

Essential Components of Mathematics Intervention

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Mathematics Intervention

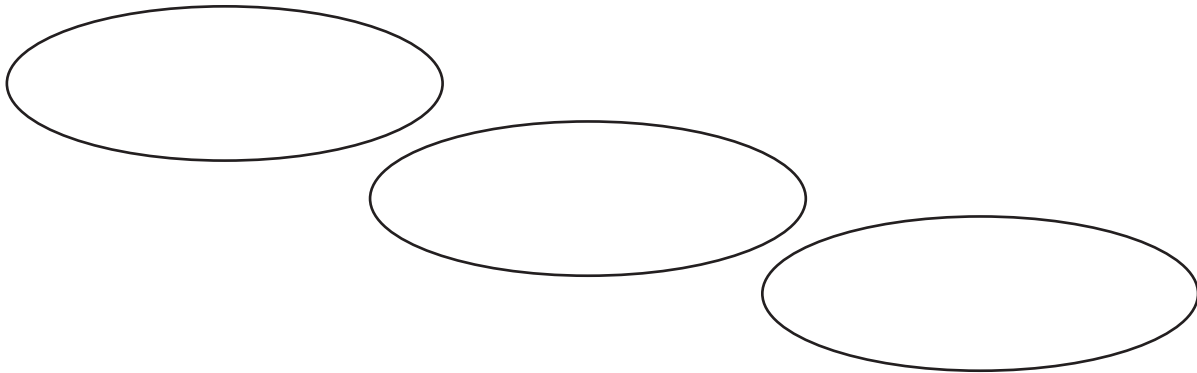
Critical Content



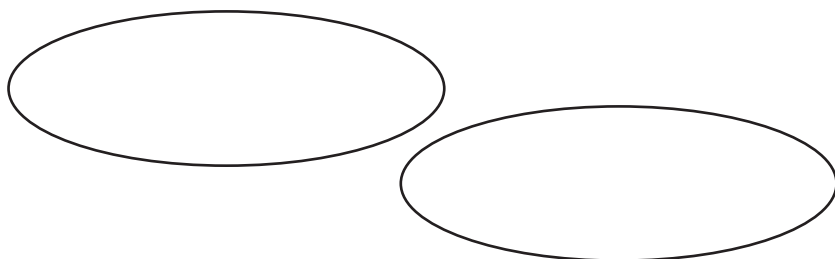
Evidence-Based Practices

Instructional Platform

Instructional Delivery



Instructional Strategies



Explicit Instruction

MODELING

PRACTICE

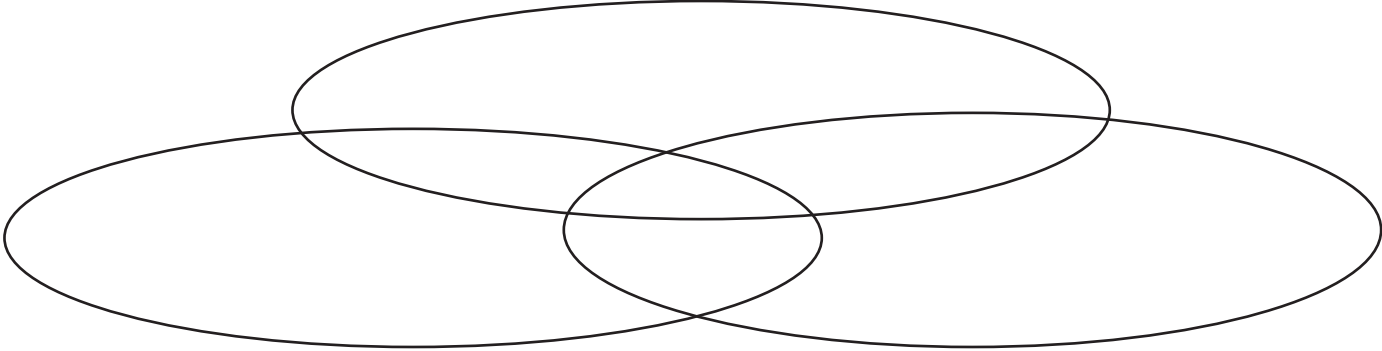
SUPPORTS

Mathematical Language

Mathematical Language

Instead of that...	Say this...

Multiple Representations




Fractions

Three Models

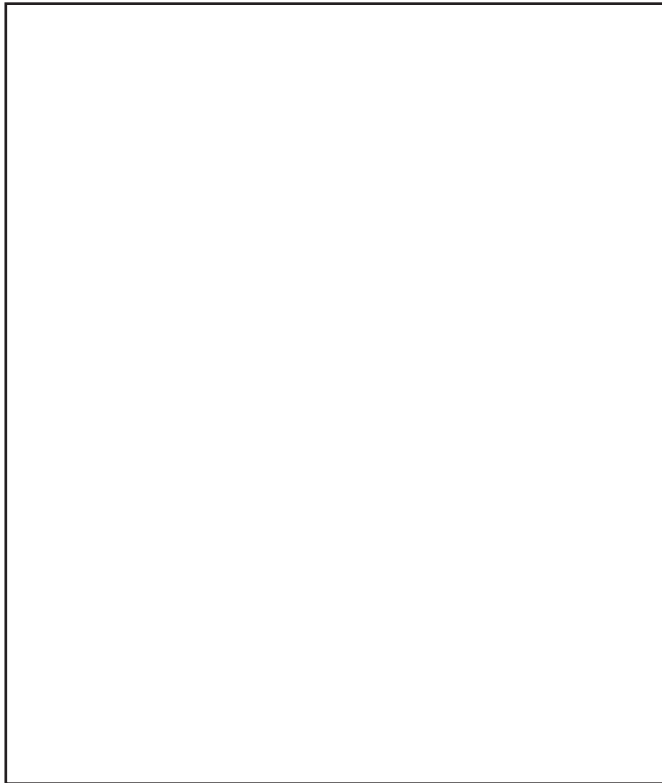
Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

Fluency

Addition



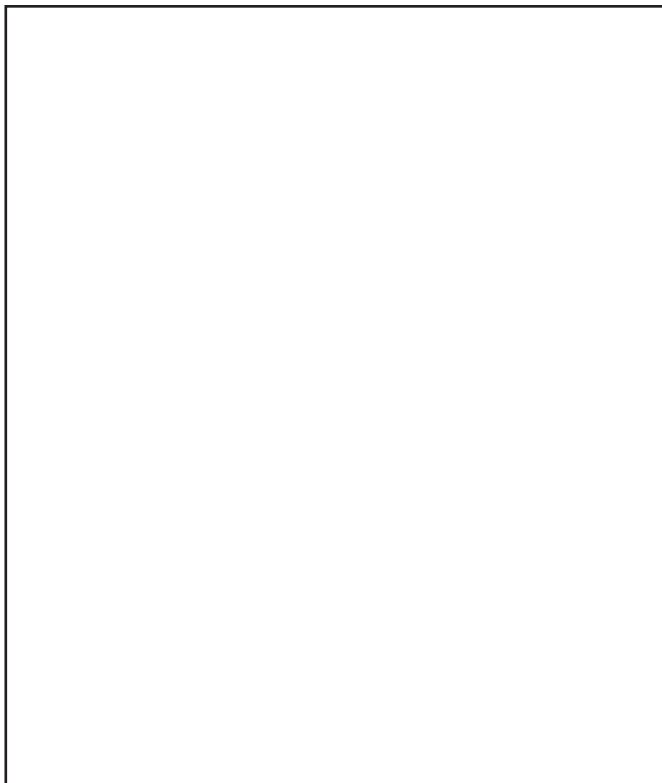
Subtraction



Multiplication

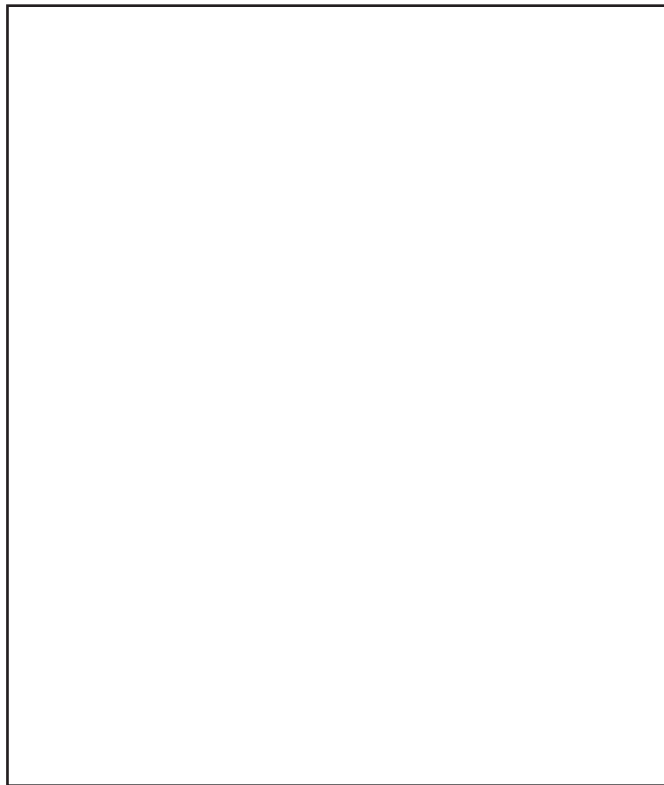


Division

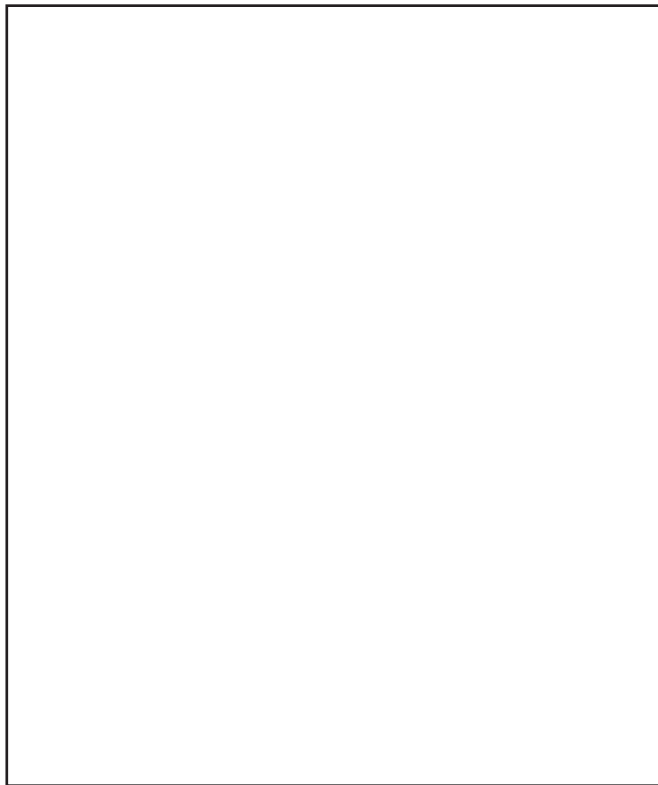


Computation

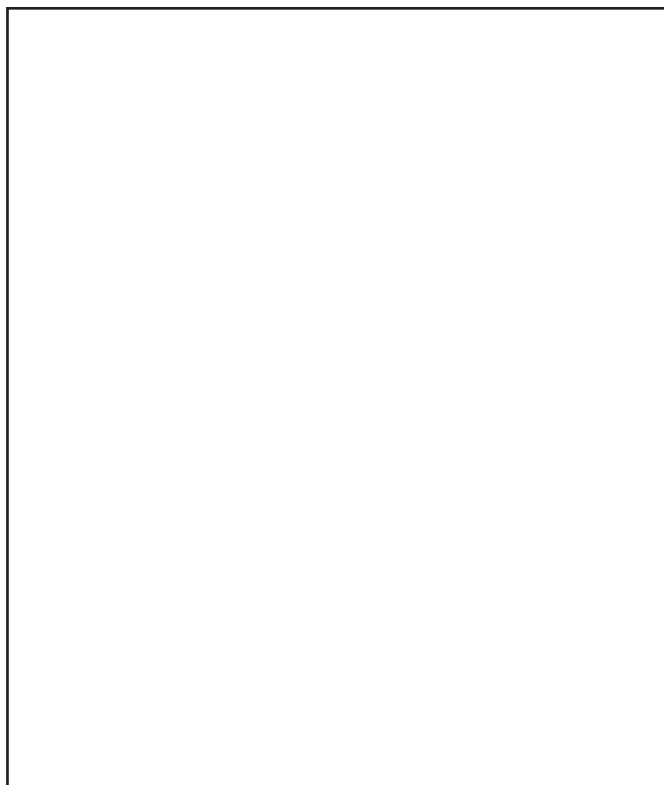
Addition



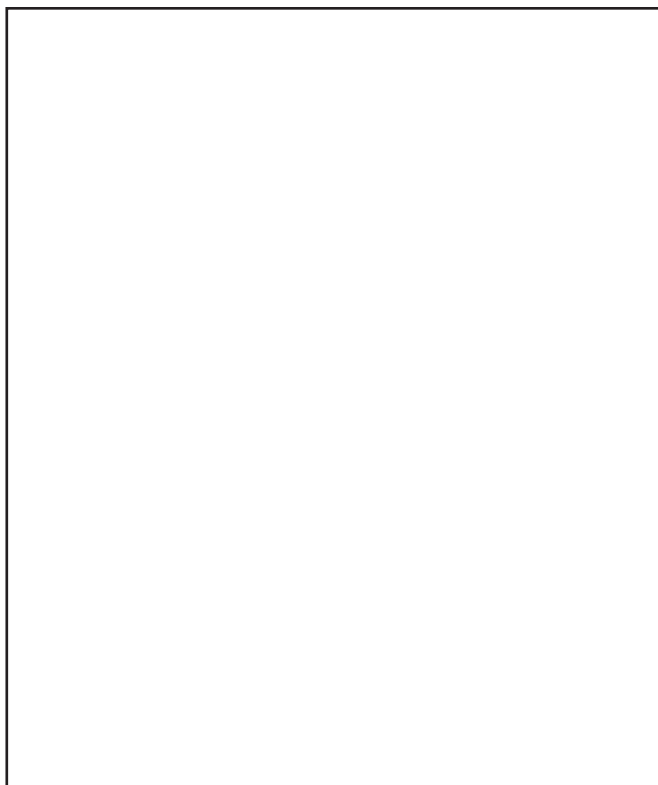
Subtraction



Multiplication



Division



Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

Solve the problem

What skills are necessary to solve this problem?

Problem-Solving Difficulties

Teaching Problem Solving

Attack Strategy

UPS✓

UNDERSTAND

Read and explain.

PLAN

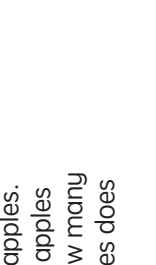
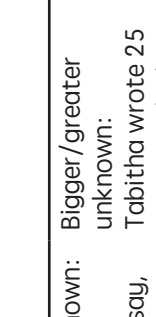
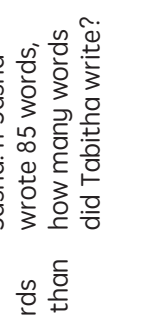

How will you solve the problem?

SOLVE

Set up and do the math!

✓CHECK

Does your answer make sense?

Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part-whole) Parts combined for a sum</p>	<p>$P1 + P2 = T$ (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	<p>$B - S = D$ (bigger - smaller = difference)</p>  <p>$G - L = D$ (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p>$ST + / - C = E$ (start +/- change = end)</p> 	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p> <p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p> <p>Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?</p> <p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>

Additive Word Problems

A.

Megan baked 38 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.

Jana has 162 wooden beads and 95 glass beads. How many more wooden beads than glass beads does Jana have?

D.

The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

Additive Word Problems

E.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?

F.

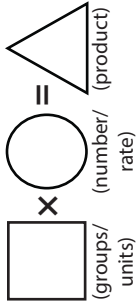
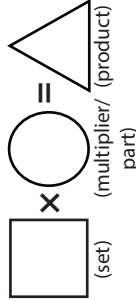
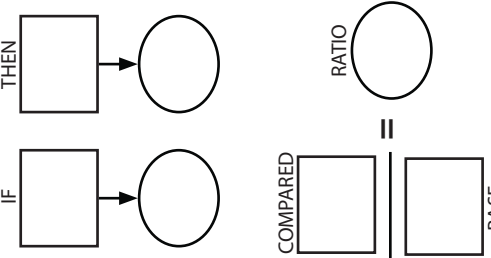
Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

G.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?

Schema and Definition	Graphic Organizers	Examples	Variations
<p>Equal Groups (Vary) A number of equal sets or units</p>		<p><i>Product unknown:</i> Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?</p> <p><i>Groups unknown:</i> Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?</p> <p><i>Number unknown:</i> Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?</p>	<p><i>With rate:</i> Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?</p>
<p>Comparison One set as a multiple or part of another set</p>		<p><i>Product unknown:</i> Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?</p> <p><i>Set unknown:</i> Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?</p> <p><i>Times unknown:</i> Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?</p>	<p><i>With fraction:</i> Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?</p>
<p>Proportions; (Unit Rate) Relationships among quantities</p> <p>Ratio</p>		<p><i>Subject unknown:</i> Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?</p> <p><i>Object unknown:</i> Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?</p> <p><i>Base unknown:</i> Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?</p> <p><i>Compared unknown:</i> Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?</p> <p><i>Ratio unknown:</i> Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?</p>	<p><i>With percentage:</i> Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?</p> <p><i>With unit rate:</i> Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?</p>

Multiplicative Word Problems

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

B.

Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.

Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.

Susan has 7 times as many books as Mo. Mo has 18 books. How many books does Susan have?

Multiplicative Word Problems

E.

The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

F.

An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

G.

Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

H.

Sam's two new aquariums each hold exactly 200 gallons of water. One aquarium will hold small fish and the other will hold large fish. Now he needs new fish for his aquarium. He will buy 5 small fish for every 10 gallons of water in the aquarium. He will buy 8 large fish for every 40 gallons of water in the aquarium. What is the total number of fish Sam will have? What will be the ratio of Sam's small fish to large fish?

Multi-Step Problems

- A.
Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?
- B.
Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?
- C.
An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons.
The small carton hold 8 eggs, and $\frac{1}{6}$ of the total cartons are small.
The medium carton holds 12 eggs, and $\frac{2}{3}$ of the total cartons are medium.
The large carton holds 18 eggs, and the rest of the total cartons are large.

Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.

Multi-Step Problems

D.

Allison is saving to buy a \$500 bicycle by working during summer vacation. The job pays her \$8 for every 1 hour worked. Allison works exactly 20 hours each week. If she works for 4 weeks and buys the bicycle, how much money will she have left over?

E.

Jessica rented 1 video game and 3 movies for a total of \$11.50. The video game cost \$4.75 to rent. The movie costs the same amount each to rent. What amount, in dollars, did Jessica pay to rent each movie?

F.

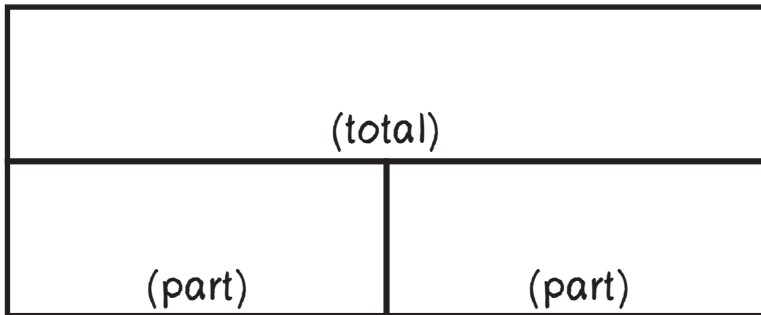
Sal exercised by stretching and jogging 5 days last week. He stretched for a total of 25 minutes during the week. He jogged for an equal number of minutes each of the 5 days. He exercised for a total of 240 minutes.

Elena also exercised by stretching and jogging 5 days last week. She stretched for a total of 15 minutes during each day. She jogged for an equal number of minutes each of the 5 days. She exercised for a total of 300 minutes.

Determine the number of minutes Sal jogged each day last week and the number of minutes Elena jogged each day last week.

TOTAL

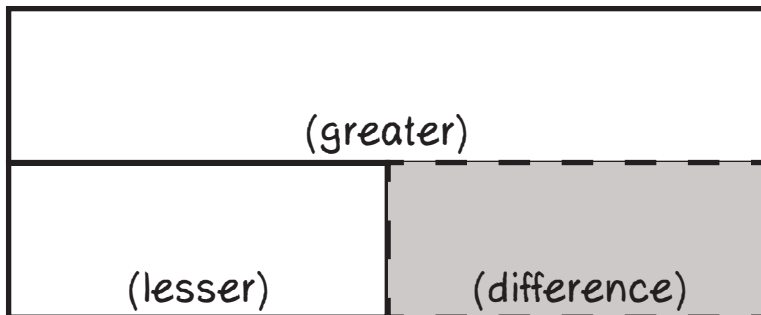
Are parts put together for a total?



$$P1 + P2 = T$$

DIFFERENCE

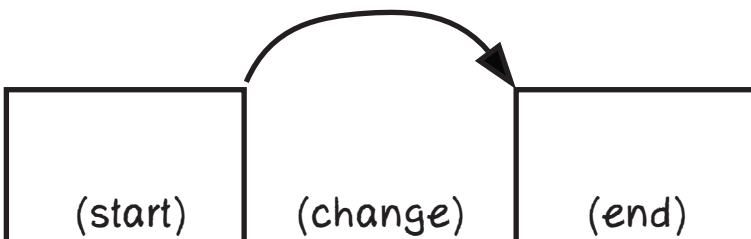
Are amounts compared for a difference?



$$G - L = D$$

CHANGE

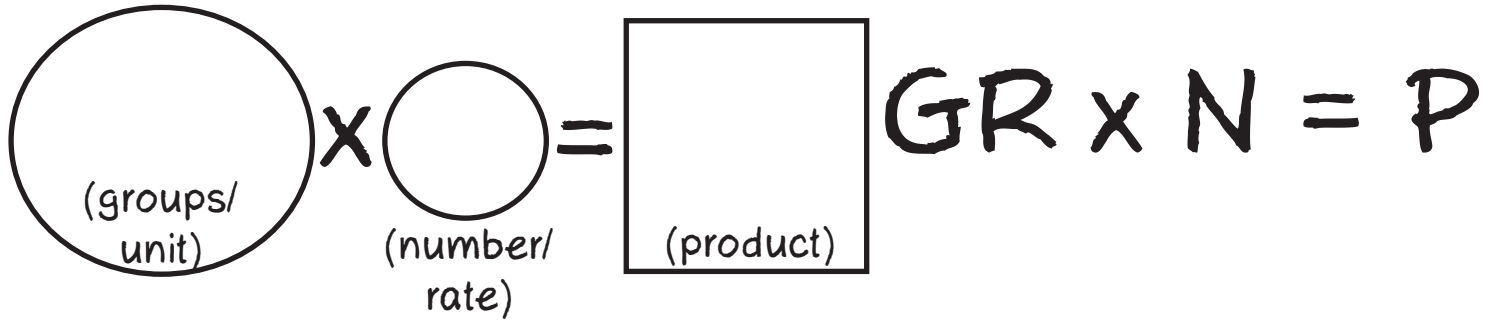
Does an amount increase or decrease?



$$ST +/- C = E$$

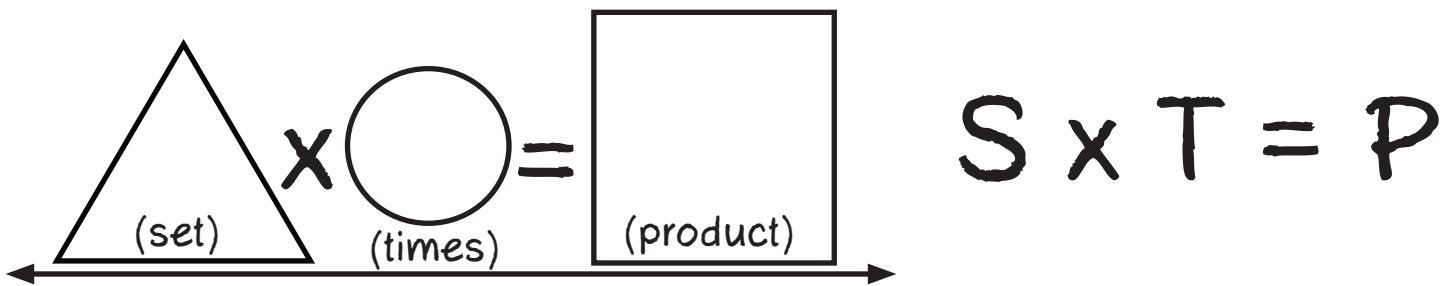
EQUAL GROUPS

Are there groups with an equal number in each group?



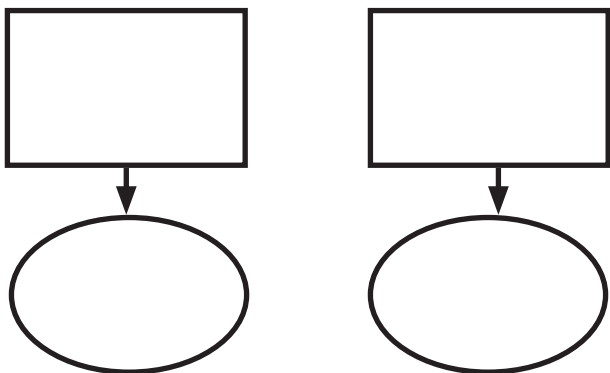
COMPARISON

Is a set compared a number of times?



RATIOS/PROPORTIONS

Are there relationships among quantities?



$$\frac{Q_1}{Q_2} = \frac{Q_3}{Q_4}$$