Chapter 4
Polynomial Functions

Section 4-8
Analyzing Graphs of Polynomial Functions
And
Section 4-9
Modeling with Polynomial Functions

Graphing Polynomial Functions

In this chapter, you have learned that zeros, factors, solutions, and x-intercepts are closely related concepts. Here is a summary of these relationships.

Concept Summary

Zeros, Factors, Solutions, and Intercepts

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ be a polynomial function. The following statements are equivalent.

Zero: k is a zero of the polynomial function f.

Factor: $x - k$ is a factor of the polynomial $f(x)$.

Solution: k is a solution (or root) of the polynomial equation $f(x) = 0$.

***x*-Intercept:** If k is a real number, then k is an x-intercept of the graph of the polynomial function f. The graph of f passes through $(k, 0)$.

The Location Principle

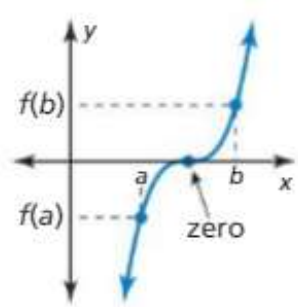
You can use the *Location Principle* to help you find real zeros of polynomial functions.

Core Concept

The Location Principle

If f is a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$, then f has at least one real zero between a and b.

To use this principle to locate real zeros of a polynomial function, find a value a at which the polynomial function is negative and another value b at which the function is positive. You can conclude that the function has *at least* one real zero between a and b.



EXAMPLE 1 Using *x*-Intercepts to Graph a Polynomial Function

Graph the function

$f(x) = \frac{1}{6}(x + 3)(x - 2)^2$.

Graph the function.

1. $f(x) = \frac{1}{2}(x + 1)(x - 4)^2$

2. $f(x) = \frac{1}{4}(x + 2)(x - 1)(x - 3)$

EXAMPLE 2 Locating Real Zeros of a Polynomial Function

Find all real zeros of

$$f(x) = 6x^3 + 5x^2 - 17x - 6.$$

SOLUTION

Step 1 Use a graphing calculator to make a table.

X	Y1
0	-6
1	-12
2	28
3	150
4	390
5	784
6	1368

X=1

Step 2 Use the Location Principle. From the table shown, you can see that $f(1) < 0$ and $f(2) > 0$. So, by the Location Principle, f has a zero between 1 and 2. Because f is a polynomial function of degree 3, it has three zeros. The only possible *rational* zero between 1 and 2 is $\frac{3}{2}$. Using synthetic division, you can confirm that $\frac{3}{2}$ is a zero.

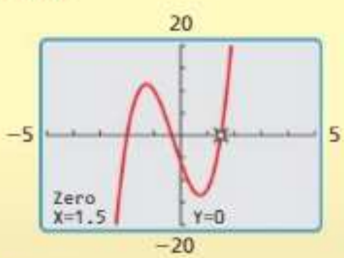
Step 3 Write $f(x)$ in factored form. Dividing $f(x)$ by its known factor $x - \frac{3}{2}$ gives a quotient of $6x^2 + 14x + 4$. So, you can factor $f(x)$ as

$$f(x) = \left(x - \frac{3}{2}\right)(6x^2 + 14x + 4)$$

$$= 2\left(x - \frac{3}{2}\right)(3x^2 + 7x + 2)$$

$$= 2\left(x - \frac{3}{2}\right)(3x + 1)(x + 2).$$

Check

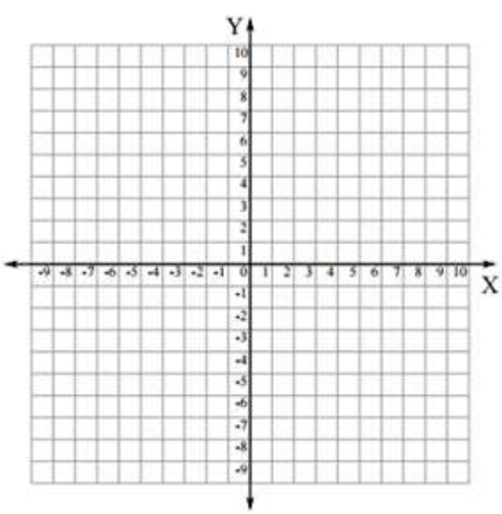


From the factorization, there are three zeros. The zeros of f are

$\frac{3}{2}$, $-\frac{1}{3}$, and -2.

Check this by graphing f.

3. Find all real zeros of $f(x) = 18x^3 + 21x^2 - 13x - 6$.



X	Y

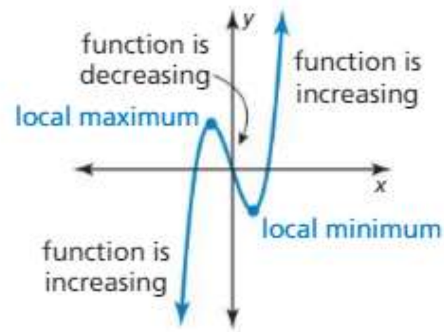
Turning Points

Another important characteristic of graphs of polynomial functions is that they have *turning points* corresponding to local maximum and minimum values.

READING

Local maximum and local minimum are sometimes referred to as *relative maximum* and *relative minimum*.

- The y-coordinate of a turning point is a **local maximum** of the function when the point is higher than all nearby points.
- The y-coordinate of a turning point is a **local minimum** of the function when the point is lower than all nearby points.



The turning points of a graph help determine the intervals for which a function is increasing or decreasing.

Core Concept

Turning Points of Polynomial Functions

1. The graph of every polynomial function of degree n has *at most* $n - 1$ turning points.
2. If a polynomial function has n distinct real zeros, then its graph has *exactly* $n - 1$ turning points.

Tutorial

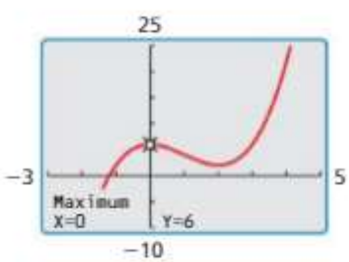
EXAMPLE 3 Finding Turning Points

Graph each function. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which each function is increasing or decreasing.

a. $f(x) = x^3 - 3x^2 + 6$ **b.** $g(x) = x^4 - 6x^3 + 3x^2 + 10x - 3$

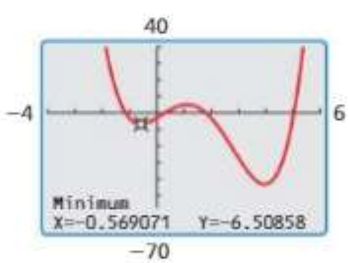
SOLUTION

a. Use a graphing calculator to graph the function. The graph of f has one x-intercept and two turning points. Use the graphing calculator's *zero*, *maximum*, and *minimum* features to approximate the coordinates of the points.



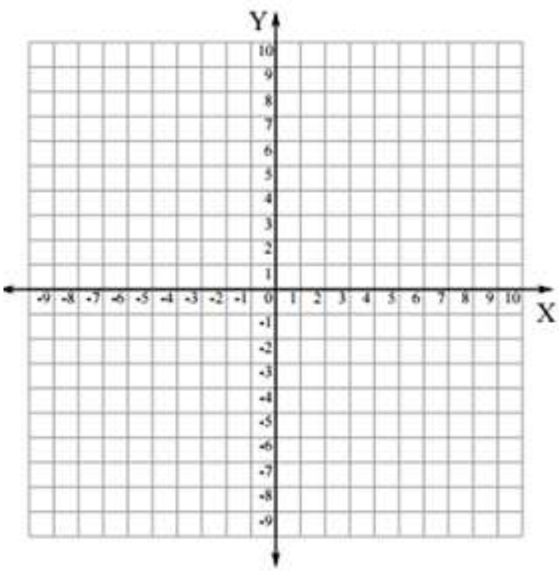
▶ The x-intercept of the graph is $x \approx -1.20$. The function has a local maximum at $(0, 6)$ and a local minimum at $(2, 2)$. The function is increasing when $x < 0$ and $x > 2$ and decreasing when $0 < x < 2$.

b. Use a graphing calculator to graph the function. The graph of g has four x-intercepts and three turning points. Use the graphing calculator's *zero*, *maximum*, and *minimum* features to approximate the coordinates of the points.



▶ The x-intercepts of the graph are $x \approx -1.14$, $x \approx 0.29$, $x \approx 1.82$, and $x \approx 5.03$. The function has a local maximum at $(1.11, 5.11)$ and local minimums at $(-0.57, -6.51)$ and $(3.96, -43.04)$. The function is increasing when $-0.57 < x < 1.11$ and $x > 3.96$ and decreasing when $x < -0.57$ and $1.11 < x < 3.96$.

4. Graph $f(x) = 0.5x^3 + x^2 - x + 2$. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing.



X	Y

Even and Odd Functions

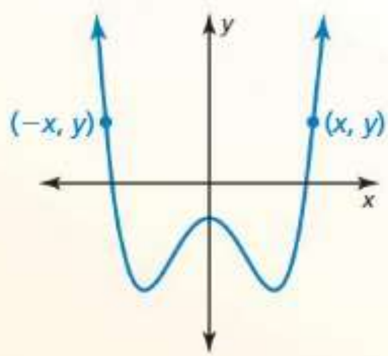
Core Concept

Even and Odd Functions

A function f is an **even function** when $f(-x) = f(x)$ for all x in its domain. The graph of an even function is *symmetric about the y-axis*.

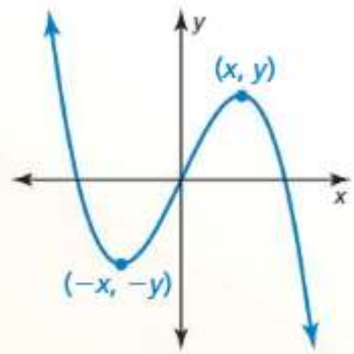
A function f is an **odd function** when $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is *symmetric about the origin*. One way to recognize a graph that is symmetric about the origin is that it looks the same after a 180° rotation about the origin.

Even Function



For an even function, if (x, y) is on the graph, then $(-x, y)$ is also on the graph.

Odd Function



For an odd function, if (x, y) is on the graph, then $(-x, -y)$ is also on the graph.

Tutorial

EXAMPLE 4 Identifying Even and Odd Functions

Determine whether each function is *even*, *odd*, or *neither*.

a. $f(x) = x^3 - 7x$ **b.** $g(x) = x^4 + x^2 - 1$ **c.** $h(x) = x^3 + 2$

SOLUTION

a. Replace x with $-x$ in the equation for f, and then simplify.

$$f(-x) = (-x)^3 - 7(-x) = -x^3 + 7x = -(x^3 - 7x) = -f(x)$$

▶ Because $f(-x) = -f(x)$, the function is odd.

b. Replace x with $-x$ in the equation for g, and then simplify.

$$g(-x) = (-x)^4 + (-x)^2 - 1 = x^4 + x^2 - 1 = g(x)$$

▶ Because $g(-x) = g(x)$, the function is even.

c. Replacing x with $-x$ in the equation for h produces

$$h(-x) = (-x)^3 + 2 = -x^3 + 2.$$

▶ Because $h(x) = x^3 + 2$ and $-h(x) = -x^3 - 2$, you can conclude that $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$. So, the function is neither even nor odd.

Determine whether the function is *even*, *odd*, or *neither*.

5. $f(x) = -x^2 + 5$ **6.** $f(x) = x^4 - 5x^3$ **7.** $f(x) = 2x^5$

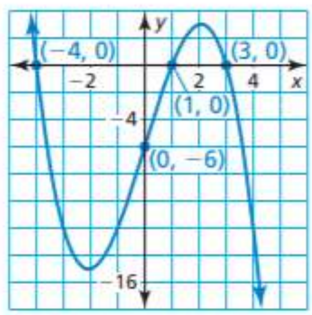
Section 4-9
Modeling with Polynomial Functions

Writing Polynomial Functions for a Set of Points

You know that two points determine a line and three points not on a line determine a parabola. In Example 1, you will see that four points not on a line or a parabola determine the graph of a cubic function.

EXAMPLE 1 **Writing a Cubic Function**

Write the cubic function whose graph is shown.



Finite Differences

When the x-values in a data set are equally spaced, the differences of consecutive y-values are called **finite differences**. Recall from Section 2.4 that the first and second differences of $y = x^2$ are:

equally-spaced x-values

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

first differences: −5 −3 −1 1 3 5

second differences: 2 2 2 2 2

Notice that $y = x^2$ has degree *two* and that the *second* differences are constant and nonzero. This illustrates the first of the two properties of finite differences shown on the next page.

Core Concept

Properties of Finite Differences

1. If a polynomial function $y = f(x)$ has degree n, then the nth differences of function values for equally-spaced x-values are nonzero and constant.
2. Conversely, if the nth differences of equally-spaced data are nonzero and constant, then the data can be represented by a polynomial function of degree n.

The second property of finite differences allows you to write a polynomial function that models a set of equally-spaced data.

EXAMPLE 2 Writing a Function Using Finite Differences

Use finite differences to determine the degree of the polynomial function that fits the data. Then use technology to find the polynomial function.

x	1	2	3	4	5	6	7
$f(x)$	1	4	10	20	35	56	84

SOLUTION

Step 1 Write the function values. Find the first differences by subtracting consecutive values. Then find the second differences by subtracting consecutive first differences. Continue until you obtain differences that are nonzero and constant.

$f(1)$ $f(2)$ $f(3)$ $f(4)$ $f(5)$ $f(6)$ $f(7)$ — Write function values for equally-spaced x-values.

1 4 10 20 35 56 84

3 6 10 15 21 28 — First differences

3 4 5 6 7 — Second differences

1 1 1 1 — Third differences

Because the third differences are nonzero and constant, you can model the data *exactly* with a cubic function.

Step 2 Enter the data into a graphing calculator and use cubic regression to obtain a polynomial function.

```
CubicReg
y=ax³+bx²+cx+d
a=.1666666667
b=.5
c=.3333333333
d=0
R²=1
```

▶ Because $\frac{1}{6} \approx 0.1666666667$, $\frac{1}{2} = 0.5$, and $\frac{1}{3} \approx 0.333333333$, a polynomial function that fits the data exactly is

$$f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{1}{3}x.$$

Section 4-8 Homework #1-21odd and #27-45 odd and 47

Section 4-9 Homework #3-13 odd