

## Corona Related $V_4$ Cordial graphs

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### Abstract

Let  $\langle A, * \rangle$  be any abelian group. A graph  $G = (V(G), E(G))$  is said to be  $A$ -cordial[6,9] if there is a mapping  $f: V(G) \rightarrow A$  which satisfies the following two conditions with each edge  $e = uv$  is labeled as  $f(u)*f(v)$ ,

$$(i) |v_f(a) - v_f(b)| \leq 1, \forall a, b \in A$$

$$(ii) |e_f(a) - e_f(b)| \leq 1, \forall a, b \in A$$

where  $v_f(a)$  = the number of vertices with label  $a$

$v_f(b)$  = the number of vertices with label  $b$

$e_f(a)$  = the number of edges with label  $a$

$e_f(b)$  = the number of edges with label  $b$

We note that if  $A = \langle V_4, * \rangle$  is a multiplicative group. Then the labeling is known as  **$V_4$  Cordial Labeling**. A graph is called a  **$V_4$  Cordial graph** if it admits a  $V_4$  Cordial Labeling. In this paper  $F_n^+$ ,  $C_n^+$ , and  $H_n^+$  are  **$V_4$  Cordial graphs**.

AMS Mathematics subject classification 2010:05C78

**Keywords and Phrases:** Cordial labeling,  $V_4$  Cordial Labeling and  $V_4$  Cordial Graph.

### 1. Introduction:

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology, we referred Harary[4]. For labeling of graphs, we referred Gallian[1].

A vertex labeling of a graph  $G$  is an assignment of labels to the vertices of  $G$  that induces for each edge  $uv$  a label depending on the vertex labels of  $u$  and  $v$ .

A graph  $G$  is said to be labeled if the  $n$  vertices are distinguished from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2]. In this paper  $F_n^+$ ,  $C_n^+$ , and  $H_n^+$  are  $V_4$  Cordial graphs.

### 2. Preliminaries

#### Definition 2.1:

Let  $G = (V, E)$  be a simple graph. Let  $f: V(G) \rightarrow \{0, 1\}$  and for each edge  $uv$ , assign the label  $|f(u) - f(v)|$ .  $f$  is called a **cordial labeling** if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

#### Definition 2.2:

Let  $\langle A, * \rangle$  be any abelian group. A graph  $G = (V(G), E(G))$  is said to be  $A$ -cordial[6,9] if there is a mapping  $f: V(G) \rightarrow A$  which satisfies the following two conditions with each edge  $e = uv$  is labeled as  $f(u)*f(v)$ .

$$(i) |v_f(a) - v_f(b)| \leq 1, \forall a, b \in A$$

$$(ii) |e_f(a) - e_f(b)| \leq 1, \forall a, b \in A$$

where  $v_f(a)$  = the number of vertices with label  $a$

$v_f(b)$  = the number of vertices with label  $b$

$e_f(a)$  = the number of edges with label a

$e_f(b)$  = the number of edges with label b

We note that if  $A = \langle V_4, * \rangle$  is a multiplicative group. Then the labeling is known as

**$V_4$  Cordial Labeling.** A graph is called a  **$V_4$  Cordial graph** if it admits a  $V_4$  Cordial Labeling.

**Definition 2.3:**

$G^+$  is a graph obtained from G by attaching a pendant vertex from each vertex of the graph G.

### 3.Main Results:

#### Theorem 3.1.

$F_n^+$  is a  $V_4$  Cordial graph.

**Proof:**

Let  $V(F_n^+) = \{u, v, u_i : 1 \leq i \leq n, v_i : 1 \leq i \leq n\}$ .

Let  $E(F_n^+) = \{(uu_i) : 1 \leq i \leq n\} \cup \{(u_i u_{i+1}) : 1 \leq i \leq n-1\} \cup \{uv\} \cup \{u_i v_i : 1 \leq i \leq n\}$ .

Define  $f : V(F_n^+) \rightarrow V_4$  by

Let  $f(u) = 1, f(v) = i$

$$f(u_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod{4} \\ -1 & \text{if } i \equiv 1 \pmod{4} \\ i & \text{if } i \equiv 2 \pmod{4} \\ 1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 0 \pmod{4} \\ 1 & \text{if } i \equiv 1 \pmod{4} \\ -i & \text{if } i \equiv 2 \pmod{4} \\ -1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

**Case (i) : when n is even**

The induced edge labelings are

$f(u)*f(v) = i$

$$f(u)*f(u_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod{4} \\ -1 & \text{if } i \equiv 1 \pmod{4} \\ i & \text{if } i \equiv 2 \pmod{4} \\ 1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(u_i)*f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(u_i)*f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n$$

**Vertex Conditions:**

$$v_f(1) = v_f(i) = \frac{n}{2} + 1 \text{ and } v_f(-i) = v_f(-1) = \frac{n}{2}$$

Hence,  $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$ .

**Edge Conditions:**

**Let  $n=4k, k \in N$**

$$e_f(1) = e_f(i) = e_f(-1) = e_f(-i) = n - k.$$

**Let  $n=4k+2, k \in N$**

$$e_f(1) = e_f(-i) = n - k - 1 \text{ and } e_f(-1) = e_f(i) = n - k.$$

Hence,  $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$ .

Hence,  $F_n^+$  is a  $V_4$  Cordial Graph .

For example, the  $V_4$  Cordial Labeling of  $F_6^+$  and  $F_8^+$  are shown in Figures 3.1.1 and 3.1.2.

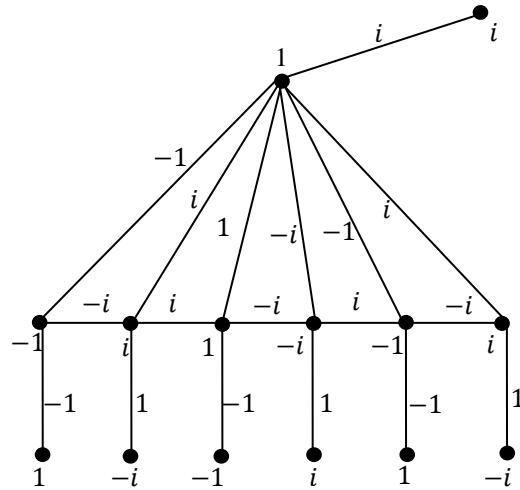


Figure3.1.1

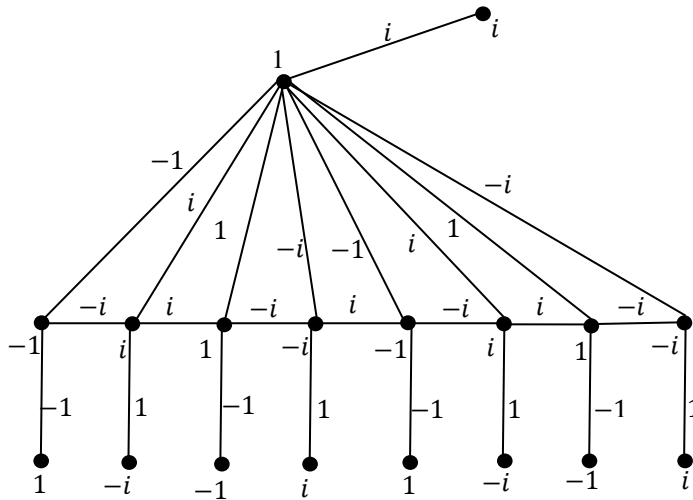


Figure3.1.2

**Case (ii) : when n is odd**

The induced edge labelings are

$$f(u) * f(v) = 1$$

$$f(u) * f(u_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod{4} \\ -i & \text{if } i \equiv 1 \pmod{4} \\ -1 & \text{if } i \equiv 2 \pmod{4} \\ i & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n - 1$$

$$f(u_i) * f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n$$

**Vertex Conditions:**

$$v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = \frac{n+1}{2}$$

Hence,  $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$ .

**Edge Conditions:**

Let  $n=4k+1, k \in \mathbb{N}$

$$e_f(1) = e_f(-1) = e_f(-i) = n - k \text{ and } e_f(i) = n - k - 1.$$

Let  $n=4k+3, k \in \mathbb{N}$

$e_f(1) = e_f(-i) = e_f(i) = n - k - 1$  and  $e_f(-1) = n - k$ .

Hence,  $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$ .

Hence,  $F_n^+$  is a  $V_4$ Cordial Graph.

For example, the  $V_4$ Cordial Labeling of  $F_5^+$  and  $F_7^+$  are shown in Figures 3.1.3 and 3.1.4.

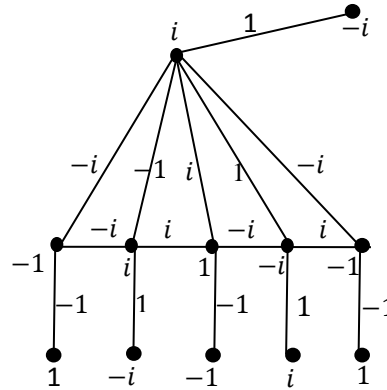


Figure3.1.3

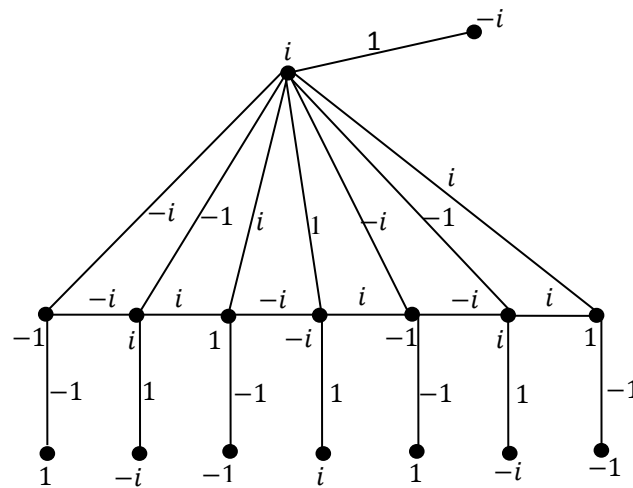


Figure3.1.4

**Theorem 3.2.**

$C_n^+$  is a  $V_4$  Cordial graph.

**Proof:**

Let  $V(C_n^+) = \{u_i: 1 \leq i \leq n, v_i: 1 \leq i \leq n\}$ .

Let  $E(C_n^+) = \{(u_i u_{i+1}): 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{(u_i v_i): 1 \leq i \leq n\}$ .

**Case(i): when  $n \equiv 0, 1 \pmod{4}$**

Define  $f: V(C_n^+) \rightarrow V_4$  by

$$f(u_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod{4} \\ -1 & \text{if } i \equiv 1 \pmod{4} \\ i & \text{if } i \equiv 2 \pmod{4} \\ 1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 0 \pmod{4} \\ 1 & \text{if } i \equiv 1 \pmod{4} \\ -i & \text{if } i \equiv 2 \pmod{4} \\ -1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

**Subcase(i): when  $n \equiv 0 \pmod{4}$**

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n$$

$$f(u_i) * f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n$$

**Vertex Conditions:**

Here,  $v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = \frac{n}{2}$

Hence,  $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$ .

**Edge Conditions:**

Here,  $e_f(1) = e_f(i) = e_f(-1) = e_f(-i) = \frac{n}{2}$

Hence,  $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$ .

Hence,  $C_n^+$  is a  $V_4$ Cordial graph.

For example, the  $V_4$ Cordial Labeling of  $C_8^+$  is shown in the Figure 3.2.1.

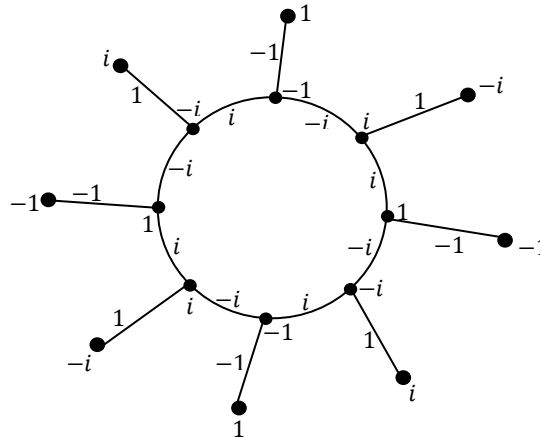


Figure 3.2.1

**Subcase(ii): when  $n \equiv 1 \pmod{4}$**

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(u_n) * f(u_1) = 1$$

$$f(u_i) * f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n$$

**Vertex Conditions:**

Here,  $v_f(1) = v_f(-1) = \frac{n+1}{2}$  and  $v_f(i) = v_f(-i) = \frac{n-1}{2}$ .

Hence,  $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$ .

**Edge Conditions:**

Here,  $e_f(1) = e_f(-1) = \frac{n+1}{2}$  and  $e_f(i) = e_f(-i) = \frac{n-1}{2}$ .

Hence,  $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$ .

Hence,  $C_n^+$  is a  $V_4$ Cordial graph.

For example, the  $V_4$ Cordial Labeling of  $C_9^+$  is shown in the Figure 3.2.2.

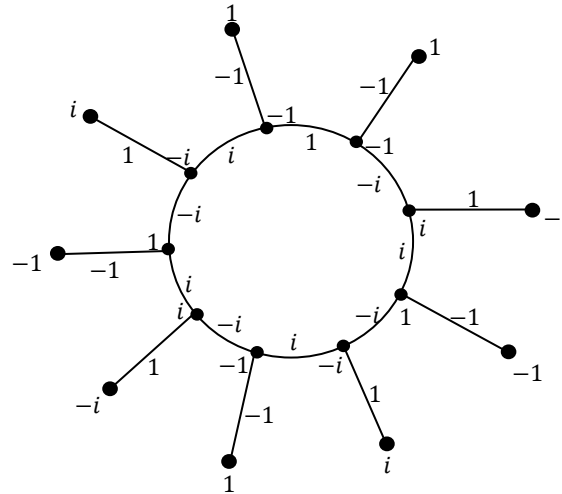


Figure 3.2.2

**Case(ii): when  $n \equiv 2(\text{mod } 4)$**

Define  $f : V(C_n^+) \rightarrow V_4$  by

$$f(u_i) = \begin{cases} -i & \text{if } i \equiv 0(\text{mod } 4) \\ -1 & \text{if } i \equiv 1(\text{mod } 4) \\ i & \text{if } i \equiv 2(\text{mod } 4) \\ 1 & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, 1 \leq i \leq n-2$$

$$f(u_{n-1}) = 1, \quad f(u_n) = -1$$

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 0(\text{mod } 4) \\ 1 & \text{if } i \equiv 1(\text{mod } 4) \\ -i & \text{if } i \equiv 2(\text{mod } 4) \\ -1 & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, 1 \leq i \leq n-2$$

$$f(v_{n-1}) = i, \quad f(v_n) = -i$$

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1(\text{mod } 2) \\ i & \text{if } i \equiv 0(\text{mod } 2) \end{cases}, 1 \leq i \leq n-3$$

$$f(u_{n-2}) * f(u_{n-1}) = -i, \quad f(u_{n-1}) * f(u_n) = -1, \quad f(u_n) * f(u_1) = 1$$

$$f(u_i) * f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1(\text{mod } 2) \\ 1 & \text{if } i \equiv 0(\text{mod } 2) \end{cases}, 1 \leq i \leq n-2$$

$$f(u_{n-1}) * f(v_{n-1}) = f(u_n) * f(v_n) = i$$

**Vertex Conditions:**

$$\text{Here, } v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = \frac{n}{2}$$

$$\text{Hence, } |v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4.$$

**Edge Conditions:**

$$\text{Here, } e_f(1) = e_f(i) = e_f(-1) = e_f(-i) = \frac{n}{2}$$

$$\text{Hence, } |e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4.$$

Hence,  $C_n^+$  is a  $V_4$ Cordial graph.

For example, the  $V_4$ Cordial Labeling of  $C_{10}^+$  is shown in the Figure 3.2.3.

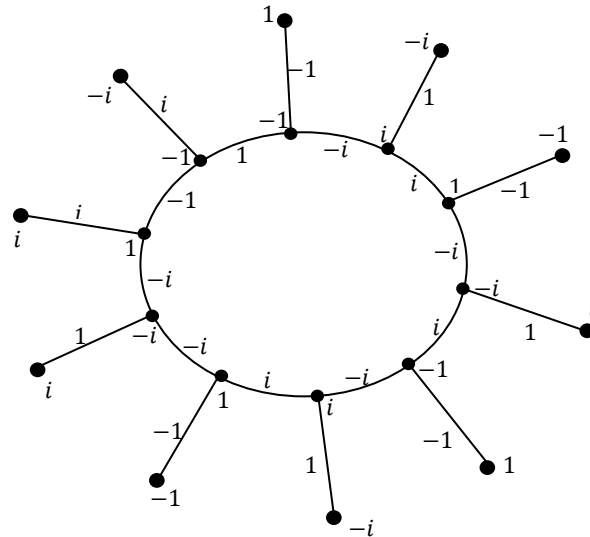


Figure 3.2.3

**Case(ii): when  $n \equiv 3 \pmod{4}$**

Define  $f : V(C_n^+) \rightarrow V_4$  by

$$f(u_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod{4} \\ -1 & \text{if } i \equiv 1 \pmod{4} \\ i & \text{if } i \equiv 2 \pmod{4} \\ 1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n-2$$

$$f(u_{n-1}) = i, \quad f(u_n) = 1$$

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 0 \pmod{4} \\ 1 & \text{if } i \equiv 1 \pmod{4} \\ -i & \text{if } i \equiv 2 \pmod{4} \\ -1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n-2$$

$$f(v_{n-1}) = f(v_n) = -i$$

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(u_n) * f(u_1) = -1$$

$$f(u_i) * f(v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(u_n) * f(v_n) = -i$$

**Vertex Conditions:**

$$\text{Here, } v_f(1) = v_f(-i) = \frac{n+1}{2} \text{ and } v_f(-1) = v_f(i) = \frac{n-1}{2}$$

$$\text{Hence, } |v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4.$$

**Edge Conditions:**

$$\text{Here, } e_f(1) = e_f(i) = \frac{n-1}{2} \text{ and } e_f(-1) = e_f(-i) = \frac{n+1}{2}$$

$$\text{Hence, } |e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4.$$

Hence,  $C_n^+$  is a  $V_4$ Cordial graph.

For example, the  $V_4$ Cordial Labeling of  $C_{11}^+$  is shown in the Figure 3.2.4 .

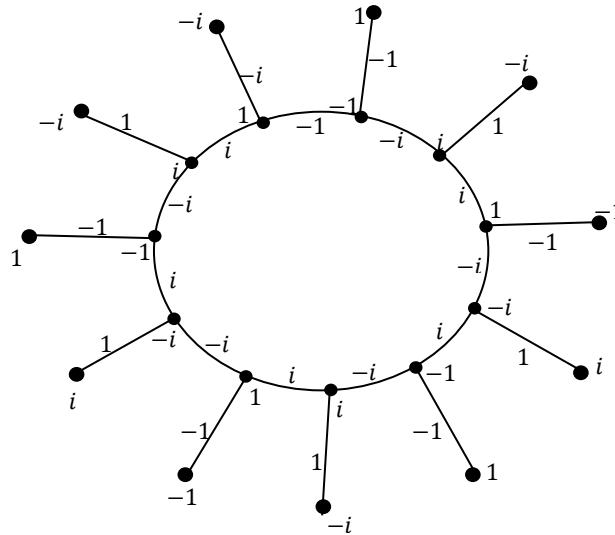


Figure 3.2.4

**Theorem 3.3.**

$H_n^+$  is a  $V_4$  Cordial graph (when  $n$  is odd).

**Proof:**

Let  $V(H_n^+) = \{u_i, v_i : 1 \leq i \leq n, u'_i, v'_i : 1 \leq i \leq n\}$ .

Let  $E(H_n^+) = \{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \leq i \leq n-1\} \cup \{u_i u'_i, v_i v'_i : 1 \leq i \leq n\} \cup \{u_{\frac{(n+1)}{2}} v_{\frac{(n+1)}{2}}\}$ .

Define  $f : V(H_n^+) \rightarrow V_4$  by

$$f(u_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod{4} \\ -1 & \text{if } i \equiv 1 \pmod{4} \\ i & \text{if } i \equiv 2 \pmod{4} \\ 1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(u'_i) = \begin{cases} i & \text{if } i \equiv 0 \pmod{4} \\ 1 & \text{if } i \equiv 1 \pmod{4} \\ -i & \text{if } i \equiv 2 \pmod{4} \\ -1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod{4} \\ i & \text{if } i \equiv 1 \pmod{4} \\ -1 & \text{if } i \equiv 2 \pmod{4} \\ -i & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(v'_i) = \begin{cases} -1 & \text{if } i \equiv 0 \pmod{4} \\ -i & \text{if } i \equiv 1 \pmod{4} \\ 1 & \text{if } i \equiv 2 \pmod{4} \\ i & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(v_i) * f(v_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1 \pmod{2} \\ i & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(u_i) * f(u'_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(v_i) * f(v'_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n-1$$

$$f(u_{\frac{(n+1)}{2}}) * f(v_{\frac{(n+1)}{2}}) = -i$$

**Vertex Conditions:**



Here,  $v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = n$

Hence,  $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$ .

**Edge Conditions:**

Here,  $e_f(1) = e_f(-1) = e_f(-i) = n$  and  $e_f(i) = n-1$ .

Hence,  $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$ .

Hence,  $H_n^+$  is a  $V_4$ Cordial graph.

For example, the  $V_4$ Cordial Labeling of  $H_5^+$  and  $H_7^+$  are shown in Figures 3.3.1 and 3.3.2.

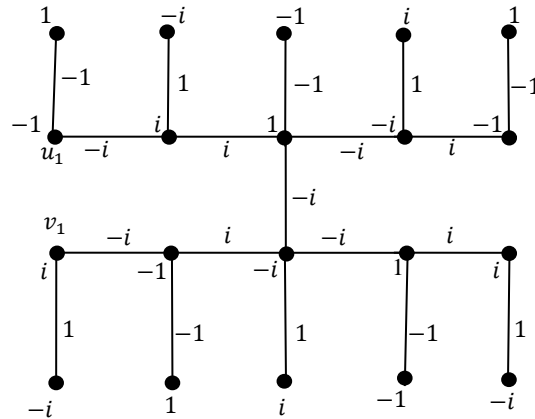


Figure 3.3.1

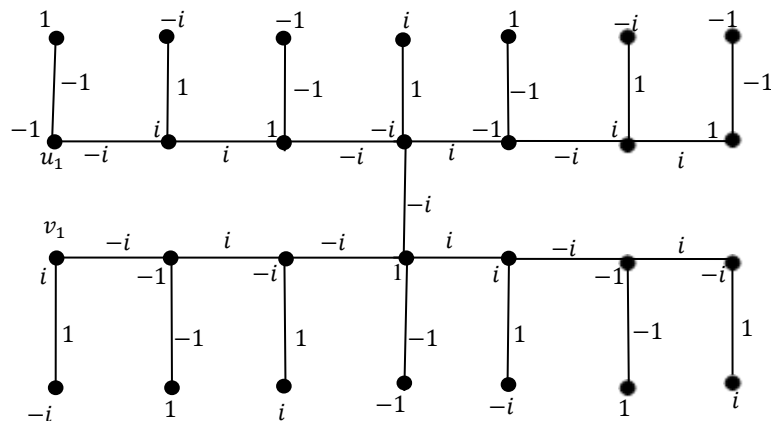


Figure 3.3.2

**Theorem 3.4.**

$H_n^+$  is a  $V_4$  Cordial graph (when n is even).

**Proof:**

Let  $V(H_n^+) = \{u_i, v_i : 1 \leq i \leq n, u'_i, v'_i : 1 \leq i \leq n\}$ .

Let  $E(H_n^+) = \{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \leq i \leq n-1\} \cup \{u_i u'_i, v_i v'_i : 1 \leq i \leq n\} \cup \{u_{(\frac{n}{2}+1)} v_{(\frac{n}{2})}\}$ .

Define  $f : V(H_n^+) \rightarrow V_4$  by

$$f(u_i) = \begin{cases} -i & \text{if } i \equiv 0 \pmod{4} \\ -1 & \text{if } i \equiv 1 \pmod{4} \\ i & \text{if } i \equiv 2 \pmod{4} \\ 1 & \text{if } i \equiv 3 \pmod{4} \end{cases}, 1 \leq i \leq n$$

$$f(u_i) = \begin{cases} i & \text{if } i \equiv 0(\text{mod } 4) \\ 1 & \text{if } i \equiv 1(\text{mod } 4) \\ -i & \text{if } i \equiv 2(\text{mod } 4) \\ -1 & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} -i & \text{if } i \equiv 0(\text{mod } 4) \\ 1 & \text{if } i \equiv 1(\text{mod } 4) \\ i & \text{if } i \equiv 2(\text{mod } 4) \\ -1 & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, 1 \leq i \leq n$$

$$f(v'_i) = \begin{cases} i & \text{if } i \equiv 0(\text{mod } 4) \\ -1 & \text{if } i \equiv 1(\text{mod } 4) \\ -i & \text{if } i \equiv 2(\text{mod } 4) \\ 1 & \text{if } i \equiv 3(\text{mod } 4) \end{cases}, 1 \leq i \leq n$$

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} -i & \text{if } i \equiv 1(\text{mod } 2) \\ i & \text{if } i \equiv 0(\text{mod } 2) \end{cases}, 1 \leq i \leq n-1$$

$$f(v_i) * f(v_{i+1}) = \begin{cases} i & \text{if } i \equiv 1(\text{mod } 2) \\ -i & \text{if } i \equiv 0(\text{mod } 2) \end{cases}, 1 \leq i \leq n-1$$

$$f(u_i) * f(u'_i) = \begin{cases} -1 & \text{if } i \equiv 1(\text{mod } 2) \\ 1 & \text{if } i \equiv 0(\text{mod } 2) \end{cases}, 1 \leq i \leq n-1$$

$$f(v_i) * f(v'_{i+1}) = \begin{cases} -1 & \text{if } i \equiv 1(\text{mod } 2) \\ 1 & \text{if } i \equiv 0(\text{mod } 2) \end{cases}, 1 \leq i \leq n-1$$

$$f(u_{\binom{n}{2}+1}) * f(v_{\binom{n}{2}}) = i$$

**Vertex Conditions:**

Here,  $v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = n$

Hence,  $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$ .

**Edge Conditions:**

Here,  $e_f(1) = e_f(-1) = e_f(-i) = n$  and  $e_f(i) = n-1$ .

Hence,  $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$ .

Hence,  $H_n^+$  is a  $V_4$ Cordial graph.

For example, the  $V_4$ Cordial Labeling of  $H_6^+$  and  $H_8^+$  are shown in Figures 3.4.1 and 3.4.2.

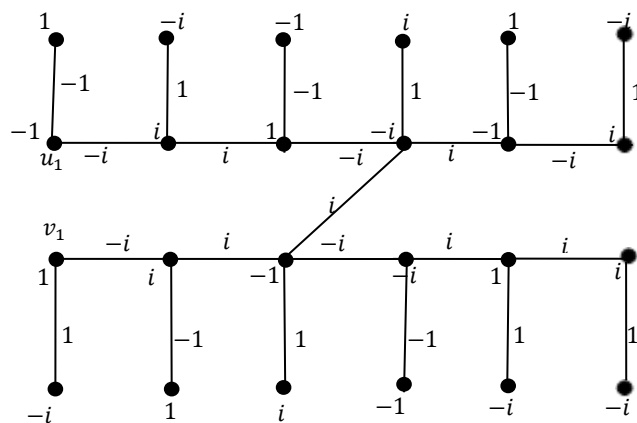


Figure 3.4.1

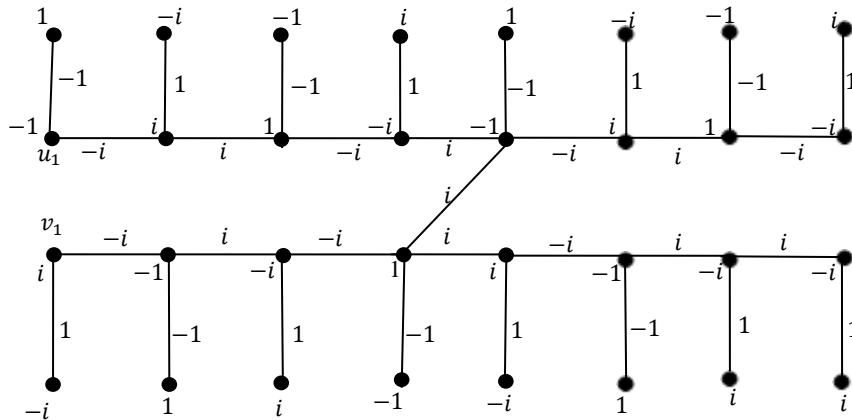


Figure 3.4.2

#### 4. Conclusion

Group Theory plays a crucial role in the field of Mathematics. Combining group theory and the labelling of a graph by defining edge labelling induced by composition of mapping will definitely a significant aspect. It may bring wider applications.

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