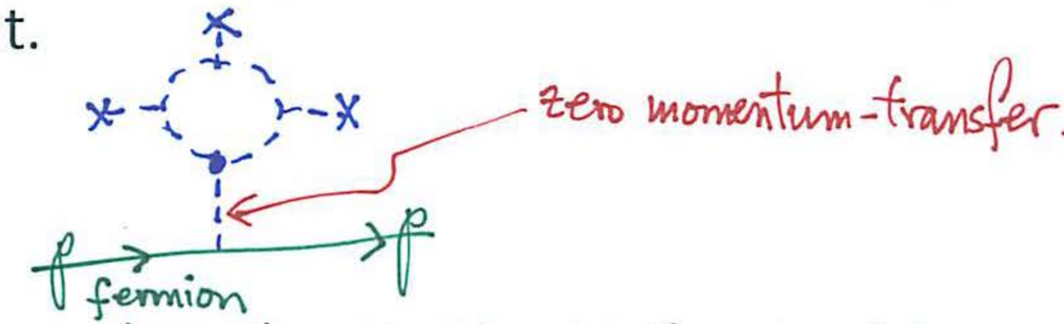
- Note: all experimental data are taken within the limiting description when $\mu \longrightarrow 0$ (very large size of the quantization box). This is similar to saying that all our laboratory experiments are taken at a temperature very small compared to the Zeldovich/QCD scales.

Consequences

- The theory "at a large momentum scale μ " is NOT directly phenomenological. It is an abstraction.
- The effective potential for Higgs condensates can be (and has to be!) scale dependent.



- Therefore tadpole contributions to fermion masses can also be scale dependent.



- Such contributions are less threatening to the precision electroweak experimental constraints on these ideas, and may be expected to play a prominent role.

III. The Mixing Matrix

- The motivation for what follows comes directly from considering the gravity sector via MacDowell and Mansouri.

- A natural question which arose was what the emergent Zeldovich scale had to do with the masses and mixings of quarks and leptons.

- I was led to work of Hong-Mo Chan and his collaborators via an endorsement by Roger Penrose in one of his books. What follows is directly based on their ideas.

- Alternatives are a variety of braneworld scenarios scattered through the literature, but I found them less promising. (This may be a big mistake on my part, but so far I have no regrets with my choice.)

Hong-Mo's Rule I for two generations

- Consider the mass matrix M for μ and τ :

$$M\psi = \begin{pmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \psi_2 \\ \psi_3 \end{pmatrix} \begin{matrix} \leftarrow \text{2nd generation} \\ \leftarrow \text{3rd generation} \end{matrix}$$

- Call the eigenvalues m_μ and m_τ .

- Introduce scale-dependence to the mass matrix M :

$$M \to M(\mu)$$

- Assume that, when $\mu > m_{top}$, M(μ) is rank 1.

- This is implemented via a simple mixing matrix:

$$M \cong m_\tau \begin{pmatrix} |\alpha|^2 & \alpha(\mu) \\ \alpha^*(\mu) & 1 \end{pmatrix} \qquad (|\alpha| \ll 1)$$

Hong-Mo's Rule II

- The way the mixing runs with scale is (approximately):

$$\alpha(\mu) \approx \sqrt{\frac{m}{\mu}} \qquad \boxed{m \approx 7\ \mathrm{MeV}}$$

- This feature is universal, i.e. the same mixing parameter occurs for the down-quark sector and the up-quark sector:

$$(M_{lepton})_{ij} = m_{tau}\alpha_i \alpha_j^*$$

$$(M_{down})_{ij} = m_b\, \alpha_i \alpha_j^*$$

$$(M_{top})_{ij} = m_{top}\, \alpha_i \alpha_j^*$$

- The third-generation masses are input, and of course their values do not share this universality feature.

Hong-Mo's Rule III

- The mass eigenvector for the physical tau lepton (which does not run with scale) is equated with the running unit eigenvector evaluated at $\mu = m_\tau$.

$$M(\mu = m_\tau)\,|tau\rangle = m_\tau\,|tau\rangle \qquad |tau\rangle \cong \begin{pmatrix} \sqrt{\frac{m}{m_\tau}} \\ 1 \end{pmatrix}$$

- The mass eigenvector for the physical muon (in this two-generation scenario) is the unit vector orthogonal to the tau eigenvector:

$$|mu\rangle \cong \begin{pmatrix} 1 \\ -\sqrt{\frac{m}{m_\tau}} \end{pmatrix} \qquad \langle mu|tau\rangle = 0$$

- By definition,

$$M(\mu) = m_\tau \begin{pmatrix} \frac{m}{\mu} & \sqrt{\frac{m}{\mu}} \\ \sqrt{\frac{m}{\mu}} & 1 \end{pmatrix}$$

- Since $|\tau au\rangle \cong \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- it follows that $|mu\rangle \approx \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- The eigenvalue condition is then (to good approximation)

$$m_\mu \equiv \langle mu \,|\, M(\mu = m_\mu) \,|\, mu\rangle \approx m_\tau \left(\frac{m}{m_\mu}\right) \Rightarrow \boxed{m_\mu \cong \sqrt{m m_\tau} \approx 110\,\text{MeV}}$$

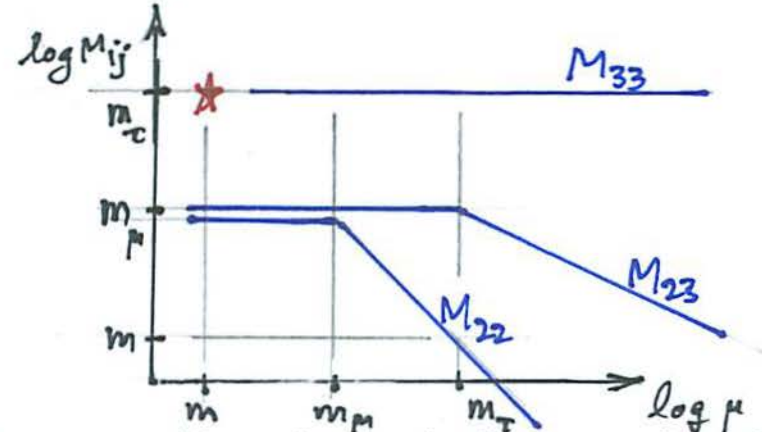
- Similar arguments for the quarks clearly lead to the results

$$\boxed{m_s \approx \sqrt{m m_b} \approx 170\ \text{MeV}} \qquad \boxed{m_c \approx \sqrt{m m_t} \approx 1.1\ \text{GeV}}$$

- The CKM mixing angle V_{cb} is dominated by the down-sector rotation, and is alredy determined at the third-generation scale, namely when $\mu = m_b$. One obtains

$$\boxed{|V_{cb}| \approx \sqrt{\frac{m}{m_b}} - \sqrt{\frac{m}{m_t}} \approx \sqrt{\frac{m}{m_b}} \approx .04}$$

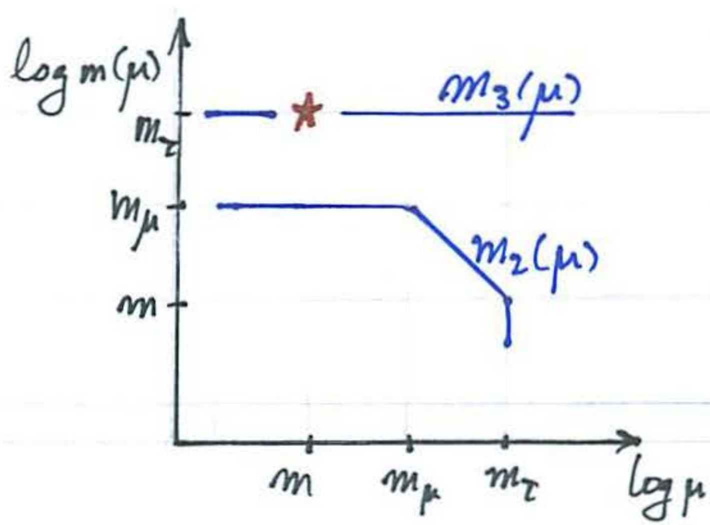
I interpret the rules in a slightly different way. Below the tau mass scale, $\mu < m_\tau$, I assume the mass matrix becomes rank 2:

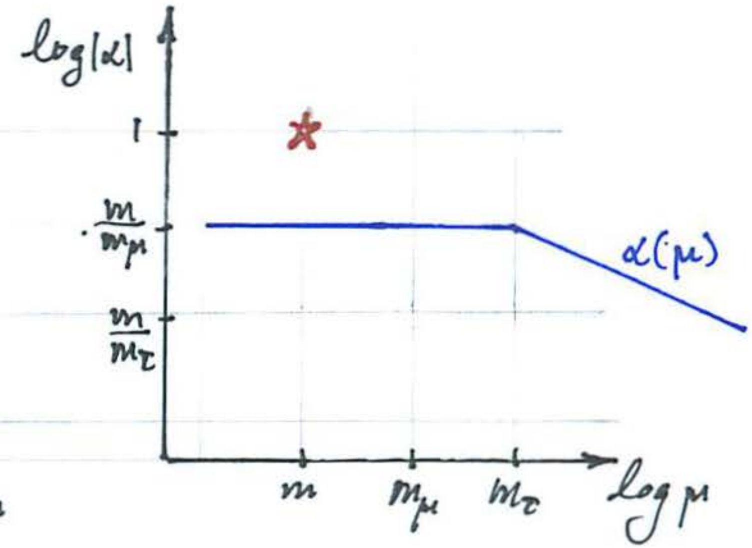


$$M \to \begin{pmatrix} m_\mu & m_\mu \\ m_\mu & m_\tau \end{pmatrix}$$

$(\mu \to 0)$

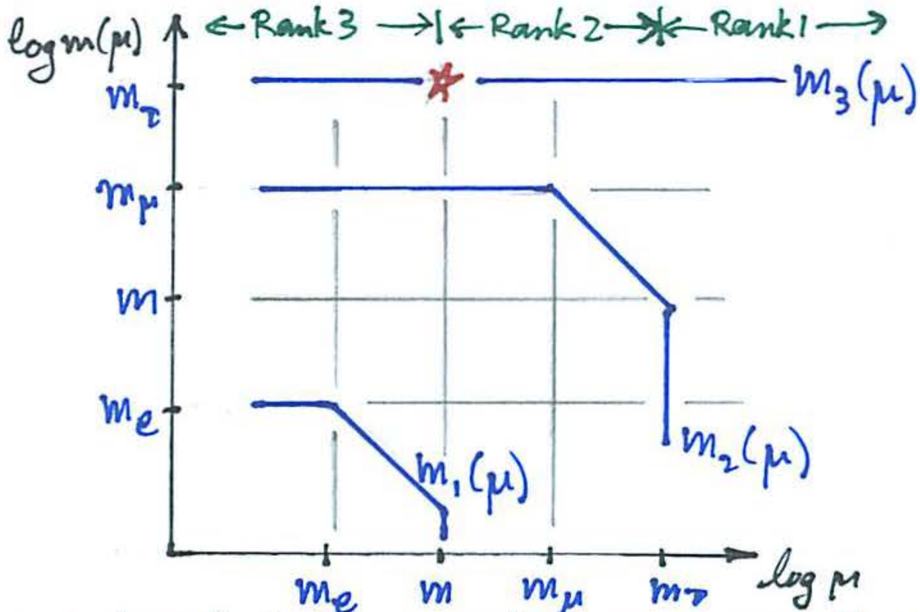
This matrix can be diagonalized and the scale dependence of the eigenvalues and the mixing parameter is determined as shown below:





Three generations

- Hong Mo et. al. have more trouble when including the first generation. (So do I!!)
- My own try remains somewhat sketchy, and deviates from theirs. But a generalization for the mass-eigenvalue plot that works is as follows:

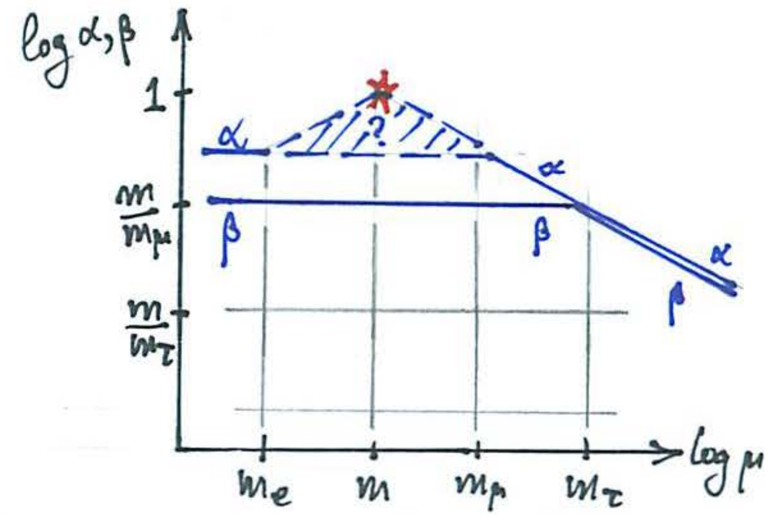


- The rank of M remains two until the scale parameter is less than 7 MeV.

- The mixing behavior is generalized as follows:

$$M(\mu) = U\, m_{Diag}\, U^{-1}$$

$$U \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \beta \\ 0 & -\beta & 1 \end{pmatrix} \begin{pmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cong \begin{pmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & \beta \\ \beta\alpha & -\beta & 1 \end{pmatrix}$$

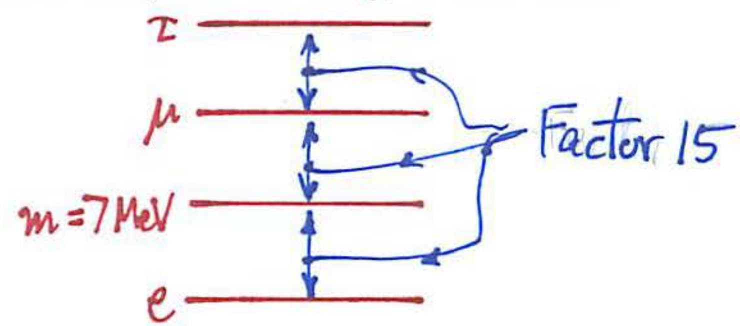


- For obscure notational reasons, the parameter beta is to be identified with the previously defined alpha. I'm sorry about that.
- Alpha eventually "freezes out" at low enough scales. What it does in the interval $m_e < \mu < m_\mu$ is problematic. We will come back to that problem later.
- Note that the physical mass eigenvectors are alread determined at the second-generation scale $\mu = m_\mu$. Therefore the rank-3 physical mass matrix for $\mu = 0$ is already fixed when $\mu = m_\mu \gg m = 7$ MeV.

- In order to get the electron mass to come out correctly, I assumed a "reciprocity relation":

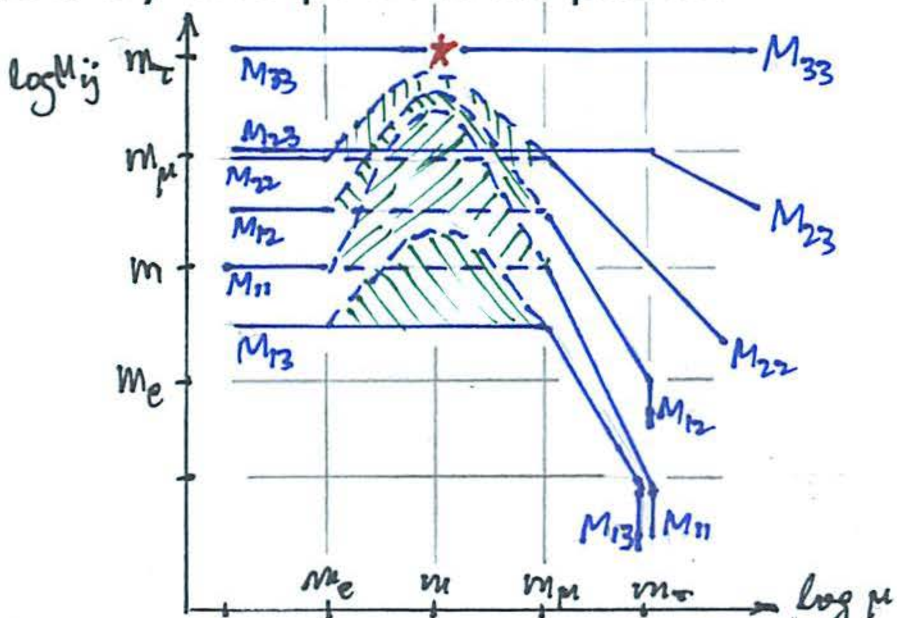
$$\alpha(\mu) = \alpha\left(\frac{m^2}{\mu}\right)$$

- This is motivated by a curious empirical fact, namely that for the charged leptons there is (on a logarithmic scale) an "equal-spacing rule":



- The reciprocity assumption successfully implements this.

- All individual elements of the mass matrix M is determined by the previous plots:



- The details of this plot seem doubtful even to me.
- In addition, the equal spacing rule give up and down quark masses much too small:

$$m_u \stackrel{?}{\doteq} \frac{m^2}{m_t} \cong 50\ \text{keV} \qquad m_d \stackrel{?}{\doteq} \frac{m^2}{m_s} \cong 300\ \text{keV}$$

- So there are some clear deficiences to be addressed.

- At this point I backed off of this approach, because it was getting too rococo. I think that if one speculates at this extreme, toxic level at all, one should strive to keep the description as simple and austere as possible.

- The motivation for the revised approach is that the numerical successes associated with the previous arguments are more provocative and suggestive of underlying simplicity than the arguments themselves.

- So the new approach just describes the end results at the $\mu = 0$ scale in as direct and simple a way as possible.

Just the facts:

- A change of language: start by assuming the previous answer for the mass matrix.
- Define

$$m \equiv 7\ \text{MeV} \qquad \Theta^4 \equiv \frac{m}{m_\tau}$$

- Assume

$$\underset{\substack{\uparrow \\ \text{(value for } \mu \to 0)}}{M} = m_\tau \begin{pmatrix} \theta^4 & \theta^3 & 0 \\ \theta^3 & \theta^2 & \theta^2 \\ 0 & \theta^2 & 1 \end{pmatrix}$$

Negotiable; should be $\ll O(\theta^3)$

- Diagonalize:

$$M = U\, m_{\text{Diag}}\, U^{-1}$$

$$U \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \theta^2 \\ 0 & -\theta^2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta & 0 \\ -\theta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad m_{\text{Diag}} \cong \begin{pmatrix} -\theta^6 & 0 & 0 \\ 0 & \theta^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau$$

- For the down sector, change the assumption slightly:

$$\text{Assume} \quad \theta^4 \cong -\left(\frac{m}{m_b}\right)$$

- Introduce a kludge factor $Z \neq 1$ in the mass matrix:

$$M \cong m_b \begin{pmatrix} Z|\theta|^4 & |\theta|^2\theta & 0 \\ |\theta|^2\theta^* & |\theta|^2 & |\theta|^2 \\ 0 & |\theta|^2 & 1 \end{pmatrix}$$

- The same matrix $U(\theta)$ as used for the leptons diagonalizes the mass matrix.

$$U \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & |\theta|^2 \\ 0 & -|\theta|^2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta & 0 \\ -\theta^* & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- However,

$$m_{Diag} \cong \begin{pmatrix} (Z-1)|\theta|^4 & 0 & 0 \\ 0 & |\theta|^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_b$$

- This leads to satisfactory results provided the kludge factor Z is order unity, but not exactly unity:

$$m_d \cong (Z_d - 1)|\theta|^4 m_b = (Z_d - 1) m = (Z_d - 1)(7\,\mathrm{MeV}) \quad \boxed{Z_d \sim 1.7}$$

- Do the same for the up sector:

$$m_u \cong (Z_u - 1)|\theta|^4 m_t = (Z_u - 1) m = (Z_u - 1)(7\,\mathrm{MeV}) \quad \boxed{Z_u \sim 1.4}$$

- Now construct the CKM matrix, which is the relative rotation between up and down species:

$$V_{CKM} = U_u^{-1} U_d = \begin{pmatrix} 1 & -\theta_u & |\theta_u|^2\theta_u \\ \theta_u^* & 1 & -|\theta_u|^2 \\ 0 & |\theta_u|^2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_d & 0 \\ -\theta_d & 1 & |\theta_d|^2 \\ |\theta_d|^2\theta_d^* & -|\theta_d|^2 & 1 \end{pmatrix}$$

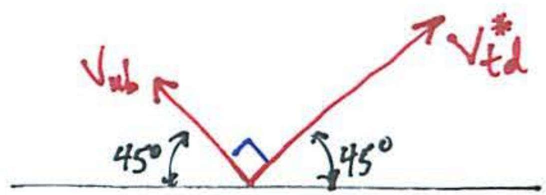
$$\cong \begin{pmatrix} 1 & (\theta_d - \theta_u) & (|\theta_u|^2 - |\theta_d|^2)\theta_u \\ (\theta_u - \theta_d) & 1 & (|\theta_d|^2 - |\theta_u|^2) \\ (|\theta_d|^2 - |\theta_u|^2)\theta_d^* & (|\theta_u|^2 - |\theta_d|^2) & 1 \end{pmatrix}$$

- The magnitudes of crucial CKM elements also come out very well.

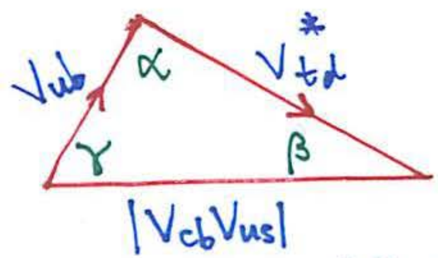
PDG

$$|V_{cb}| \approx |\theta_d|^2 \approx .04 \quad (.042)$$
$$|V_{ub}| \approx |\theta_d|^2|\theta_u| \approx .003 \quad (.0036)$$
$$|V_{td}| \approx |\theta_d|^2|\theta_d| \approx .008 \quad (.0087)$$

- In addition, the phases of the corner elements must be some fourth root of -1, thanks to the original definition of θ_{up} and θ_{down}.
- Therefore it is easy to choose successful phases:



- This is the unitarity triangle, but drawn upside down.



PDG

$\alpha = 90°$ $(88° \pm 6°)$

Hong-Mo's Rule IV

- What is going on?
- Hong Mo suggests that the mass matrix is influenced by the presence of strong CP violation.
- In particular, if θ_{QCD} were zero, the unitarity triangle would be degenerate, and there would be no CKM induced CP violation.
- I interpret this as follows:

The dark sector contains a Peccei-Quinn-like mechanism that removes CP violation from the strong sector.

The properties of the mass matrix are a function of the input θ_{QCD}.

In the limit $\theta_{QCD} = 0$, the kludge factor Z equals 1 to good accuracy, and the up and down quark masses take on the unacceptably small values associated with equal spacing.

Only for large input values of θ_{QCD} does Z deviate significantly from 1, and the up and down masses move upward near the 7 MeV scale.

Consequently, in this picture the input value of θ_{QCD} is indeed of order maximal.

Neutrinos

- Assume

$$M_{neutrino} = M^{\dagger}_{Dirac} \, M^{-1}_{Majorana} \, M_{Dirac}$$

- This assumption is made for real-life applications: the renormalization scale factor $\mu = 0$.

- But what is the scale dependence?

- Maybe it is like the other mass matrices, but where the third generation Dirac mass scale is near the 7 MeV scale.

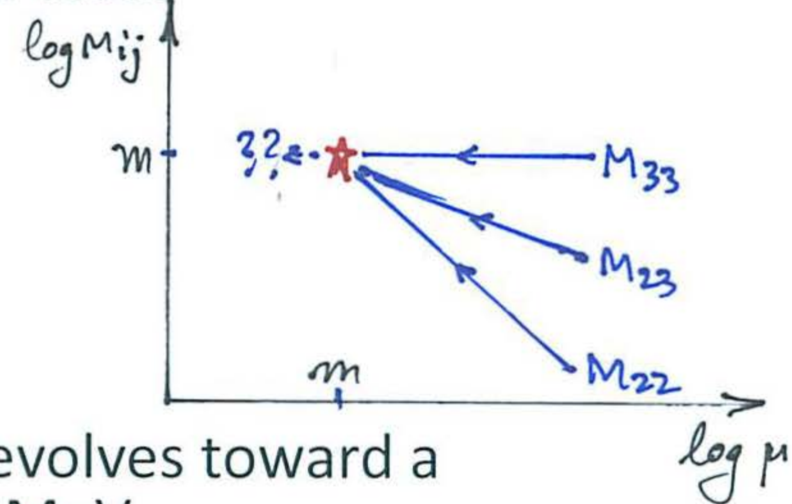
- The motivation for this is that mixings in the neutrino sector are large, and that there is no second-generation stopping point for $\mu \gg m = 7\,\text{MeV}$.

- A scenario for the two-generation case:

$$M(\mu) \approx \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} m \qquad \mu \gg m$$

$$(m = 7\,\text{MeV})$$

$$M(\mu) \to \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} m \qquad \mu \to m$$



- MAYBE, the 3-generation matrix evolves toward a democratic mass matrix at μ = 7 MeV.

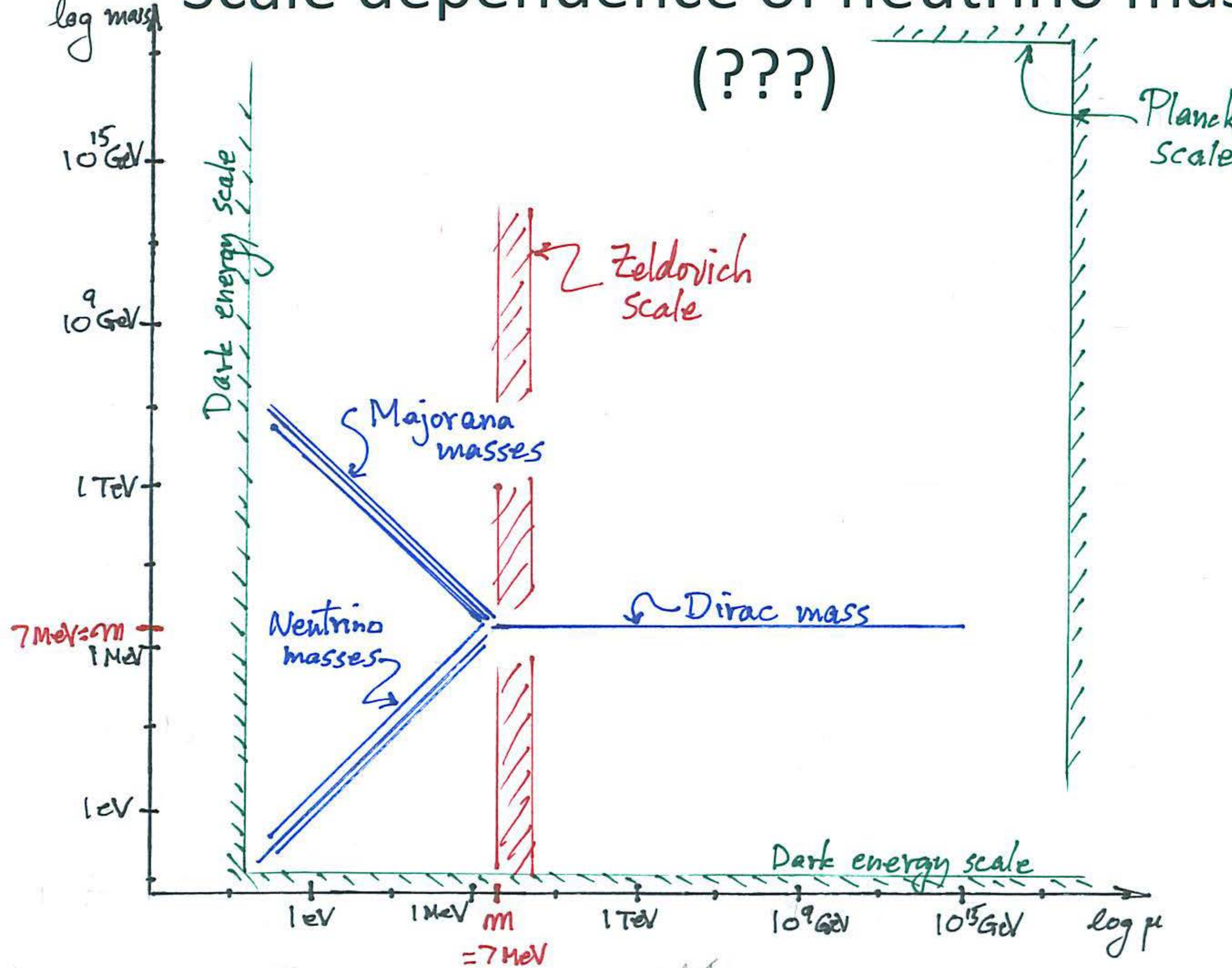
$$M(\mu) \xrightarrow[\mu \to m]{???} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

- If so, the rotation matrix would be tribimaximal.

Where is the seesaw mechanism?

- MAYBE it only is originated below the 7 MeV scale.
- A motivation is that CP violation is only manifested when the mass matrix becomes rank 3, when the scale factor $\mu < 7$ MeV. Maybe this is the case for lepton number violation as well.
- If so the scale dependence of neutrino masses would be very dramatic.

Scale dependence of neutrino masses (???)



Implications

- The Majorana masses increase inversely with scale as the scale parameter μ decreases.
- The neutrino masses decrease linearly with scale as μ decreases.
- Perhaps it is some property of the "cosmological vacuum" which interrupts the running when the scale factor μ approaches the dark energy scale of 2.4 meV, thereby freezing the neutrino masses at their observed values.
- Perhaps above the 7 MeV scale the Majorana mass matrix and the Dirac mass matrix are the same. Then in fact the seesaw mass matrix also is the same as the other two.
- It is very likely that such strong scale dependence must also occur in the bosonic "dark sector". Therefore the behavior of the neutrino mass matrix may provide the most direct guideposts on how to construct models of the dark sector.
- In any event, if the seesaw mechanism is confined to the infrared, there is no question that very serious things must happen near the 7 MeV "phase boundary."

Summary: my homework problems

- There is no shortage of big ideas.
- There is a severe shortage of serious computations and modeling to back them up.

- MacDowell – Mansouri:
  Geometry of the six extra dimensions (Calabi-Yau?)
  Understanding the "density of topology"
  Gauss-Bonnet flow (rift and subduction zones,etc.)
  Including standard model degrees of freedom .

- Hong-Mo Chan:
  Is there a link to the gravity description?
  Working out the ABC model.
  Convincingly coupling the bosons to the quarks and leptons.
  Exploring low energy theorems and current algebra constraints.
  Confronting the model with precision electroweak data .

Thanks again for listening and for your criticisms!

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