Import Tariff and Exchange Rate Transmission in a Small Open Economy

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Abstract

This paper presents a small open economy DSGE model with internal and external sticky prices, based on Monacelli (2005), which considers an incomplete exchange rate pass-through. On the top of this framework, import tariff was included as another variable, besides the exchange rate, to affect the Law of One Price. The responses of key macroeconomic variables to an import tariff shock have been analyzed, considering the parameters for the Brazilian economy. The conclusion was that the import-tariffs shock entails deviations on the Law of One Price since import prices become higher. There is an increase on the total inflation, which leads to a raise on the interest rates. The long run effect is a worsening on the terms of trade, since the exchange rate appreciation which follows the shock cancels the initial effect of the increase of import tariffs.

Keywords: DSGE Model; Open Economy; Import Tariff; Exchange Rate.

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1. INTRODUCTION

From 2010 to 2013 the Brazilian government imposed high import tariffs to protect domestic manufacturers of some sectors from foreign competition. Tariff advocates claimed that high import duties were necessary not only to deal with international unfair trade practices and imbalances derived from international environment, but also to create a steady demand in the home market for domestic goods. On the other hand, tariff critics charged that those import duties raised the cost of living for consumers. In fact, a body of literature has grown on the effects of import tariffs on prices (Dornbusch, 1987; Dixit, 1988; Feenstra, 1989; Levinsohn, 1993; Harrison, 1994; Hay, 2001; Lisboa et al., 2010). Basically, these authors argue that high import tariffs imply less imports and domestic producer prices above marginal costs. In fact, the inflation rate in Brazil through the 12 months ended December 2013 was 5.91%. However, some products categories, for example, plastic, steel and chemical, which were subject to higher import tariffs, recorded 9% to 15% over the same period.

Considering that changes in ad valorem tariffs may affect supply and demand and, as a consequence, producer and consumer prices, one should expect other macroeconomic variables to be affected, such as the terms of trade and the real exchange rate. Although these effects make sense in emerging economies (given that developed countries apply lower import tariffs), there is a lack of studies concerning the transmission effects of an import tariff shock in the economy with the dynamic stochastic general equilibrium (DSGE) modeling. At this point lies the main motivation of this work.

We consider a small open economy model with imperfect competition and nominal price rigidities, similar to that in Clarida et al. (2001). The basic assumptions, in particular the functional form of the household preferences and of the technology of firms, follow the literature related to these models, as the initial work of Obstfeld and Rogoff (1995) - called the redux model - and more recently Galí and Monacelli (2005). Our framework is based on Monacelli (2005) which introduces incomplete exchange rate pass-through on import prices. The setting presented by Monacelli (2005) can be reduced to a tractable compact form easily comparable to the canonical New Keynesian model previously adopted by the literature. One key difference is that, in addition to having exchange rate, import tariffs are included to generate deviations from the law of one price\(^1\).

\(^1\) The law of one price states that in the absence of trade barriers, whether natural or imposed by the government, goods should be sold at the same price in all countries. However, a large number of empirical studies have
One limitation of the redux model is the assumption that the pass-through of the exchange rate to prices is complete and therefore the law of one price holds continually. This lies in contrast with a well established empirical fact: the failure of the law of one price for tradables (FROOT; KLEMPERER, 1989; ENGEL, 1993; GOLDBERG; KNETTER, 1997). Given these empirical evidences, many authors have introduced international market segmentation in the original model since firms segment markets by country and set prices in local currency of sale, a practice referred to as pricing-to-market (DEVEREUX; ENGEL, 1998; BETTS; DEVEREUX, 2000; CHARI et al., 2002). Others have examined the monetary policy implications of allowing an incomplete pass-through in an inflation targeting framework (ADOLFSON, 2001; BENIGNO, 2004; DEVEREUX, 2004, CORSETTI; PESENTI, 2001; SUTHERLAND, 2005; DIVINO, 2009). Assuming domestic price stickiness, as a consequence of firms facing costs of changing prices, the domestic currency (import) price will not be fully altered even if exchange rate affect the marginal cost. This implies that import prices do not move immediately and in a one-to-one relation with the exchange rate (incomplete exchange rate pass-through), allowing deviations from the law of one price.

In Brazil, few papers lay out a small open economy version of a dynamic stochastic general equilibrium model (VALLI; CARVALHO, 2009; VEREDA; CAVALCANTI, 2010). Silveira (2006a, 2006b) is the contribution more in line with the present paper. In order to reproduce the Brazilian macroeconomic series more accurately, these authors suggest that DSGE models for Brazil be estimated considering some structural hypotheses, such as incomplete exchange rate pass-through. The Brazilian DSGE model currently used by the Central Bank of Brazil is the one of Castro et al. (2011), called SAMBA (Stochastic Analytical Model with Bayesian Approach). This model combines the building blocks of the standard DSGE models (price and wage rigidities and adjustment costs) with the following features that better describe the Brazilian economy: i) fiscal authority pursuing an explicit target for the primary surplus; ii) administered or regulated prices as part of consumer prices; iii) external financing for imports; iv) imported goods used in the production function of differentiated goods. Moreover, exchange rate pass-through is incomplete in the short run.

We analyze the responses of key macroeconomic variables to an import tariff shock, considering the parameters for the Brazilian economy. We conclude that the import tariff
shock entails deviations from the law of one price since import prices become higher. There is an increase on domestic prices as well as on the total inflation, which leads to a raise on the interest rates. The long run effect is a worsening on the terms of trade, since the exchange rate evaluation which follows the shock cancels the initial effect of the increase of import tariffs. Finally, the output gap reveals a contraction of the economic activity.

The rest of the paper is organized as follows. Section 2 presents the model. The log linearized equations of the model are presented in Section 3. Section 4 describes the equilibrium conditions. Section 5 provides the details and the results. Finally, Section 6 is dedicated to the concluding remarks.

2. THE MODEL

We model the world economy as a continuum of small open economies represented by the unit interval. Since each economy is of measure zero, its domestic policy decisions do not have any impact on the rest of the world. We assume that they share identical preferences, technology and market structure, as presented by Obstfeld and Rogoff (1996), Galí and Monacelli (2005) and Monacelli (2005). Regarding the hypotheses related to monetary policy, these were based on Taylor (1993).

The small open economy is inhabited by a representative household who seeks to maximize his utility, which in turn depends upon consumption and leisure. Money does not appear in either the budget constraint or the utility function since monetary policy will be specified in terms of an interest rate rule. Households consume domestic and foreign goods that are imperfect substitutes. The domestic good is a composite of a continuum of differentiated goods.

In the rest of the world, a representative household faces a problem identical to the one outlined above. Hence a set of analogous optimality conditions characterize the solution to the consumer’s problem in the world economy. As in Galí and Monacelli (2005), however, the size of the small open economy is negligible relative to the rest of the world, an assumption that allows the treatment of the latter as if it was a closed economy.

In this economy there is a continuum of monopolistic competitive firms (owned by consumers). Each firm faces a demand curve with constant relative price elasticity. Production is linear in labor, the only input, and takes into consideration productivity shocks. Stock variations are not considered in order to simplify the derivation of the model. Firms set nominal prices on a staggered basis, following Calvo (1983). Firms changing price in the
current period choose an optimal price based on the expected path of marginal cost. Firms not changing price simply adjust output to meet demand. For simplicity we assume that export price of the domestic good is flexible and determined by the law of one price.

In addition to domestic producers we assume that the domestic market is populated by local retailers who import differentiated goods and sell them to households. Import price inflation rises as the world price of imports exceeds the local currency price of the same good. In other words, a nominal depreciation determines a wedge between the price paid by the importers in the world market and the local currency price applied in the domestic market. This wedge acts as an increase in their real marginal cost and therefore boosts foreign goods inflation. This generates deviations from the law of one price in the short run. The price paid by importers is affected by the exchange rate and the import tariff which are considered of incomplete pass-through.

The only destination of expenditures is transfer to households. Consequently, the government runs a balanced budget each period as in Obstfeld e Rogoff (1996). This assumption eliminates the possibility of inflationary pressure as a result of government budget deficits since monetary policy is not able to promote price stability in an environment characterized by fiscal imbalances. Besides, in this economy there are no costs of financial intermediation and therefore the interest rate is identical for buying and selling bonds. The central bank sets the nominal interest rate for monetary policy purposes, aiming at stabilizing inflation and the output gap.

2.1 HOUSEHOLDS

All individuals throughout the world have identical preferences over a consumption index. The small open economy is inhabited by a representative household who maximizes the present value of utility. Preferences are determined by consumption \( C_t \) and leisure \( 1 - N_t \). The intertemporal utility function of a typical home agent is given by:

\[
U_0 = E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1 - \sigma} - \frac{N_{t+i}}{1 + \gamma_n} \right],
\]

where \( \beta \in (0,1) \) denotes a subjective discount factor, \( \sigma \) and \( \gamma_n > 0 \) represent the inverse of the intertemporal elasticity of substitution for consumption and labor, respectively. The first term refers to aggregate consumption and the second captures the disutility of labor. Notice that money does not appear in either the budget constraint or the utility function because we
specify monetary policy in terms of an interest rate rule; hence, we do not need to introduce money explicitly in the model (GALÍ; MONACELLI, 2005). That modelling strategy has been adopted in much recent research on monetary policy. In it money can be thought of as playing the role of a unit of account only.

Household consumption is a constant-elasticity-of-substitution (CES) composite of home and foreign goods (OBSTFELD; ROGOFF, 1995):

\[
C_t = \left[ (1 - \alpha) \frac{1}{\eta} \left( C_{H,t} \right)^{\frac{\eta - 1}{\eta}} + \alpha \frac{1}{\eta} \left( C_{F,t} \right)^{\frac{\eta - 1}{\eta}} \right]^{\frac{1}{\eta - 1}},
\]

where \( C_{H,t} \) represents an index of consumption of domestic goods produced in home country, \( C_{F,t} \) is an index of consumption of imported goods, \( \alpha \in (0,1) \) is the degree of home bias in preferences, and is thus a natural index of openness. Parameter \( \eta > 1 \) measures the substitutability between domestic and foreign goods, from the point of view of the domestic consumer.

The indexes above are in turn given by CES aggregators of the quantities consumed of each type of good (DIXIT; STIGLITZ, 1977):

\[
C_{H,t} = \left[ \int_0^1 C_{H,t}(j)^{\frac{\varepsilon - 1}{\varepsilon}} \, dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}; \quad C_{F,t} = \left[ \int_0^1 C_{F,t}^{\frac{\gamma - 1}{\gamma}} \, dj \right]^{\frac{\gamma}{\gamma - 1}}; \quad C_{i,t} = \left[ \int_0^1 C_{i,t}(j)^{\frac{\gamma - 1}{\gamma}} \, dj \right]^{\frac{\gamma}{\gamma - 1}},
\]

where \( j \in \{0,1\} \) denotes the varieties of good, \( C_{H,t} \) is the consumption index for domestic goods, \( C_{F,t} \) is an index of imported goods and \( C_{i,t} \) is an index of goods imported from country \( i \) and consumed by domestic households. Parameter \( \varepsilon > 1 \) denotes the elasticity of substitution between varieties (produced within any given country) while \( \gamma > 1 \) measures the substitutability between goods produced in different foreign countries.

Prices are defined as follows: \( P_{H,t} \) is the domestic price index (i.e, the prices of domestically produced goods), \( P_{i,t} \) is a price index for goods imported from country \( i \) (expressed in domestic currency) and \( P_{F,t} \) is the price index for imported consumption goods, also expressed in domestic currency, for all \( i \in \{0,1\} \).

\[
P_{H,t} = \left( \int_0^1 P_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}; \quad P_{i,t} = \left( \int_0^1 P_{i,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}; \quad P_{F,t} = \left( \int_0^1 P_{F,t}^{1-\gamma} \, di \right)^{\frac{1}{1-\gamma}}
\]

The problem of households is solved in two stages. First, given a level of consumption \( C_t \), the domestic representative individual chooses a combination of domestic and imported goods which minimizes his costs:
Min_{C_H,t, C_F,t} P_{H,t} C_{H,t} + P_{F,t} C_{F,t}

subject to

\[ C_t = \left(1 - \alpha \right)^{\frac{1}{\eta}} \left( C_{H,t} \right)^{\frac{\eta-1}{\eta}} + \alpha \eta \left( C_{F,t} \right)^{\frac{\eta-1}{\eta}} \]

The first order conditions of the Lagrangian function (L) imply:

\[ \frac{C_{H,t}}{C_{F,t}} = \frac{1 - \alpha}{\alpha} \left( \frac{P_{H,t}}{P_{F,t}} \right)^{-\eta}, \]  

(5)

\[ C_{H,t} = 1 - \alpha \left( \frac{P_{H,t}}{\lambda_t} \right)^{-\eta} C_t, \]  

(6)

\[ C_{F,t} = \alpha \left( \frac{P_{F,t}}{\lambda_t} \right)^{-\eta} C_t, \]  

(7)

\[ \lambda_t = \left[ \left( 1 - \alpha \right) \left( P_{H,t} \right)^{1-\eta} + \alpha \left( P_{F,t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} = P_t \]  

(8)

The equations above represent, in the order in which they appear, the relative demand, the demand for domestic goods, the demand for imported goods and the consumer price index. The optimal allocation of expenditures across types of goods implies standard demand functions:

\[ C_{H,t}(j) = \left( \frac{P_{H,t}}{P_{F,t}} \right)^{-\eta} C_{H,t}; \quad C_{i,t}(j) = \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\eta} C_{i,t}; \quad C_{i,t} = \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \]  

(9)

In order to complete the specification of the individual’s problem, we introduce the agent’s budget constrain (OBSTFELD; ROGOFF, 1996, p. 663):

\[ C_t + \frac{1}{1 + r_t} \left( B_t + \frac{B_{t-1}}{P_t} N_t + \frac{B_{t-1}}{P_{t-1}} + \Pi_t + TR_t \right) \]  

(10)

In the equation above, $B_t$ refers to a riskless nominal bond, $W_t$ is the nominal wage paid for hours worked, $N_t$, $1+r_t$ is the real gross interest rate, $\Pi_t$ refers to the profits received from firms (since consumers are the owners of firms) and $TR_t$ denotes lump sum taxes (or governmental transfers if $TR_t > 0$). All the previous variables are expressed in terms of domestic currency. It is worth to mention that the Fischer parity equation $1 + i_{t+1} = $
implies that in the equilibrium, the gross real rates of return on real and nominal bond must be the same (OBSTFELD; ROGOFF, 1996).

The consumer problem becomes to maximize (1) subject to (10). The solution can be obtained by the dynamic optimization technique of Bellman (2003). The first order conditions can be written as:

\[
\frac{\partial u_0}{\partial B_t} \Rightarrow C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} (1 + r_t), \text{ which replaced by the Fischer parity equation becomes:}
\]

\[
\frac{\partial u_0}{\partial B_t} \Rightarrow C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} (1 + i_t) \left( \frac{P_t}{P_{t+1}} \right) \tag{11}
\]

\[
\frac{\partial u_0}{\partial N_t} \Rightarrow \frac{N_t^{\gamma_n}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{12}
\]

The standard first-order consumption Euler equation (11) defines the optimal path of consumption of individuals; intertemporal consumption smoothing occurs through the real interest rate. The labor-leisure trade off is given by equation (12), a standard intratemporal condition. The marginal rate of substitution between them equals the real wage.

In this economy, individuals cannot get to the end of their lives either with debt \((B_{t+T+1} < 0)\) or unused resources \((B_{t+T+1} > 0)\). Therefore, equilibrium also requires the transversality condition, derived by iterating the period budget constraint:

\[
\lim_{T \to \infty} \left( \frac{1}{1 + r_{t+T}} \right)^T B_{t+T} P_{t+T} = 0
\]

In the rest of the world, a representative household faces a problem identical to the one outlined above. Hence a set of analogous optimality conditions characterize the solution to the consumer’s problem in the world economy. The size of the small open economy is negligible relative to the rest of the world, an assumption that allows the treatment of the latter as if it were a closed economy (GALÍ; MONACELLI, 2005).

2.2 FIRMS

In the domestic goods market, there are two types of firms:

a) Domestic producers which use labor supplied by households and produce homogeneous final consumer goods with "j" inputs imperfect substitutes;

b) Importing firms which transform a commodity bought in the foreign market and turn it into a differentiated good that is sold to residents.
2.2.1 Domestic Producers

The domestic production process is divided into two types of firms. In each period \( t \) a group of firms acquires labor \( (N_t) \) from individuals, who receive a wage \( (W_t) \) and produce an intermediate good \( (y_t(j)) \). Another group of firms uses a continuum of these intermediate goods to produce homogeneous final consumer goods to domestic households (which will form the basket of domestic consumption \( C_{H,t} \)).

The production function of the domestic firm producing the final good assumes the following form:

\[
Y_t = \left( \int_0^1 Y_t(j) \frac{\varepsilon - 1}{\varepsilon} \, dj \right)^{\frac{\varepsilon}{\varepsilon - 1}},
\]

where the parameter \( \varepsilon > 1 \) refers to the elasticity of substitution of different intermediate goods\(^2\). The objective of this firm is to choose \( y_t(j) \) such that its profit is maximized, considering the aggregate price of the basket \( (P_{H,t}) \) and its inputs \( (P_{H,t}(j)) \). Therefore the function that maximizes this firm's profit will be:

\[
Max_{y_t(j)} \pi_t = P_{H,t} Y_t - P_{H,t}(j) Y_t(j)
\]

By substituting (13) above and solving the maximization problem, we obtain the following demand curve for intermediate goods:

\[
Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t
\]

A typical firm in the home economy produces a differentiated intermediate good with a linear technology represented by the production function below, where \( A_t \) represents home productivity:

\[
y_t(j) = A_t N_t(j),
\]

where \( A_t = A_0^{\rho_a} e^{\varepsilon t} \) is a productivity shock; \( j \in [0,1] \) represents a firm specific index; \( 0 \leq \rho_a \leq 1 \) is a persistent parameter and \( e_t \) is an i.i.d shock. Therefore the problem of this firm becomes:

\[\]
The term \( \frac{W_t}{P_{H,t}} \) refers to the real cost of labor \((N_t(j))\) and \( \phi_t \) to the real marginal cost common to all firms (the Lagrange multiplier). The first order conditions of this minimization problem lead to the equation below:

\[
\phi_t = \frac{W_t}{P_{H,t}} \cdot \frac{1}{A_t} \tag{18}
\]

Now we need to define the price of intermediate goods produced by domestic firms under monopolistic competition. We assume that firms set prices in a staggered fashion, as in Calvo (1983). Hence, a measure \((1-\theta)\) of randomly selected firms sets new prices \((P_{H,t}(j))\) each period, with an individual firm’s probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. Thus, each period a measure \((1-\theta)\) of (randomly selected) firms reset their prices, while a fraction \(\theta\) keep their prices unchanged. Despite producing differentiated goods, firms have the same production technology and the same demand curve. Thus, the only difference between them is the fact that they do not adjust their prices simultaneously in all time periods. All firms that adjust their prices in \(t\) will choose the same price \(\bar{P}_{H,t}\) (we can ignore the subscript "j", since the price is the same). All firms that adjust their prices in \(t\) are maximizing the expected discounted value of current and future profits. In a future period \(t + k\), their profits will be affected by the choice made at \(t\), provided that in the meantime there has not been any price increase. Therefore, according to Calvo (1983), \(P_{H,t+k}(j) = \bar{P}_{H,t}(j) = \bar{P}_{H,t}\), with probability \(\theta^K\), for \(K=0, 1, 2, 3, \ldots\).

The maximization problem of the domestic firm which produces intermediary goods can be written as follows:

\[
Max_{p_{H,t}} \sum_{k=0}^{\infty} \theta^K E_t \left\{ \Delta_{t,t+k} Y_{t+k}(j) \left[ \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right) - MC_{t+k} \right] \right\}
\]

subject to

\[Y_{t+k}(j) = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k},\]
where \( \Lambda_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right) \) is stochastic discount factor associated to the consumption decision of the households since they are the owners of the firms.\(^3\) The term \( MC_{t+k} \) refers to real marginal cost. The expression above shows that when setting a new price \( \bar{P}_{H,t} \) in period \( t \) firms seek to maximize the current value of its dividend stream, conditional on that price being effective.

The first order condition results in the equation below.

\[
\sum_{k=0}^{\infty} \theta^K H E_t \left\{ \Lambda_{t,t+k} Y_{t+k} \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right) + \left( \frac{\varepsilon}{1-\varepsilon} \right) MC_{t+k} \right\} = 0
\]  \( (19) \)

Isolating \( \bar{P}_{H,t} \) we obtain the optimal price equation:

\[
\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^K H E_t \left\{ \Lambda_{t,t+k} \left( \frac{1}{P_{H,t+k}} \right)^{1-\varepsilon} Y_{t+k} MC_{t+k} \right\}}{\sum_{k=0}^{\infty} \theta^K H E_t \left\{ \Lambda_{t,t+k} \left( \frac{1}{P_{H,t+k}} \right)^{1-\varepsilon} Y_{t+k} \right\}}
\]  \( (20) \)

The expression (18) can be rewritten as:

\[
\bar{P}_{H,t} = \mu E_t \sum_{k=0}^{\infty} \omega_{t,t+k} MC_{t+k} \bar{P}_{H,t+k}
\]  \( (21) \)

The *markup* \( \mu = \frac{\varepsilon}{\varepsilon - 1} > 1 \) indicates a price adjustment factor above marginal cost, since the firm has market power. Therefore, the optimal price \( \bar{P}_{H,t} \) is a *markup* over a weighted average of expected future marginal costs.

### 2.2.2 Importing Firms

Campa and Goldberg (2002) estimate import pass-through elasticities for a range of countries of the Organisation for Economic Co-operation and Development (OECD). They find that: (i) the degree of pass-through is partial in the short-run and becomes gradually complete only in the long-run; (ii) the sensitivity of prices to exchange rate movements is much larger at the *wholesale* import stage than at the *consumer* stage. According to the authors, one explanation for the degree of pass-through is the composition of trade in each country.

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\(^3\) This discount factor came from equation (11) of the consumer problem, where \((1 + \iota_t)\) was replaced by the Fisher equation.
As proposed by Monacelli (2005), we assume that there is a local retailer who import differentiated goods at a cost \( \epsilon_t \) \( P_{F,t}^* (j) \), where \( \epsilon_t \) refers to the nominal exchange rate, \( P_{F,t}^* (j) \) is the foreign currency price of the imported good. Our contribution to this setting is the introduction of the import tariff \((1 + \tau_t(j))\). Like local producers, importing firms set prices \( \bar{P}_{F,t} (j) \) in a staggered fashion, as in Calvo (1983). The parameter \( \theta_F \) governs the degree of pass-through, generating deviations from the law of one price in the short run. Thus, the problem of the importing firm becomes:

\[
\begin{align*}
\max_{\bar{P}_{F,t} (j)} & \ E_t \left\{ \sum_{k=0}^{\infty} \theta_F^k (t) A_{t+k} \right\} \left[ \bar{P}_{F,t} (j) - \epsilon_{t+k} P_{F,t+k}^* (j)(1 + \tau_t(j)) \right] \\
\text{subject to} & \\
C_{F,t+k} (j) & = \left( \frac{\bar{P}_{F,t} (j)}{P_{F,t+k}} \right)^{-\epsilon} C_{F,t+k} \\
\end{align*}
\]

The first order conditions of this problem yields:

\[
\bar{P}_{F,t} (j) = \frac{\epsilon \frac{E_t \left\{ \sum_{k=0}^{\infty} \theta_F^k (t) A_{t+k} \right\} \epsilon_{t+k} P_{F,t+k}^* (j) (1 + \tau_t(j))}{E_t \left\{ \sum_{k=0}^{\infty} \theta_F^k (t) A_{t+k} \right\}}}{E_t \{ \sum_{k=0}^{\infty} \theta_F^k (t) A_{t+k} \}} 
\]

(22)

Import price inflation rises as the world price of imports exceeds the local currency price of the same good. In other words, a nominal depreciation or an increase of the import tariff determines a wedge between the price paid by importers in the world market and the local currency price applied in the domestic market. This wedge acts as an increase in their real marginal cost and therefore boosts foreign goods inflation.

Goods market equilibrium is given by:

\[
Y_t (j) = C_{H,t} (j) + \int_0^1 C_{H,t}^i (j) di, 
\]

(23)

where \( C_{H,t}^i (j) \) represents the demand of country \( i \) for the good \( j \) produced in the domestic economy\(^4\), assuming symmetric preferences between countries.

\[\text{footnote}^4 \ C_{H,t}^i (j) = \alpha \left( \frac{p_{H,t} (j)}{p_{H,t}} \right)^{-\varepsilon} \left( \frac{p_{H,t} (j)}{\epsilon_{i,t} p_{F,t}} \right)^{-\gamma} \left( \frac{p_{H,t}^i (j)}{p_{i,t}^i} \right)^{-\eta} C_i^i \]
3. LOG-LINEARIZATION

The analysis of DSGE models may require solving nonlinear equations. The general procedure to solve these models is to log-linearize the necessary equations characterizing the equilibrium of the system to make equations approximately linear in the log deviations from the steady state (UHLIG, 1997). Woodford (1986) demonstrated that, under certain conditions, economy floats around steady-state. Thus, for sufficiently small deviations from this steady state, the log-linearized equations are sufficiently close to the equilibrium.

3.1 HOUSEHOLDS

The solution of the consumer problem is given in two steps. First, given a \( C_t \) level, consumers decide to buy a combination of domestic and imported goods to minimize their costs. Then the relative demand, the demand for domestic and imported goods and the aggregate level of consumer prices were found. In the second stage we obtain the solution to the dynamic problem of maximizing utility (1) subject to the budget constraint (10), which was derived from the Euler equation and the optimal intratemporal condition between leisure and consumption.

The log-linearized equations of each stage are found below. Equations (2), (5), (6), (7) e (8) become:

\[
\begin{align*}
\hat{c}_t &= (1 - \alpha) \hat{H}_t C_{\text{H},t} + \alpha \hat{H}_t C_{\text{F},t} \\ 
C_{\text{H},t} - C_{\text{F},t} &= -\eta(p_{\text{H},t} - p_{\text{F},t}) \\ 
C_{\text{H},t} &= c_t - \eta(p_{\text{H},t} - p_t) \\ 
C_{\text{F},t} &= c_t - \eta(p_{\text{F},t} - p_t) \\ 
p_t &= (1 - \alpha)p_{\text{H},t} + \alpha p_{\text{F},t}
\end{align*}
\]

Equations (11) and (12) evolve according to:

\[
\begin{align*}
\hat{c}_t &= E_t[c_{t+1}] - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1})] \\
\text{where } \pi_{t+1} &= p_{t+1} - p_t \\
\gamma \hat{\pi}_t + \sigma \hat{c}_t &= w_t - p_t
\end{align*}
\]
Equation (29) shows that intertemporal consumption smoothing occurs through the interest rate. The labor-leisure trade off is given by equation (30).

3.2 FIRMS

The log-linearized equations of domestic producers are:

\[ y_t(j) - y_t = -\varepsilon [p_{H,t}(j) - p_{H,t}] \]  \hspace{1cm} (31)

\[ y_t(j) = a_t + n_t(j) \]  \hspace{1cm} (32)

From the cost minimization problem we obtain the marginal cost \( \hat{\phi}_t \) which is common across producers:

\[ \hat{\phi}_t = w_t - p_{H,t} - a_t \]  \hspace{1cm} (33)

From the maximization problem of the domestic firm which produces intermediate goods under a Calvo (1983) price-setting structure, we obtain the log-linearized version of the price setting rule of domestic producers in terms of expected nominal marginal costs, weighted by the probability of occurrence of this price in later periods:

\[ \tilde{p}_{H,t} = (1 - \theta_h \beta) E_t \left\{ \sum_{k=0}^{\infty} (\theta_h \beta)^k (m_{t+k} + p_{H,t+k}) \right\} \]  \hspace{1cm} (34)

Like domestic producers, a local retail importing goods when setting the domestic currency price of these goods (\( \tilde{P}_{F,t}(j) \)) solve an optimal (dynamic) markup problem. The parameter \( \theta_F \) governs the degree of pass-through, generating deviations from the law of one price in the short run. The log-linearization of (22) follows the one of equation (20). The marginal cost is replaced by \( \varepsilon_{t+K} P_{F,t+K}^* (j)(1 + \tau_t(j)) \). For log-linearization purposes, we have \( \varepsilon_{t+K} P_{F,t+K}^* (j)(1 + \tau_t(j)) = \psi_{F,t+K} P_{F,t+K} \) which represents the domestic currency cost of the imported good. Therefore the log-linearized form of equation (22) is:

\[ \tilde{p}_{F,t} = (1 - \theta_F \beta) E_t \left\{ \sum_{k=0}^{\infty} (\theta_F \beta)^k (\tilde{\psi}_{F,t+k} + p_{F,t+k}) \right\} , \]  \hspace{1cm} (35)

where \( \tilde{\psi}_{F,t+K} \) is the marginal cost of the imported good and depends on the exchange rate \( (e_t) \), on the national \((p_{F,t})\) and international prices \((p_t^*)\) and on the import tariffs \((\bar{\tau}_t)\). It can be written as:

\[ \tilde{\psi}_{F,t} = (e_t + p_t^* - p_{F,t}) + \theta_t \bar{\tau}_t , \]  \hspace{1cm} (36)
where $\theta_\tau = \frac{\tau}{1+\tau}$.

Equation (36) denotes deviations from international prices relative to current domestic import prices, thus measuring deviations from the law of one price.

3.3 PHILLIPS CURVE

From equation (34) we can find the dynamics of domestic inflation in terms of the real marginal cost. This equation can be written as:

$$\bar{p}_{H,t} = (1 - \theta_H \beta)(mc_t + p_{H,t}) + \theta_H \beta E_t \bar{p}_{H,t+1}$$

(37)

And, in terms of inflation, we have:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{(1 - \theta_H)(1 - \theta_H \beta)}{\theta_H} mc_t$$

(38)

where $\lambda_H = \frac{(1 - \theta_H)(1 - \theta_H \beta)}{\theta_H}$.

Equation (38) is the New Keynesian Phillips Curve that, in its primitive form, relates domestic inflation to real marginal cost, $mc_t$, and expected future inflation. Since firms follow a Calvo (1983) price setting, the current period price is chosen based on the expected path of marginal cost, which in turn depends on real domestic wages.

From (35) similar equations can be obtained for importing firms:

$$\bar{p}_{F,t} = (1 - \theta_F \beta)(\bar{p}_t + p_{F,t}) + \theta_F \beta E_t \bar{p}_{F,t+1}$$

(39)

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \frac{(1 - \theta_F)(1 - \theta_F \beta)}{\theta_F} \bar{\psi}_{F,t}$$

(40)

where $\lambda_F = \frac{(1 - \theta_F)(1 - \theta_F \beta)}{\theta_F} e \bar{\psi}_{F,t}$ is the log-linearized version of $\psi_{F,t+K} = e_{t+K} \frac{p_{F,t+K}}{p_{F,t+K}}(1 + \tau_{c}(t))$

Therefore import price inflation rises as the world price of imports exceeds the local currency price of the same good. In other words, a nominal depreciation of the exchange rate or a nominal raise of the import tariff determine a wedge between the price paid by the importers in the world market and the local currency price applied in the domestic market. This wedge acts as an increase in their real marginal cost and therefore boosts foreign goods inflation. The parameter $\theta_F$ responds for price stickiness.
3.4 OTHER IDENTITIES

Before proceeding with our analysis of equilibrium, we introduce several assumptions and definitions as well as a number of identities that are extensively used in the small open economy literature.

First we define the bilateral terms of trade between the domestic economy and country \( i \) as \( S_{i,t} = \frac{P_{j,t}}{P_{H,t}} \), i.e. the price of country \( i \)'s goods in terms of home goods. The effective terms of trade are thus given by:

\[
S_t = \frac{P_{F,t}}{P_{H,t}} = \int_0^1 S_{i,t}^{1-\gamma} di, \tag{41}
\]

which, in a log-linear form, becomes:

\[
s_t = P_{F,t} - P_{H,t} \tag{42}
\]

The above equation holds independently of the degree of pass-through. Log-linearization of the consumer price index (8) around a symmetric steady state, with \( P_{H,t} = P_{F,t} \), yields:

\[
p_t = P_{H,t} + \alpha s_t \tag{43}
\]

It follows that domestic inflation and consumer price inflation are linked according to:

\[
\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{44}
\]

Equation (43) defines the gap between the two measures of inflation proportional to the percentage change in the terms of trade, with the coefficient of proportionality given by the degree of openness, \( \alpha \), defined as a ratio between imports and Gross Domestic Product.

The treatment of the rest of the world as an closed economy, with goods produced in the small economy representing a negligible fraction of the world consumption basket, implies that \( p^*_t = p^*_{F,t} \), \( \pi^*_t = \pi^*_{F,t} \), for all \( t \). Hence, equivalence between domestic and consumer price inflation holds in the world economy (MONACELLI, 2005).

Let’s define \( e \) as the nominal exchange rate. Assuming that the law of one price holds:

\[
P_{i,t} (j) = \epsilon_{i,t} p_{i,t}^j (j), for all i, j \in [0,1] \tag{45}
\]

where \( \epsilon_{i,t} \) refers to the bilateral nominal exchange rate, \( p_{i,t}^j (j) \) is the price of good \( j \) from country \( i \) expressed in the domestic currency of the producer country. According to (44), the law of one price can be written as \( P_{F,t} = \epsilon_t P^*_t \), which log-linear form is \( p_{F,t} = e_t + p^*_t \).

Combining this result with the definition of terms of trade, yields:
The real exchange rate in a log-linear form becomes:

\[ s_t = e_t + p_t^* + p_{H,t} \]  \hspace{1cm} (46)

The real exchange rate \( q_{t,t} = \frac{e_{t,t}p_t^i}{p_t} \) in a log-linear form becomes:

\[ q_t = e_t + p_t^* - p_t \]  \hspace{1cm} (47)

Combining equations (42) and (43), one get:

\[ q_t = (1 - \alpha)s_t \]  \hspace{1cm} (48)

However under incomplete pass-through, the law of one price does not hold (MONACELLI, 2005). In this case, we have:

\[ q_t = \tilde{p}_{F,t} + (1 - \alpha)s_t \]  \hspace{1cm} (49)

Clearly, two are the sources of deviation from aggregate Purchasing Power Parity (PPP) in this framework. The first lies in the heterogeneity of consumption baskets between the small economy and the rest of the world, an effect captured by the term \((1 - \alpha)s_t\), as long as \(\alpha < 1\). For \(\alpha \to 1\), in fact, the two aggregate consumption baskets coincide and relative price variations are not required in equilibrium. The second source of deviation from PPP is due to the deviation from the law of one price, captured by movements in \(\tilde{p}_{F,t}\). Hence, with incomplete pass-through, the law of one price gap contributes to the volatility of the real exchange rate.

The assumption of complete markets at the international level leads to a simple relationship linking domestic consumption with world consumption and the terms of trade (GALI; MONACELLI, 2005). Observe that equation (11) can be written as:

\[ q_{t,t+1} = \beta \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \]  \hspace{1cm} (50)

An analogous first order condition must also hold for the representative household in any other country, say country \(i\):

\[ q_{t,t+1} = \beta \left( \frac{P_t^i}{P^i_{t+1}} \right) \left( \frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \left( \frac{e_t^i}{e^i_{t+1}} \right) \]  \hspace{1cm} (51)

Combining (50) and (51):

\[ C_t = v_t C_t^i Q_{t,t}^{\frac{1}{2}} \]  \hspace{1cm} (52)

and where \(v_t\) is a constant that will generally depend on initial conditions regarding relative net asset positions and \(Q\) is the real exchange rate. Henceforth, and without loss of generality, we assume symmetric initial conditions, in which case \(v_t = v = 1\) for all \(i\). In the symmetric
perfect foresight steady state, we also have that $C = C^i = C^*$. Considering the incomplete exchange rate pass-through of equation (49), we have the following log-linear form of (52):

$$ c_t = c_t^* + \frac{1}{\sigma} [\bar{\psi}_{F,t} + (1 - \alpha)s_t] $$

(53)

Hence, deviations from the law of one price, by affecting movements of the real exchange rate, also affect the relative consumption baskets. Under complete international asset markets, it is also possible to derive a standard log-linear version of the uncovered interest parity condition:

$$ i_t - i_t^* = E_t \{\Delta e_{t+1}\} + w_t $$

(54)

where $w_t$ is an exogenous risk premium shock, such that $w_t = \rho w_{t-1} + \xi_{wt}$, $\rho^w \in (0,1)$ and $\xi_{wt} \sim i.i.d \ (0, \sigma^w_{\xi})$.

By combining (29), (33) and (53), imposing market clearing ($c_t^* = y_t^*$) in the world economy, where $y_t^*$ is the world output, and substituting the production function, one obtains, after aggregation, an equilibrium equation for the domestic real marginal cost, which also expresses the equilibrium in the labor market:

$$ \tilde{\phi}_t = (w_t - p_t) + \alpha s_t - a_t = y_t n_t - (1 + n) a_t + \sigma y_t^* + s_t + \bar{\psi}_{F,t} $$

(55)

Notice the open economy elements that affect the real marginal cost: world output ($y_t^*$) and a “relative price effect” captured by $s_t$ and $\bar{\psi}_{F,t}$. Regarding internal factors, an increase in $y_t$ obliges firms to hire more labor to increase production, causing an increase in marginal cost. In its turn, $a_t$ reduces the marginal cost because it makes the worker more productive.

4. EQUILIBRIUM

4.1 GOODS MARKET

Goods market equilibrium implies equality between production and consumption, as stated in equation (23). Considering also equation (3) and writing it in a log-linear form:

$$ y_t = (1 - \alpha) c_{h,t} + \alpha c_{H,t}^* $$

(56)

Considering equations (25), (26), (42) and (46), we have $c_{H,t} = c_t + \eta s_{t} + \bar{\psi}_{F,t}$ and $c_{F,t} = c_t - \eta (1 - \alpha) s_t$. According to these equations, export demand ($c_{H,t}^*$) rises both when the terms of trade depreciate (i.e., the price $p_{H,t}$ falls relative to $p_{F,t}$)
and when the domestic currency price of foreign goods \( p_{F,t} \) falls relative to the world price (i.e., \( \Phi^*_F = \Phi_F \) rises). Considering \( c^*_t = y_t^* \) and substituting this into (56):

\[
y_t^* = (1 - \alpha)c_t + (2 - \alpha)\eta as_t + \alpha y_t^* + \eta \Phi_F \tag{57}
\]

By substituting (57) into (53) we have:

\[
y_t - y_t^* = \frac{1}{\sigma} [w_s s_t + w_{\Phi} \Phi_F] \tag{58}
\]

where \( w_s = 1 + \alpha(2 - \alpha)(\sigma \eta - 1) > 0 \), \( w_{\Phi} = 1 + \alpha(\sigma \eta - 1) > 0 \) \((w_s > w_{\Phi})\) since \( \alpha \in (0, 1) \) are, respectively, the elasticities of relative output to the terms of trade and the law of one price gap.

4.2 AGGREGATE DEMAND

The IS curve represents the demand side of this economy, which is usually written in terms of the output gap. We start by assuming that domestic producer prices are flexible. This is useful to formally derive two results. First, the nominal exchange rate volatility is linked to the degree of pass-through. Second, for a sufficiently low degree of pass-through, the law of one price gap must respond positively to a productivity shock (MONACELLI, 2005).

By imposing a constant markup \((mc_t = \Phi_t = 0)\) and substituting equation (58) into (55) we obtain an expression for the domestic flexible price level of output, where a hat sign over the variable denotes a flexible price environment:

\[
\hat{y}_t = \left( \frac{w_s (1 + \gamma_n)}{\sigma + \gamma_n w_s} \right) a_t + \left( \frac{\sigma (1 - w_s)}{\sigma + \gamma_n w_s} \right) y_t^* - \left( \frac{w_s - w_{\Phi}}{\sigma + \gamma_n w_s} \right) \hat{\Phi}_F \tag{59}
\]

where \( \left( \frac{w_s (1 + \gamma_n)}{\sigma + \gamma_n w_s} \right) a_t + \left( \frac{\sigma (1 - w_s)}{\sigma + \gamma_n w_s} \right) y_t^* = \hat{y}_t^n \tag{60} \)

\( \hat{y}_t^n \) denotes the natural level of output, i.e., the one that we would obtain in the case of both flexible domestic prices and complete pass-through. By definition, the output gap, \( h_t \), is the percent deviation of observed output \( y_t \) from \( \hat{y}_t^n \), that is:

\[
h_t = y_t - \hat{y}_t^n \tag{61}
\]

The IS curve can be obtained by isolating \( s_t \) in equation (53) and substituting the result into equation (57):
\[ y_t = \frac{w_s}{1 - \alpha} c_t - \frac{\alpha \eta}{1 - \alpha} \bar{\psi}_{F,t} + \left(1 - \frac{w_s}{1 - \alpha}\right) y_t^* \]  
(62)

Since (62) is valid for all \( t \), we can isolate \( c_t \) and substitute it into (29):

\[ y_t = E_t y_{t+1} + \left[\left(\frac{\alpha \eta}{1 - \alpha}\right) E_t \bar{\psi}_{F,t+1} - \bar{\psi}_{F,t}\right] + \left[\left(\frac{w_s}{1 - \alpha} - 1\right) E_t y_{t+1} - y_t^*\right] - \frac{w_s}{\sigma} \left[i_t - E_t(\pi_{H,t+1})\right] \]  
(63)

Considering equations (43) and (58) and isolating \( s_t \) we obtain:

\[ y_t = E_t y_{t+1} + \left[\left(\frac{\alpha \eta}{\sigma}\right)(1 - \alpha)(\sigma \eta - 1) (E_t \bar{\psi}_{F,t+1} - \bar{\psi}_{F,t})\right] + [(w_s - 1)(E_t y_{t+1} - y_t^*)] - \frac{w_s}{\sigma} \left[i_t - E_t(\pi_{H,t+1})\right] \]  
(64)

Usually the IS curve is presented in terms of the output gap instead of the level of output. Applying the definition (61), we can write:

\[ h_t = E_t h_{t+1} - \frac{w_s}{\sigma} \left[i_t - E_t(\pi_{H,t+1}) - \tilde{i}_t^\pi\right] + \Gamma \psi E_t \Delta \bar{\psi}_{F,t+1} \]  
(65)

where \( \Gamma \psi = \left(\frac{\alpha (1-\alpha)(\sigma \eta - 1)}{\sigma}\right) \) and \( \tilde{i}_t^\pi = \sigma \left(\frac{\gamma_n (w_s - 1)}{\sigma + \gamma_n w_s}\right) E_t (\Delta y_{t+1}^*) - \left(\frac{\alpha (1-\alpha)(1+\gamma_n)}{\sigma + \gamma_n w_s}\right) a_t. \)

Notice that the natural interest rate \( \tilde{i}_t^\pi \) depends not only on domestic productivity, but also on the expected growth in world output. Equation (65) shows that, to the extent that \( \sigma \eta > 1 \), expected changes in the output gap are negatively related to expected future changes in the law of one price gap.

### 4.3 Aggregate Supply

By considering the output gap as the percentage deviation of the current output from the natural level and combining equations (55), (58) and (59), the equilibrium real marginal cost can be expressed as:

\[ mc_t = \left(\gamma_n + \frac{\sigma}{w_s}\right) h_t + \left(1 - \frac{w_s}{w_s}\right) \bar{\psi}_{F,t} \]  
(66)

Therefore, the presence of incomplete pass-through breaks down the proportionality relationship between the real marginal cost and the output gap which typically characterizes the canonical sticky-price model with imperfect competition. By substituting equation (66) into (37) we obtain the forward looking Phillips curve for domestic goods:

\[ \pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_{1,H} h_t + \kappa_{1,\psi} \bar{\psi}_{F,t} + u_t \]  
(67)
where \( \kappa_{1,H} = \lambda_H \left( \gamma_n + \frac{\sigma}{w_s} \right) \), \( \kappa_{1,\psi} = \lambda_H \left( 1 - \frac{w_s}{w_s} \right) \) and \( u_t \) is a supply shock, written as \( u_t = \rho^u u_{t-1} + \xi_{ut} \), \( \rho^u \in (0,1) \) and \( \xi_{ut} \sim i.i.d (0, \sigma_u^2) \).

The elasticity of the output gap in relation to inflation is determined by structural parameters of the economy, such as subjective discount factor (\( \beta \)), intertemporal elasticity of substitution in consumption (\( \sigma \)) and labor elasticity (\( \gamma_n \)), degree of openness (\( \alpha \)), elasticity of substitution between domestic and imported (\( \eta \)) goods. The same can be observed about incomplete pass-through, in which the degree of openness of the economy (\( \alpha \)) and the elasticity of substitution between domestic and imported consumption goods (\( \eta \)) determine the influence of the external sector on domestic inflation. Furthermore, equation (67) shows the influence of the exchange rate and the import tariff, captured by \( \bar{\psi}_{F,t} \).

Considering that equation (28) is valid for all \( t \), the consumer inflation \( \pi_t \) can also be expressed in terms of a convex combination between domestic inflation \( \pi_{H,t} \) and inflation of imported goods \( \pi_{F,t} \):

\[
\pi_t = (1 - \alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{68}
\]

By substituting (39) and (67) into (68) we obtain:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa_{2,H} h_t + \kappa_{2,\psi} \bar{\psi}_{F,t} \tag{69}
\]

where \( \kappa_{2,H} = (1 - \alpha) \kappa_{1,H} \) and \( \kappa_{2,\psi} = (1 - \alpha) \kappa_{1,\psi} + \alpha \lambda_F \).

A rise in the law of one price gap, for a given output gap, causes a rise in consumer price inflation. Therefore, a full stabilization of inflation would require a fall in the output gap.

The two Phillips curves derived in this section characterize the supply side of this economy. They differ from the traditional Phillips curve because they incorporate deviation from the LOP in the optimal behavior of the economic agents.

### 4.4 MONETARY POLICY

The real marginal cost equilibrium in the rest of the world is given by:

\[
\bar{\phi}_{t}^* = w_t^* - p_t^* - a_t^* \tag{70}
\]

If \( w_t^* - p_t^* = \gamma_n n_t^* + \sigma c_t^* + c_t^* = y_t^* \) we have:

\[
\bar{\phi}_{t}^* = (\gamma_n + \sigma)y_t^* - (1 + \gamma_n)a_t^* \tag{71}
\]
In fact, equation (71) is a closed economy version of equation (55), which was obtained when \( \alpha = 0 \). The natural level of output for the world economy is obtained when \( \tilde{\phi}^* = 0 \), which in turn implies \( \pi^*_t = 0 \). Thus we have:

\[
y_t^* = \left( \frac{1 + \gamma_n}{\sigma + \gamma_n} \right) \alpha_t
\]

(72)

The output gap will be completely stabilized under fully flexible prices, i.e., \( h^*_t = y_t^* - \tilde{y}_t^\pi = 0 \). We assume, therefore, that the monetary authority of the rest of the world aims to replicate allocation under flexible prices, stabilizing simultaneously both inflation and output gap.

In the small open economy, we assume that the domestic monetary authority sets policy according to the following Taylor rule:

\[
i_t = \rho i_{t-1} + \left( 1 - \rho^i \right) \left[ \tau_1 \pi_t + \tau_2 h_t \right] + \xi_{zt}
\]

(73)

where \( \xi_{zt} \sim i.i.d \left( 0, \sigma_x^2 \right) \) is an exogenous monetary policy shock.

Alternative specifications of this rule are also used in the literature. Such variations include the absence of interest rate smoothing \( (\rho^i = 0) \) or a reaction of the monetary authority to past and future deviations of the variables of interest. We adopted the Taylor rule with interest rate smoothing because: a) the inertia of monetary policy increases with \( \rho^i \); b) the most parsimonious rules tend to be more robust; c) the differences in the model results are marginal when using \( \pi_t \) and \( h_t \) variables in \( t-1 \) or \( t+1 \). Therefore the nominal interest rate reacts to deviations from zero inflation steady state and from potential output. The autoregressive coefficient \( (\rho^i) \) aims to smooth the effects of \( \pi_t \) and \( h_t \) fluctuations over the monetary policy instrument.

4.5 LOG-LINEARIZED EQUILIBRIUM

The equilibrium of this economy is fully described by the endogenous equations (40), (54), (65), (67), (69), (73), with the respective process for the exogenous shocks, and the following complimentary equations:

\[
s_t = \frac{1}{\alpha} \left[ \pi_t - \pi_{H,t} \right] + s_{t-1}
\]

(74)

\[
\tilde{\psi}_{t} = \tilde{\psi}_{t-1} + e_t - e_{t-1} + \pi^*_t - \pi_{F,t} + \left( \tilde{r}_t - \tilde{r}_{t-1} \right)
\]

(75)
with the exogenous import tariff shock given by \( \tau_t = \rho^\tau \tau_{t-1} + \xi_{\tau t} \), \( \rho^\tau \in (0,1) \) and \( \xi_{\tau t} \sim i.i.d \ (0, \sigma_\tau^2) \).

Equations (67) and (69) refer to the forward-looking Phillips curves and represent the supply side of the economy. Notice that consumer inflation \( (\pi_t) \) is a function of consumer expectations \( (E_t \pi_{t+1}) \), of the level of economic activity given by the output gap \( (h_t) \) and of deviations from the law of one price \( \tilde{p}_{F-t} \).

Equation (65) represents the IS curve, i.e., the demand side of the economy. The output gap \( (h_t) \) is a positive function of its expectations \( (E_t h_{t+1}) \) and of the expected deviations from the law of one price \( (E_t \Delta \tilde{p}_{F,t+1}) \). The nominal interest rate affects negatively \( h_t \). Deviations from the natural interest rate cause economic activity variations in an opposite direction.

The main difference from previous models, such as Monacelli (2005), is the inclusion of equation (75) that presents the sources of deviations in the law of one price, which in turn arise not only from changes in the exchange rate, but also from variations on import tariffs.

The other equations are added considering the relative nominal price rigidity in the external sector. Equation (54) is the uncovered interest rate parity in nominal terms. Equation (74) shows that the gap between consumer inflation \( (\pi_t) \) and domestic inflation \( (\pi_{H,t}) \) is affected by variations in the economy terms of trade \( (\alpha \Delta s_t) \). Equation (40) is equivalent to the Phillips curve, but for imported goods. Finally, equation (73) represents the monetary policy rule followed by the monetary authority of the small open economy.

The dynamics of the model comes from exogenous shocks in equations (54), (67), (73) and (75). These exogenous disturbances correspond to risk premium shock, supply shock, monetary policy shock, and import tariff shock, respectively.

5. RESULTS

5.1 CALIBRATION

The calibration of the artificial model requires choosing values for the structural parameters based on other related studies and compatible with data from the real economy (KYDLAND; PRESCOTT, 1982). For the Brazilian economy, it is advised to use data for the period after July 1994, when the country reached a relative economic stability after the edition of the Real Plan. Table 1 summarizes the parameter values used in the simulation exercises.
The intertemporal discount factor, $\beta$, is calibrated by taking into account that $\left(\frac{1}{\beta} - 1\right)$ is equal to the value assumed by the average real interest rate observed in a sufficiently long interval of time (WOODFORD, 2003). Kanczuk (2002) works with $\beta = 0.98$. Cavalcanti and Vereda (2011) advocate an admissible range between 0.98 and 0.99, noting that this range generates an average real interest rate between 4.1% and 8.4% per year, values that seem compatible with the Brazilian economy in the period. We chose to fix the discount factor $\beta$ at 0.989, as suggested by Castro et al. (2011).

**TABLE 1: Calibrated parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibration</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.989</td>
<td>Time discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Coefficient of risk aversion</td>
</tr>
<tr>
<td>$\gamma_n$</td>
<td>0</td>
<td>Elasticity of marginal disutility of labor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>6</td>
<td>Elasticity of substitution between domestic and foreign goods</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>3</td>
<td>Elasticity of substitution between goods produced in the same country</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.10</td>
<td>Degree of openness</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>0.74</td>
<td>Probability of domestic firms not change prices</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>0.64</td>
<td>Probability of importing firms not change prices</td>
</tr>
<tr>
<td>$\rho^i$</td>
<td>0.79</td>
<td>Interest rate smoothing</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>1.5</td>
<td>Reaction to changes in the rate of inflation</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>0.5</td>
<td>Reaction to changes in the output gap</td>
</tr>
</tbody>
</table>
Regarding the intertemporal elasticity of consumption \(\left(\frac{1}{\sigma}\right)\), Woodford (2003) suggests the unit value in case of models from the real business cycle literature, or absolute values significantly higher than 1 in order to capture the magnitude of the empirically observed effects of an identified monetary policy shock on aggregate demand. In the Brazilian case, Kanczuk (2002), Carneiro and Duarte (2001), and Carvalho and Valli (2010) assume \(\left(\left|\frac{1}{\sigma}\right|\right) = 1\). Araújo et al. (2006) considers \(\left(\left|\frac{1}{\sigma}\right|\right) = 2.5\). Silveira (2008) carries out an econometric exercise in which the point estimation found is 2.09, with a 90% chance of the parameter being in the range [1.46, 2.86]. The results obtained in the Brazilian and international literature, according to Cavalcanti and Vereda (2011), suggest an admissible range between 1 and 3. We use \(\left(\left|\frac{1}{\sigma}\right|\right) = 1\), meaning that the coefficient of risk aversion is \(\sigma = 1\).

The elasticity of marginal disutility of labor \(\gamma_n\), as suggested by Cavalcanti and Vereda (2011), can take values between 0 and 3. Kanczuk (2002) and Ellery Junior, Gomes and Sachsida (2002) adopt a utility function with \(\gamma_n = 0\), implying that the utility varies linearly and negatively with respect to the amount of labor supplied by the representative individual. Carneiro and Duarte (2001) pick a parameter close to zero. Silveira (2008) finds \(\gamma_n = 0.77\), with a 90% chance of the parameter being in the interval [-0.11, 1.91]. Following most of the literature for the Brazilian economy, we also use \(\gamma_n = 0\).

As for parameter \(\eta\), which measures the substitutability between domestic and foreign goods, it is required from equation (58) that \((\sigma\eta - 1) > 0\) and so that \(\eta > \frac{1}{\sigma}\). Given that \(\sigma = 1\), we chose to set \(\eta = 3\).

According to Cavalcanti and Vereda (2011), the values usually found in national and international literature for the parameter that measures the elasticity of substitution between differentiated goods are such that the markup of the steady state belongs to the interval [10%, 25%]. Carvalho and Valli (2010) define \(\varepsilon = 6\), consistent with a steady state markup of 20% \(\left(\mu = \frac{\varepsilon}{\varepsilon-1} = 1.2\right)\). We follow this suggestion and also use \(\varepsilon = 6\). The parameter \(\mu\) is the markup over the price that would prevail in the absence of nominal rigidities.

Concerning the degree of openness of the economy, Silveira (2008) considers \(\alpha = 0.13\) based on Brazilian data over the period 1999-2005. In this work, we set \(\alpha = 0.10\), a value obtained from the average imports over GDP for the Brazilian economy using quarterly data for the period from 2000 to 2012.
For the parameters that refer to the probability of price adjustment, we use the estimates by Castro et al. (2011) and assume that $\theta_H = 0.74$ and $\theta_F = 0.64$.

Finally, in relation to the interest rate rule parameters, Carvalho and Valli (2010) define 0.9 for the coefficient associated with the lagged interest rate and a value between 1.7 and 1.8 for the coefficient of response to inflation. Regarding the coefficient of the output gap, the evidence is less robust. Some studies get positive but small estimates (around 0.2), while others do not get estimates statistically different from zero. The literature, according to Cavalcanti and Vereda (2011) suggests the interval $[0.7, 0.9]$, $[1.5, 3]$ and $[0, 0.5]$ for the parameters $\rho^i, \tau_1 e \tau_2$, respectively. Based on this lack of consensus, we define $\rho^i = 0.79$, $\tau_1 = 1.5$ and $\tau_2 = 0.5$, which are in line the standard Taylor rule.

For the coefficients measuring the persistence of the shocks $(\rho^r, \rho^u, \rho^g)$, we follow Svensson (2000) and assume the same value of 0.8 for all of them.

5.2 IMPULSE-RESPONSE FUNCTIONS

The performance of the model is analyzed in terms of impulse-response functions. These functions describe how the economy responds to exogenous shocks usually presented in the form of autoregressive vectors at the time of shock and in subsequent periods. It is worth mentioning that it is not our objective to analyze moment conditions or variance decomposition. Our purpose is to assess how the different transmission mechanisms are reproduced in the model. We analyzed impulse-response functions through four shocks in the following variables: import tariffs, domestic inflation, monetary policy, and foreign interest rate. We consider, first, a shock of 20% on import tariffs. Figure 1 presents the results.
Figure 1: Impulse responses to an import tariff shock

The results show that the shock of 20% on the import tariff ($\tau = \text{tau}$) reproduced the expected effects on the economy. The domestic currency cost of the imported good ($\tilde{\psi}_{F,t} = \text{psif}$) becomes higher, increasing deviation from the law of one price and consequently the cost of imported goods ($\pi_{F,t} = pif$). These results show incomplete pass-through rate since the magnitude of $\pi_{F,t}$ is less than 20%. There is an increase in domestic inflation ($\pi_{H,t} = pih$). The immediate effect is that the overall domestic inflation rate ($\pi_t = pi$) increases. In order to control inflation, the interest rate rises, affecting aggregate demand ($h$). The difference between national and international interest rates ($i > i^*$) leads to the appreciation of the nominal exchange rate ($e$). Considering that the higher cost of imported goods reduces imports in the domestic country, the terms of trade suffer an immediate positive impact. However, in the long run, there is a deterioration of the terms of trade because the appreciation of the exchange rate that follows the shock stimulates imports, offsetting the initial effect of the tariff.

Figure 2 shows the effects of a positive supply shock of 1%. The aggregate supply curve, represented by the Phillips curve, shows a direct relationship between domestic inflation expectations, the output gap and deviations from the law of one price. Immediately
after the shock there is an increase in domestic inflation \((\pi_{H,t} = pih)\) and consumer inflation \((\pi_t = pl)\). Inflation encourages economic activity because companies envision the possibility of increasing profits through higher prices. Given that the monetary authority follows a Taylor rule, the domestic interest rate rises to control the inflationary pressure. The response rate is "hump-shaped" due to the lagged effects that it generates in the economic activity. The rise of interest rates stimulates capital inflows and, consequently, exchange rate appreciation, which in turn reduces the price of imported goods\((\pi_{F,t} = pif)\). The increase on imports leads to a deterioration of the terms of trade. The domestic currency cost of the imported goods \((\bar{\psi}_{F,t} = psif)\) responds as expected, but with a delay since it depends on the past domestic currency cost \(\bar{\psi}_{F,t-1}\).

![Figure 2: Impulse responses to a supply shock](image)

The effects of a monetary policy shock, with an increase of the domestic interest rate of approximately 1%, is shown in Figure 3. The immediate effect is a reduction of domestic consumption and its transmission through aggregate demand \((h)\), causing a contraction in the economic activity. Given the uncovered interest parity (UIP), the increase in the domestic interest rate attracts capital, leading to the appreciation of the exchange rate \((e)\).
Consequently, there is an incentive to import since there is a fall in the prices of imported goods ($\pi_{F,t} = pif$), making the terms of trade ($s$) negative. The combination of low economic activity with currency appreciation leads to lower domestic and consumer prices. The domestic currency cost of the imported goods ($\psi_{F,t} = psif$) responds as expected, but again with a delay because it depends on the past domestic currency cost $\psi_{F,t-1}$.

Finally, we present the effects of a 1% shock on the international interest rate in Figure 4. This shock makes the world interest rate higher than the domestic one ($i^* > i$), causing capital outflow from the small domestic economy. Given the UIP, the transmission for the economy is given by the devaluation of the exchange rate. Imports then become more expensive ($\pi_{F,t} = pif$ raises), since the domestic currency cost of the imported goods ($\psi_{F,t} = psif$) raises. Considering that equations respond simultaneously, there are two immediate effects: domestic prices fall and consumer inflation rises given the rising cost of imports. These effects lead to a decrease in the economic activity. The fall in the output gap is greater than the increase in consumer inflation, which allows explaining the fall in the domestic interest rate. In response to movements of the exchange rate, there is an improvement in the terms of trade.

![Diagram](image)

**Figure 3: Impulse responses to a domestic interest rate shock**
In general, the inclusion of import tariffs in the deviation of the law of one price and incomplete pass-through contributed to enrich the dynamic of the small open economy endogenous variables. Specifically, the model was able to illustrate how an increase in import tariffs can be transmitted to the economy as whole, through increase in inflation and reduction in the economic activity. This finding confirms that protectionist policies might adversely affect the social welfare in the small open economy.

![Figure 4: Impulse responses to an international interest rate shock](image)

### 6. CONCLUDING REMARKS

The model presented in this paper considers a small open economy in which economic fluctuations do not affect the rest of the world. Households own firms and consume imported and domestically produced goods. Domestic production meets domestic and foreign demand. Prices of domestic and imported goods have nominal rigidities and are adjusted according to the time-dependent Calvo (1983) rule. Our framework is based on Monacelli (2005) which
introduces incomplete exchange rate pass-through on import prices in the short term. One key difference is that, in addition to having exchange rate, import tariffs are included to generate deviations from the law of one price. The government runs a balanced budget each period which allows establishing a monetary policy rule of the Taylor type (1993) as an optimal reaction to inflation deviations from its equilibrium level. Structural equations which define equilibrium were calibrated for the Brazilian economy. The calibrated parameter values were already used by other authors based on microeconomic evidence consistent with the Brazilian data.

Shocks affecting four economic variables were allowed. Specifically, we consider exogenous disturbances in import tariffs, domestic inflation, monetary policy, and foreign interest rate. The impulse-response functions generated by the model simulations were analyzed. In general, the effects of shocks on the endogenous variables evolved according to the expected and contributed to enrich the dynamics of the small open economy endogenous variables.

The results suggest that the import tariff shock affects positively the domestic price of imported goods as well as consumer prices. There is an incomplete pass-through of import tariff to the economy. Therefore, protectionism, understood here as the increase of import tariffs, has a high social cost due to its effects on inflation, which in turn may negatively affect other public policies, especially those related to income distribution.

Regarding the supply shock, there is an increase in domestic and consumer prices. The domestic interest rate rises to control the inflationary pressure, according to the Taylor rule. It stimulates capital inflows and, consequently, exchange rate appreciation, which in turn reduces the price of imported goods. The domestic interest rates shock leads to an immediate reduction of domestic consumption, causing a contraction in the economic activity. The exchange rate appreciation due to capital inflows stimulates imports, making the terms of trade negative. Finally, the international interest shock showed significant effects on exchange rate and terms of trade, affecting then the real side of the small open economy.

Despite this set of results, there are limitations to the model. Although it is an open economy, the roles of financial markets and government financing of imports were not considered. An extension of this model with the inclusion of these variables would be relevant because of their effects on exchange rates and on trade flows, which in turn could affect the import tariffs dynamic. Finally, it would be important to estimate the proposed model to the Brazilian economy through Bayesian techniques.
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