Math 3331 - ODE's

Finding sol^n's of

$$ay'' + by' + cy = f(x)$$

sol^n $y = y_c + y_p$ y_c - complementary sol^n

y_p - particular sol^n

y_c: sol^n of $ay'' + by' + c = 0$

y_p particular sol^n

Method (1) Undet. Coeff. (M of UD)

(2) Variation of Parameters

(3) Reduction of Order (VP)

Today V of P

if $y_c = c_1 y_1 + c_2 y_2$

$y_p = u y_1 + v y_2$ u & v to be determined

As there are 2 unknowns, we will need 2 eq^n's

$\underline{Ex}$ $y'' + y = \sec x$

$y_c = c_1 \cos x + c_2 \sin x$

look for sol^n of the form

$y = u \cos x + v \sin x$

$\$c$ $y' = u' \cos x - u \sin x + v' \sin x + v \cos x$

Set $u' \cos x + v' \sin x = 0$ — (1)

$y' = -u \sin x + v \cos x$

$y'' = -u' \sin x - u \cos x + v' \cos x - v \sin x$

Sub $y'' + y = \sec x$

$\Rightarrow$ $-u' \sin x - \cancel{u \cos x} + v' \cos x - \cancel{v \sin x} = \sec x$

$+ \cancel{u \cos x}$ $\quad$ $+ \cancel{v \sin x}$

$\underline{Eq's}$

$$\left.\begin{aligned} u' \cos x + v' \sin x &= 0 \\ -u' \sin x + v' \cos x &= \frac{1}{\cos x} \end{aligned}\right\} \text{solve for } u', v'$$

$u' = -\dfrac{\sin x}{\cos x}$ $\qquad$ $v' = 1$

$u = \ln|\cos x| + c_1$ $\qquad$ $v = x + c_2$

$$y = u\cos x + v\sin x$$

$$= (\ln|\cos x| + c_1)\cos x + (x + c_2)\sin x$$

$$= \underbrace{c_1\cos x + c_2\sin x}_{y_c} + \underbrace{\cos x\ln|\cos x| + x\sin x}_{y_p}$$

So we can neglect the constants of integration when finding u & v.

Ex $y'' - y = 1 + 2x - x^3$

M of UC $y_p = Ax^2 + Bx + C$ (easiest)

$y_c = c_1 e^x + c_2 e^{-x}$

Reduct Order $y = ue^x$

<u>V of P</u> $y = ue^x + ve^{-x}$

$y' = u'e^x + ue^x + v'e^{-x} - ve^{-x}$

Set $u'e^x + v'e^{-x} = 0$

So $y' = ue^x - ve^{-x}$

$y'' = u'e^x + ue^x - v'e^{-x} + ve^{-x}$

<u>sub</u> $\quad y'' - y = 1 + 2x - x^2$

$$u'e^x + ue^x - v'e^{-x} + ve^{-x} - ue^x - ve^{-x} = 1 + 2x - x^2$$

$\Rightarrow \quad u'e^x - v'e^{-x} = 1 + 2x - x^2$

& $\quad u'e^x + v'e^{-x} = 0$

so $\quad u' = \frac{1}{2}(1 + 2x - x^2)e^{-x}$

$\qquad v' = \frac{1}{2}(-1 - 2x + x^2)e^{x}$

$u = \dfrac{(x^2-1)e^{-x}}{2}, \quad v = \dfrac{(3 - 4x + x^2)e^{x}}{2}$

$y_p = e^x u + e^{-x} v = \dfrac{x^2 - 1}{2} + \dfrac{3 - 4x + x^2}{2}$

$\quad = x^2 - 2x + 1$

<u>GS</u> $\quad y = c_1 e^x + c_2 e^{-x} + x^2 - 2x + 1$