Optimization and Simulation of Dispersion Characteristics for Dielectric Rod Waveguide: TLBO Algorithm

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Abstract. *The dielectric rod waveguide is an open waveguide structure that shows surface wave propagation characteristics. In this paper, analytical theory of dielectric rod waveguide has been formulated and dispersion relationship is numerically investigated for both lower order transverse modes (TE and TM mode) and higher order Hybrid modes (HE and EH mode) for guided mode investigation. The characteristic equation was employed to solve the various modes with Muller complex root search method. The appropriate value of dielectric constant and radius were found out using Teaching Learning Based Optimization (TLBO). The outlined structure has wide single mode propagation Bandwidth. Also this optimization is validated in FEM based computational method and results compared with analytical findings.*

Keywords - *Dispersion, Dielectric Rod, Transverse Mode, Hybrid Mode, Optimization*

I. INTRODUCTION

Circular geometry structures are very popular, such as coaxial lines, rod waveguide, fibers and tube waveguide. These cylindrical structures maintain an even cross section along their length, however, these metallic based structures are unfeasible in the high frequency region due to Ohmic and conductor losses. So at higher frequency, the better option is to use a dielectric waveguide [1–3]. This waveguide has guided modes with a purely real propagation constant. In 1966, Kao [4] presented a paper on the theory of circular open waveguides showing the symbolized application as an optical fiber which is used in high speed internet services and in a microwave transmission satellite link for high power applications. Several decades later, Kim [5-6] studied the guided and leaky mode characteristics and demonstrated several lower orders of the modes by using the Davidenko's complex root finding algorithm. S. Yang [7] also reported the detail study of proper and improper mode of guided and leaky wave for dielectric rod waveguide. However, through examining these articles [8-9], we found that Kim assumed the value of dielectric constant and radius as there is no particular equation to find out the cutoff value for mode in terms of radius. So there are still many ambiguous parts left unsolved in the optimization of wave propagation model.

In the following work a rigorous investigation of the modal and dispersion characteristics of dielectric rod guide is presented with a Muller numerical method. In dielectric circular waveguide dimensions such as radius and dielectric medium are two important aspects for designing the waveguide. The value of radius and dielectric constant are optimized by a new efficient Teaching Learning Based Optimization (TLBO) algorithm. This method is proposed to evaluate the characteristics equation of dielectric rod as an error function, and the optimized parameter is radius and dielectric constant in order to achieve the maximum value of wide mono mode propagation bandwidth. The numerical results were verified by comparing with Kim's results [5], and the designed structure has also been electromagnetically simulated using HFSS (High Frequency Structure Simulator) to verify the analytical findings.

II. STRUCTURE AND CHARACTERISTICS EQUATION OF DIELECTRIC ROD WAVEGUIDE:

The schematic diagram of dielectric rod waveguide is shown in Fig. 1, where it has been profiled in two regions namely, the dielectric medium as the core, the free space as the cladding outermost region.

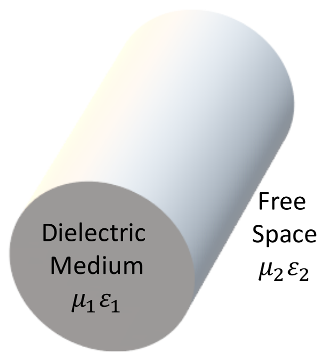


Fig. 1. The geometry of dielectric rod waveguide structure

The electric and magnetic field component in different region is expressed as:

$$E_{z1} = AJ_m(k_1 r)P_m \quad (1)$$

$$H_{z1} = BJ_m(k_1 r)Q_m \quad (2)$$

$$E_{z2} = CK_m(k_1 r)P_m \quad (3)$$

$$H_{z2} = DK_m(k_1 r)Q_m \quad (4)$$

Where, $P_m = cosm\theta e^{i(\omega t-\beta z)}$ and $Q_m = sinm\theta e^{i(\omega t-\beta z)}$. Here axial components in Z direction is used to define wave propagation characteristics. The radial and azimuthal field components can be calculated in terms of the z components using Maxwell's equations. By applying the continuity conditions of tangential fields to the circular dielectric waveguide with radius r_1 inserted in an infinitely homogeneous dielectric medium, the characteristic equation for lower order TM_{0n} and TE_{0n} mode can be expressed as [8, 9]:

For TM mode:

$$\frac{\varepsilon_{r1} J_1(k_1 r_1)}{k_1 J_0(k_1 r_1)} + \frac{\varepsilon_{r2} K_1(k_2 r_1)}{k_2 K_0(k_2 r_1)} = 0 \quad (5)$$

For TE mode:

$$\frac{\mu_{r1} J_1(k_1 r_1)}{k_1 J_0(k_1 r_1)} + \frac{\mu_{r2} K_1(k_2 r_1)}{k_2 K_0(k_2 r_1)} = 0 \quad (6)$$

J_m and K_m (m=0,1) are Bessel function of first kind and modified Bessel function of the second kind, respectively, where m is the azimuthal Eigen value of the fields, K_1 and K_2 are the propagation constants in the transverse direction and expressed as follows:

$$k_1 = k_0 \sqrt{\mu_1 \varepsilon_1 - \beta^2} \quad (7)$$

$$k_2 = k_0 \sqrt{\beta^2 - \mu_2 \varepsilon_2} \quad (8)$$

III. TEACHING LEARNING BASED OPTIMIZATION (TLBO)

A novel effective optimization method, called Teaching Learning-Based Optimization (TLBO) was developed by Rao et al. [10]. This algorithm is like other nature inspired evolutionary algorithm which works on the typical classroom teaching method. TLBO is also a population-based method and the population is considered as a group of learners. The TLBO is working in two phases: (1) Teacher Phase (2) Learner Phase. In the teacher phase, learners first acquire information from a teacher and he creates an effort to increase the mean result of the class. The best solution is observed as the teacher (X_{mean}). In the learner phase, learners learn from the teacher and the teacher tries to enhance the result of other individual (X_i) by increasing the mean result of the classroom (X_{mean}) towards his position ($X_{teacher}$). Two random numbers are generated in the range of 0 and 1 which is stored in r. [10]:

$$X_{new} = X_i + r.(X_{teacher} - T_f .X_{mean}) \quad (9)$$

where X_{new} and X_i are the new and existing solution of i, and T_f is a teaching factor which can be either 1 or 2. The second phase, algorithm simulates the learning of students through interaction among themselves. The students can also gain information by discussing and interacting with other students. A learner will learn new information if the other learners have more knowledge than him. During this stage, the student X_i interacts with another student X_j randomly in order to enhance his/her knowledge. In the case that X_j is improved than X_i, X_i is moved toward X_j.

$$X_{new} = X_i + r(X_i - X_j) \text{ if } f(X_i) < f(X_j) \quad (10)$$

$$X_{new} = X_i + r(X_j - X_i) \text{ if } f(X_j) < f(X_i) \quad (11)$$

In the new solution, X_{new} is improved and it is accepted in the population. The algorithm will continue until the best fitness condition is met. Due to its simple concept and high efficiency, TLBO has become a very popular optimization algorithm and has been productively applied to many real world problems.

The implementation steps of the TLBO are summarized as below:

Step 1: Initialize the population: All the students are initialized. The number of students has been taken as 6 and optimized parameters (i. e number of subjects) are radius (r_1) and dielectric constant (ε_1) for designing the waveguide.

Step 2: Error function is a characteristics equation and it is minimized with the optimized parameters which are radius and dielectric constant. Our observation frequency is 1 GHz to 50GHz therefore optimized radius should be between 5 to 10mm and dielectric constant should be 2 to 5.

Step 3: Select the best learner of each subject as a teacher for that subject and calculate the mean result of learner in subject.

Step 4: Evaluate the difference between the current mean and best mean result by utilizing the teaching factor TF value which is 1. Update the learner's knowledge with the help of teacher's knowledge.

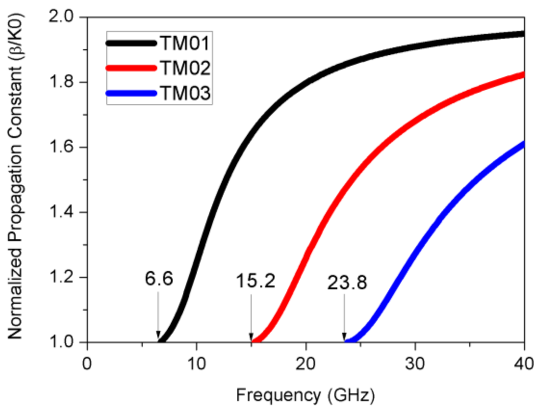
Step 5: Update the learner's knowledge by utilizing the knowledge of other learners.

Step 6: Repeat the process until the error should be minimized and final outcome has been achieved with wide single mode propagation bandwidth by subtracting the cutoff value of TM_{01} mode and TM_{02} mode from dispersion graph.

IV. COMPUTATIONAL RESULTS AND DISCUSSION

TLBO algorithm was used to optimize the Dielectric rod guided mode characteristics. Here the population (No. of Students) is 6 and design parameters are radius and dielectric constant of the rod. These two design parameters are optimized in TLBO algorithm to find out the best possible output for different operating modes (like TM and TE). The main objective is to find out wide single mode propagation bandwidth.

Kim [5] had analyzed the dispersion characteristics of the circularly symmetric modes of circular dielectric rod waveguides by Devindenko complex root search numerical method. The example used in his paper was solved using Muller complex root search method [11] for a circular dielectric rod waveguide with a radius of 10 mm and dielectric constant of 4.0. The normalized propagation constant increases as the operating frequency becomes higher. The cutoff frequencies for the TM_{0n} and TE_{0n} modes were identical, i.e., 6.63 GHz for the TM_{01} and TE_{01} modes, 15.22 GHz for the TM_{02} and TE_{02} modes, 23.86 GHz for the TM_{03} and TE_{03} modes, and so on. Since the dielectric constant of the dielectric cylinder is assumed to be 4.0, the normalized propagation constant approaches as the operating frequency increases. However, it cannot exceed the upper limit value of $\sqrt{\mu_1 \varepsilon_1}$ and modes with dissimilar order cannot cross each other.



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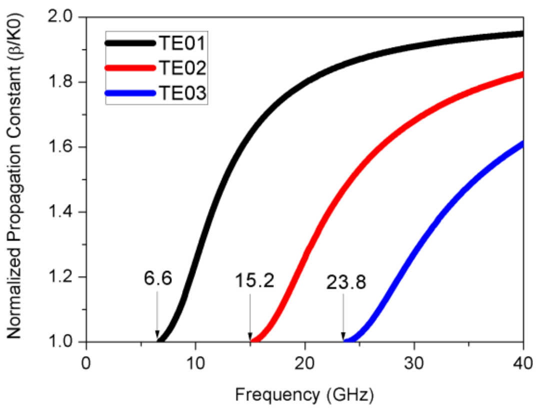


(b)

Fig. 2 Dispersion characteristics of dielectric rod waveguides with r_1=10mm, ε_1=4 according to Kim work (a) TM_{0n} mode and (b) TE_{0n} mode.

Research work done by Kim about dielectric rod waveguide was compared with Muller complex root search numerical method and implemented in MATLAB. The result shown in Fig. 2 are the same as previously reported Kim [5] result. So we can verify that Muller method is also efficient to define propagation characteristics of rod and tube waveguide.

V. *SIMULATION RESULTS AND DISCUSSIONS*

V.1 *Comparative analysis of Numerical and Analytical Findings*

The MATLAB analytical simulation is validated with actual physical structure modelled in HFSS and results compared. In waveguide design we have chosen a circular dielectric rod waveguide with a radius of 10 mm and the dielectric material is FR4 whose dielectric constant is 4 and length of the waveguide is 50mm. This model is simulated in driven mode with accuracy delta of 0.001. The wave propagation constant is obtained for traverse magnetic TM_{01} and TM_{02} mode. In Fig 3, it shows comparative analysis of analytical solution found in MATLAB and computational HFSS. The value of cutoff frequency is same for both the analysis but the behavior of the wave propagation in the waveguide is different.

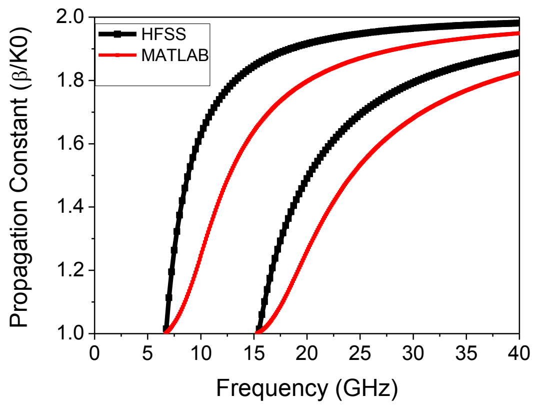


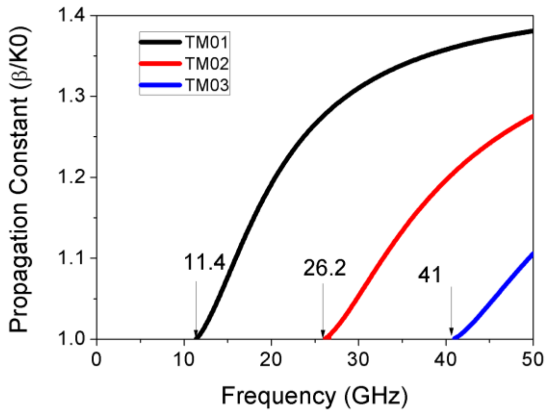
Fig. 3 Propagation constant of dielectric rod waveguides with r_1=10mm, ε_1= 4 for TM_{0n} mode

V.2 Simulation and Optimization of Lower Order Transverse Mode

TLBO algorithm is applied to dielectric rod modal equation to find out the optimum value of a dielectric rod radius and dielectric constant. Here, no of students (population size) is 6 and design variables are rod radius (upper limit-15mm and lower limit-10mm) and dielectric constant (upper limit-5 and lower limit- 2). Within 20 iterations we can get the optimum result with dielectric rod radius (r_1) as 10mm and dielectric constant (ε_1) as 2. This parameter was obtained in MATLAB and a dispersion characteristics plot in Fig 3. In Table 1 it is shown that Single mode propagation bandwidth is 14.8 which shows a 58% increment in propagation bandwidth compared to Kim's result. Therefore, proper selection of the value of dielectric constant we can have large single mode propagation bandwidth.

Table 1. Comparative table for TM guided mode of dielectric rod waveguide

Comparison	**Radius (r_1) (mm)**	**Dielectric Constant(ε_1)**	**Bandwidth (GHz)**
Kim [5]	10	4	8.59
TLBO Optimization	10	2	14.8



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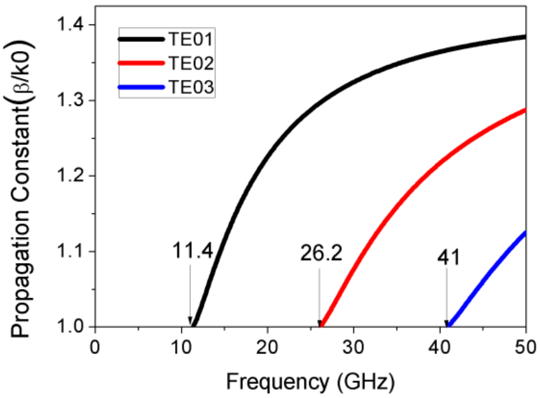
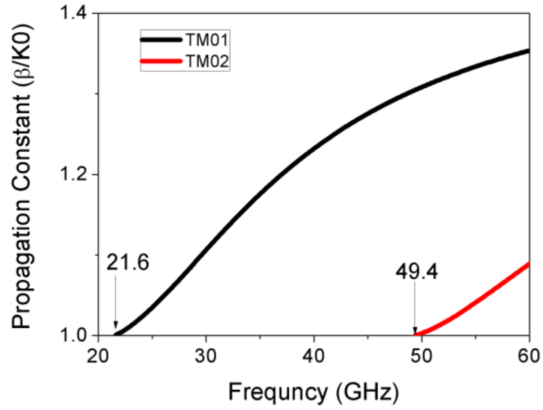


(b)

Fig. 4 Dispersion characteristics of dielectric rod waveguides with r_1=10mm, ε_1=2 using TLBO optimization. (a) TM_{0n} mode and (b) TE_{0n} mode

TLBO algorithm is also applied to a dielectric rod equation with changes in the lower and upper limit of radius and observe the variation in bandwidth and cutoff frequency. Here, no of students (population size) is 6 and design variable are rod radius (upper limit-10mm and lower limit-5mm) and dielectric constant (upper limit-5 and lower limit-2). Within 20 iterations we can get the optimum result with dielectric rod radius (r_1) as 5mm and dielectric constant (ε_1) as 2. Practically, Teflon material is available and its dielectric constant is 2 [11]. Table 2 shows the value of radius and dielectric constant at each iteration. From the optimization observation, we can conclude that lower the values of radius and dielectric constant, there is an increase in the value of single mode bandwidth. In Fig. 5 shows that the cutoff frequencies for the TM_{0n} and TE_{0n} modes are identical, i.e., 21.6 GHz for the TM_{01} and TE_{01} mode and 49.4 GHz for the TM_{02} and TE_{02} modes so on.



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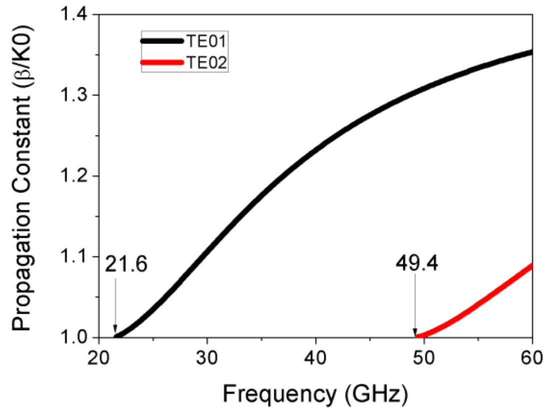


(b)

Fig. 5 Dispersion characteristics of dielectric rod waveguides with r_1=5mm, ε_1=2 using TLBO (a) TM_{0n} mode and (b) TE_{0n} mode.

Table 2. The value of rod radius and dielectric constant of each iteration using TLBO

		TM mode		TE mode	
No. of Iteration	**Bandwidth**	**Radius (r_1) (mm)**	**Dielectric Constant (ε_1)**	**Radius (r_1) (mm)**	**Dielectric Constant (ε_1)**
1	24	5.878	2.2344	5.578	2.3246
2	26	5.57	2.247	5.57	2.270
3	26	5.57	2.247	5.57	2.270
4	26	5.57	2.247	5	2.151
5	28	5	2.142	5	2.151
6	28	5	2.142	5	2.142
7	29	5	2.0	5	2.0
8	29	5	2.0	5	2.0
9	29	5	2.0	5	2.0
10	29	5	2.0	5	2.0

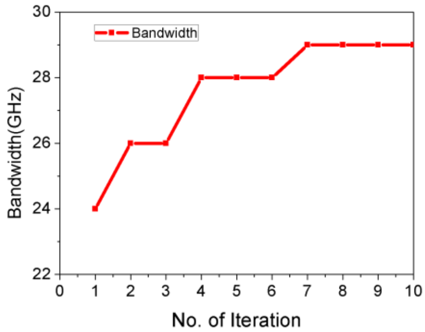


Fig. 6 Bandwidth Vs. No. of iteration for Dielectric TM and TE mode using TLBO

Next, the effects of design parameter in TLBO optimization, it is observed during optimization, radius and dielectric constant value always converge at lower limits. Therefore, by changing the lower limit of radius taken as 9mm, 8mm, 7mm, 6mm and 5mm, it is observed that the tendency of dispersion as shown in Fig. 7 shows that the normalized propagation constant curves for both the TM_{01} and TM_{02} modes shifted toward a higher frequency regime with a decrease in the radius of the rod.

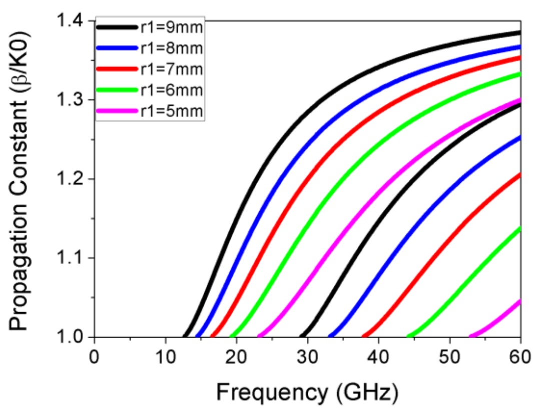


Fig. 7 Parametric variation in the radius lower limit as 5mm to 9mm with increment step size of 1mm

Table 3: Cutoff value for Dielectric rod waveguide (TM_{0n} mode) with variation in radius using TLBO

TM_{01}	TM_{02}	Radius (r_1) (mm)	Dielectric Constant (ε_1)	Bandwidth (GHz)
12.7	29.1	9	2	16.4
14.5	33.2	8	2	18.7
16.6	37.9	7	2	21.3
19.3	44.1	6	2	24.8
23.2	53	5	2	29.8

As a result, circular dielectric rod waveguides are regarded as one of the simplest nonplanar guiding structures and extensively used for microwave, millimeter wave and optical frequencies. This is an open waveguide structure so it can be used as a dielectric leaky wave antenna [6]. For this purpose, it is used in satellite communication link where frequency range is in Ku Band (12-18 GHz), K Band (18-27 GHz), Ka Band (27-40 GHz).

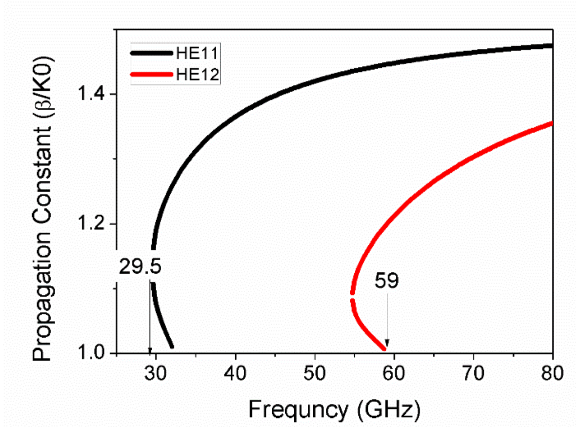
V.3 *Simulation and Optimization of Higher Order Hybrid Modes*

The azimuthal Eigen value for Higher order hybrid mode is m=1 and modes are designated as HE_{11}, HE_{12}, HE_{13} …. and EH_{11}, EH_{12}, EH_{13} …. For higher order modes, characteristics equation for dielectric rod is:

$$\left(\frac{\mu_2}{\mu_1}+\frac{\varepsilon_2}{\varepsilon_1}\right)\frac{Q}{2} \mp \sqrt{\left\{\left(\left(\frac{\mu_2}{\mu_1}+\frac{\varepsilon_2}{\varepsilon_1}\right)\frac{Q}{2}\right)\right\}+\frac{R}{\mu_1\varepsilon_1}} - P = 0 \quad (8)$$

where, $P=\frac{1}{k_1 r_1}\left(\frac{J_{m-1}(k_1 r_1)}{J_m(k_1 r_1)}-\frac{m}{k_1 r_1}\right)$, $Q=\frac{1}{k_2 r_1}\left(\frac{K_{m-1}(k_2 r_1)}{K_m(k_2 r_1)}+\frac{m}{k_2 r_1}\right)$, $R=\frac{m^2\bar{\beta}^2}{r_1^4}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)^2$

To find higher order modes this characteristics equation is applicable and same TLBO algorithm method can be used. Fig. 8 shows the dispersion characteristics of (a) HE_{1n} mode (b) EH_{1n} mode.



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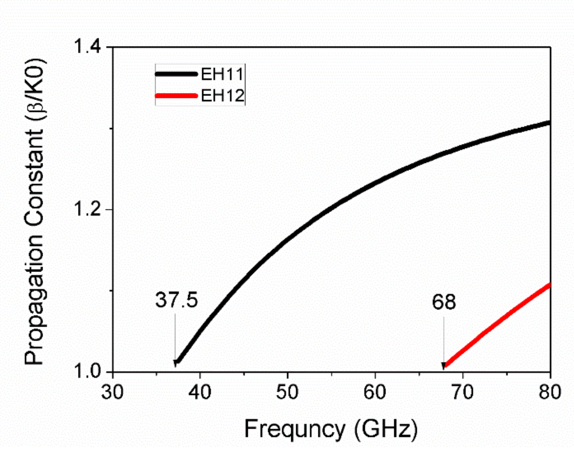

(b)

Fig. 8 Dispersion characteristics of dielectric rod waveguides with r_1=5mm, ε_1=2 using TLBO (a) HE_{1n} mode and (b) EH_{1n} mode.

The normalized propagation constant increases as the operating frequency becomes higher. The cutoff frequencies for the HE_{1n} modes are 29.5GHz and 59GHz and EH_{0n} modes are 37.5GHz and 68GHz. The single mode propagation bandwidth for HE mode is 29.5 GHz and EH mode is 30.5 GHz.

VI. CONCLUSION

The guided mode characteristics of dielectric rod waveguide was theoretically investigated for lower order Transverse Modes (TM-TE) and higher order Hybrid Modes (HE-EH) which were defined by the real value of phase constant with the numerical Muller method. With TLBO Algorithm, optimization was performed on the dielectric rod waveguide with design parameter such as radius and dielectric constant to obtained wide single mode propagation bandwidth. From the optimization, it is found that there is a 58% improvement in single mode propagation bandwidth compared with Kim result by proper choosing the dielectric constant value. It was also analyzed that optimization of waveguide always converges at the lower limits of radius and dielectric constant.

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