The Electromagnetic Impulse Pendulum and Momentum Conservation.

P. GRANEAU

Massachusetts Institute of Technology - Bldg. NW17-284, *Cambridge,* MA 02139

P. N. GRANEAU

King's College London, Physics Department - The Strand, London WC2R 2LS

(ricevuto il 14 Maggio 1985)

Summary. – Largely quantitative experiments by Pappas have indicated that the momentum imparted to an electrodynamic impulse pendulum was not balanced by an equal and opposite momentum change of field energy as required by the special theory of relativity. The authors repeated Pappas' experiment using discharge currents from a capacitor bank which contained a known amount of stored energy. It turned out that, for momentum conservation, the magnetic-field energy required would have been 1000 to 2000 times as large as the energy that was actually stored in the capacitors. In the second part of the paper the pendulum experiments are interpreted in terms of Ampère's force law. It is shown that the Ampère force exerted on the pendulum is almost exactly the same as the Lorentz force, but it arises in different parts of the pendulum conductor. Furthermore, the Ampère reaction force does not reside in the field but in the stationary part of the circuit which supplies current to the pendulum. Hence in the Ampère electrodynamics the momentum is definitely conserved. The experimental and analytical findings confirm the work by Pappas. A new and important experimental fact emerged from the present investigation. The momentum imparted to the pendulum was found to be significantly smaller than the calculated mechanical impulse given by the Lorentz and Ampère force laws. The Ampère force distribution offers an explanation of this observation in terms of the elastic distortion of the pendulum structure. The Lorentz force distribution could not produce this distortion.

PACS. 41.10. – Classical electromagnetism.

1. – Pappas' experiment.

Ever since the elastic properties of the ether were abandoned to accomodate the special theory of relativity, it was felt that another mechanism for absorbing reaction forces in the field (vacuum) was required. In the situations considered in this paper, the reaction forces counteract Lorentz forces exerted on current-carrying metallic circuits. It seems to have become the generally accepted view [1] that the rate of change of electromagnetic momentum in the field can support a force which accelerates or decelerates magnetic-field energy to and from the velocity of light c. The energy-momentum density is related to the Poynting vector by

$$\boldsymbol{p} = (1/c^2)(\boldsymbol{E}\times\boldsymbol{H}), \tag{1}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric- and magnetic-field strengths at a point. The volume integral of the rate of change of this momentum density over all space gives the vacuum reaction force

$$\boldsymbol{F}_{\text{vac}} = \int(\mathrm{d}\boldsymbol{p}/\mathrm{d}t)\,\mathrm{d}v = (1/c^2)\int(\mathrm{d}/\mathrm{d}t)(\boldsymbol{E}\times\boldsymbol{H})\,\mathrm{d}v, \tag{2}$$

where t stands for time and v for volume. When the integral is not taken over *all space*, the rate of change of momentum may be smaller than indicated by eq. (2). It is customary to make up the difference by the surface integral of Maxwell's stress tensor over the finite volume of the momentum integral. In the context of the present investigation the integral of eq. (2) will always be taken over infinite space to avoid any complication with Maxwell stresses. Then for any instant in time the vacuum reaction force may be written as

$$\boldsymbol{F}_{\text{vac}} = (\mathrm{d}/\mathrm{d}t)(m_e c), \tag{3}$$

where m_e is the equivalent electromagnetic mass of the magnetic energy stored in the field. Since c is a constant, the vacuum force will only exist when m_e changes with time, that is when magnetic energy is radiated into the field, or absorbed from the field by a conducting body. The amount of field energy U_f that must at any time be associated with the vacuum reaction force is

$$U_f = m_e c^2. \tag{4}$$

This is the famous mass-energy relation of special relativity.

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(1) W. H. K. PANOFSKY and M. PHILLIPS: *Classical Electricity and Magnetism* (Addison-Wesley, Reading, Mass., 1962).

If $\boldsymbol{F}_{vac}$ is at all times the simultaneous reaction force to the Lorentz force $\boldsymbol{F}_L$ when the latter accelerates a metallic conductor object of real mass m from zero velocity to the velocity u, then momentum conservation is expressed by

$$mu = \int p \, \mathrm{d}v = m_e c \,. \tag{5}$$

The last equation implies that Newton's third law is obeyed and the electromagnetic mass m_e is dynamically equivalent to real mass. The field energy which has to be radiated or absorbed by the conductor body to comply with eq. (5) is

$$U_f = m_e c^2 = muc \,. \tag{6}$$

PAPPAS [2] invented an impulse pendulum experiment to check whether this amount of energy did actually exist in the field of infinite extent. He found this could not be the case because the amount of field energy specified by eq. (6) was far greater than the amount of energy available from his source. The Pappas experiment is a variation of Ampére's original hairpin experiment [3,4]. It challenges the field-energy momentum concept only in so far as *metallic* conductors are concerned. The authors repeated Pappas' experiment with a different energy source and made quantitative measurements. In our case the energy available for Joule heating and conversion to kinetic and electromagnetic energy was accurately known, because it was drawn from previously charged capacitors. We measured the mechanical momentum of the pendulum and the magnitude and shape of the current pulse to evaluate energy exchanges and see to what extent the pendulum swing was consistent with the relativistic and the Ampére electrodynamics [3-9].

Figure 1 is a simplified diagram of the experimental circuit used by PAPPAS. *ABCDEF* was a horizontal, rectangular, copper circuit and *B* and *E* were two mercury cups. The portion *BCDE* formed the impulse pendulum hanging from the laboratory ceiling. The remainder of the circuit was fixed to the laboratory frame and contained a battery and a switch in branch *AF*. When the switch was closed, an instantaneous current i would flow around the copper circuit and the pendulum was observed to swing in the direction x indicated in fig. 1. This swing interrupted current flow in the mercury cups.

(2) P. T. PAPPAS: *Nuovo Cimento B*, **76**, 189 (1983).
(3) P. GRANEAU: *Nature (London)*, **295**, 311 (1982).
(4) A. M. HILLAS and P. GRANEAU: *Nature (London)*, **302**, 271 (1983).
(5) P. GRANEAU: *J. Appl. Phys.*, **53**, 6648 (1982).
(6) P. GRANEAU: *Phys. Lett. A*, **97**, 253 (1983).
(7) P. GRANEAU: *IEEE Trans. Mag.*, MAG-**20**, 444 (1984).
(8) F. F. CLEVELAND: *Philos. Mag.*, **21**, 416 (1936).
(9) P. GRANEAU: *Nuovo Cimento B*, **78**, 213 (1983).

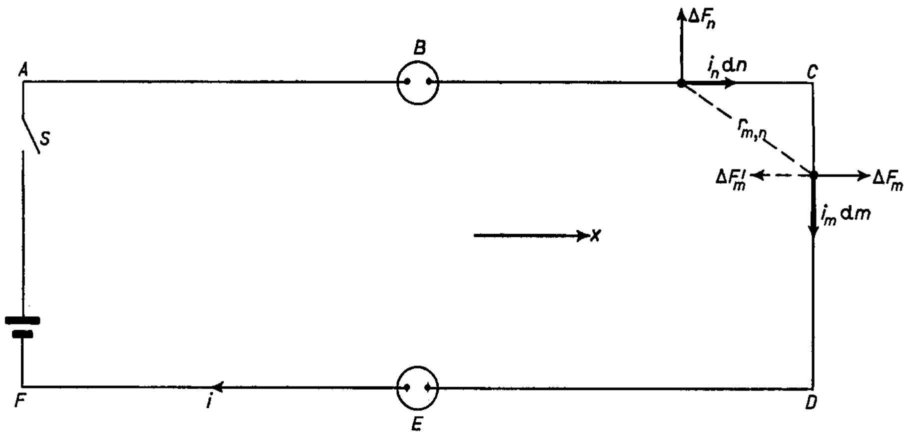

Fig. 1. – Pappas' impulse pendulum circuit.

By field theory, the force responsible for producing the pendulum motion is the magnetic component of the Lorentz force on branch *CD*. This force arises, via the Biot-Savart law, from interactions of current-element pairs governed by the Grassmann formulae

$$\Delta \boldsymbol{F}_m = (\mu_0/4\pi)(i^2/r^2_{m,n})\, \mathrm{d}\boldsymbol{m} \times (\mathrm{d}\boldsymbol{n} \times \mathbf{1}_r)\,, \tag{7}$$

$$\Delta \boldsymbol{F}_n = (\mu_0/4\pi)(i^2/r^2_{m,n})\, \mathrm{d}\boldsymbol{n} \times (\mathrm{d}\boldsymbol{m} \times \mathbf{1}_r)\,, \tag{8}$$

where $i \cdot \mathrm{d}m$ is a current element in branch *CD* and $i \cdot \mathrm{d}n$ is a current element anywhere else in the remainder of the circuit. Because of the inverse square law contained in eq. (7), most of the acceleration force on the pendulum arises from the magnetic field of current elements in the legs *BC* and *DE*, that is in the manner indicated in fig. 1 by ΔF_m.

According to field theory as taught today, ΔF_m and similar contributions to the pendulum acceleration force should be counteracted by a vacuum reaction force indicated by $\Delta F'_m$ in fig. 1. The latter force will ensure compliance with Newton's third law and linear-momentum conservation. The vacuum reaction force would either decelerate incoming energy which is then converted to Joule heat, or it could accelerate energy which is being radiated from the metal element dm. The incoming energy would be transmitted from the battery through space by the Poynting vector mechanism, while the outgoing energy radiated from dm must first reach the element somehow through the metallic conductor. Both energy streams have to be added together to arrive at the energy subtracted from the battery or other source. The vacuum reaction force acts on the space occupied by the metal element dm. If it acted on the metal itself, it would cancel the pendulum acceleration force.

The rate of change of current in the pendulum experiments is insufficient to produce a significant amount of energy reflection on dm. It will further be assumed that no electromagnetic mass is being created in midspace, away from the metal, which might somehow contribute to momentum conservation. As the field energy travels with constant velocity c, all of the vacuum reaction force $\Delta F'_m$ must then arise at the location of dm. We nevertheless extend the integration in eq. (2) over all space to eliminate all possible Maxwell stresses.

At some finite time t after the current has been switched on, the mechanical impulse imparted to the pendulum by the Lorentz force should be equal to the mechanical momentum aquired by the pendulum. Furthermore, for momentum conservation, this mechanical momentum should be equal to the field energy momentum change. Hence we may write

$$\int_0^t F_\mathrm{L}\,\mathrm{d}t = m\int_0^t (\mathrm{d}u/\mathrm{d}t)\,\mathrm{d}t = c\int_0^t (\mathrm{d}m_\mathrm{e}/\mathrm{d}t)\,\mathrm{d}t = mu = m_\mathrm{e}c\,, \tag{9}$$

where

$$F_\mathrm{L} = \sum_m \sum_n (\Delta F_m) \tag{10}$$

with the summation over m extending throughout the circuit branch CD and the summation over n covering the remainder of the circuit. Since m, u and c are known, we may use eq. (9) to calculate m_e, the electromagnetic mass, in the field which must be furnished by the battery or other energy source. The total amount of energy that must have been subtracted from the source up to the time t is specified by eq. (4). This is the minimum energy of eq. (6) required by field theory to accelerate the pendulum and cover the Joule heating in the branch CD of the circuit.

The Pappas experiment permits a comparison to be made of

1) the Lorentz force impulse with the mechanical momentum acquired by the pendulum, and

2) the minimum required field energy of eq. (6) with the energy available from the power source.

PAPPAS concluded that, in the time available, his battery would have supplied much less energy than was required by eq. (6) and that the Lorentz force on the circuit branch CD was, therefore, unlikely to be the force accelerating the pendulum. He pointed out that his experiment favors the older Ampére force law which explains the pendulum motion by Newtonian repulsion between the fixed and moving parts of the circuit.

2. – MIT experiment.

Figure 2 shows the experimental set-up of the electrodynamic impulse pendulum used by the authors. The «harpin» was made of a copper strip 0.5 in. high and 0.05 in. thick. The strip formed two 1 m long sides and one 30 cm short side of an open rectangle. The pendulum conductor was mounted on a rigid frame (not shown in fig. 2) of insulating material. The assembly weighed 0.815 kg. The pendulum was suspended from the laboratory ceiling by four 2.56 m long cotton threads. The horizontal displacement of the pendulum was measured with the cardboard slide *C* resting lightly on a flat-table top.

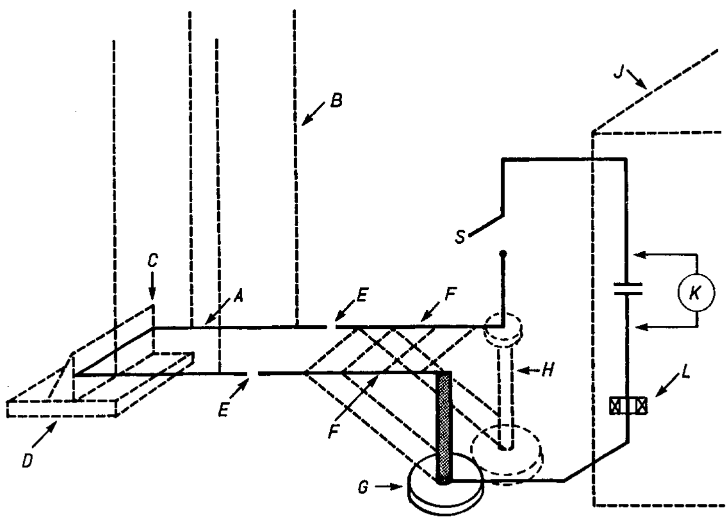

Fig. 2. – Electrodynamic hairpin pendulum. *A*) Hairpin pendulum (dielectric frame not shown), *B*) pendulum suspension on cotton threads, *C*) cardboard slide, *D*) flat table top, *E*) arc gaps in air, *F*) current rails on dielectric frame, *G*) metal stand, *H*) dielectric stand, *S*) mechanical high-voltage switch, *J*) capacitor bank (8 μF, 100 kV), *K*) charging set (200 kV), *L*) Rogowski coil for pulse current measurement.

The pulse current was derived from an 8 μF high-voltage capacitor bank which could withstand voltage reversals up to ±100 kV. The discharge was initiated by dropping the mechanical switch *S*. Two parallel current rails *F*, of the same copper strip of which the pendulum was made, brought the current to the hairpin via two, 1 mm long, arc gaps in air. The rails were mounted on a rigid frame and two heavy stands, weighed down with lead (not shown in fig. 2), to absorb the recoil impulse, predicted by the Ampère law, with a minimum of deflection. The rails were carefully aligned with the horizontal legs of the hairpin pendulum.

To perform a momentum experiment, the capacitor bank was charged to a voltage between 30 and 80 kV. The switch was then dropped, causing arcing across the short gaps between current rails and pendulum legs. The damped oscillatory current pulse was recorded with the aid of the Rogowski coil L of fig. 2 and an oscilloscope. The current pulse did cause the pendulum to swing away from the current rails and move the cardboard slide through a distance s, subsequently measured with a ruler. The duration of the current pulse was a fraction of a millisecond. Almost all the pendulum displacement occurred after the current had ceased to flow.

The maximum linear momentum, mu, imparted to the pendulum by the capacitor discharge may be calculated from the pendulum length $R = 2.56$ m, its mass $m = 0.815$ kg and the measured cardboard slide displacement s.

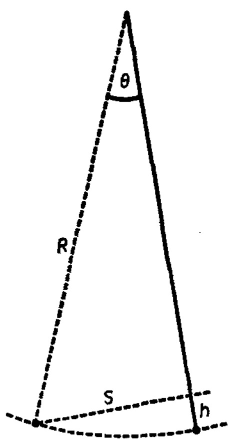

Fig. 3. – Pendulum parameters.

With the aid of fig. 3 it can be seen that

$$mgh = \tfrac{1}{2} mu^2\,, \qquad \text{or} \qquad u = \sqrt{2gh}\,, \tag{11}$$

where g is the acceleration due to gravity, u the maximum horizontal velocity that would be attained in the limit when the impulse duration tends to zero and h is the maximum vertical lift of the pendulum. The height h may be derived from the two simultaneous equations

$$h = R(1 - \cos\theta)\,, \tag{12}$$

$$s = R\sin\theta\,. \tag{13}$$

The solution is

$$h = R\left(1 - \sqrt{1 - (s/R)^2}\right) \simeq s^2/2R\,. \tag{14}$$

The approximation is accurate to three significant figures and may be used in eq. (11) to calculate u.

The electrodynamic impulse imparted to the pendulum by the Lorentz force $F_L = 10^{-7}ki^2$ may be written as

$$P_i = 10^{-7}k\int_0^\infty i^2\,dt\,, \tag{15}$$

where i is the instantaneous current and k (N/A^2) must be determined from the geometry of the circuit. A typical discharge oscillogram is reproduced in fig. 4. It shows that the instantaneous current is of the form

$$i = \exp[-t/T]\, I_0 \sin \omega t\,, \tag{16}$$

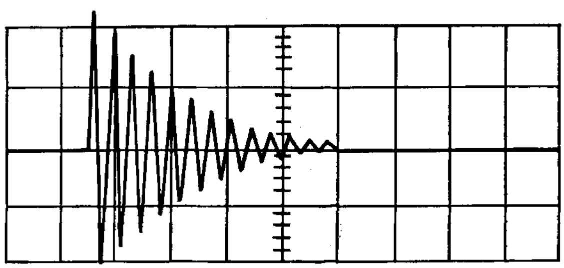

Fig. 4. – Discharge current oscillogram for 2 μF and 60 kV. $y = 15$ kA/V and 0.5 V/cm, $x = 0.1$ ms/cm.

where T is the time constant with which the oscillation decays. The full amplitude I_0 would be reached if the circuit contained negligible resistance. It is possible to integrate eq. (16), as required by (15), and obtain

$$P_i = 10^{-7}kI_0^2\{(T/4) - (1/T)/[(2/T)^2 + (2\omega)^2]\} \tag{17}$$

with $\omega = 2\pi f$ being the radian frequency. As far as the pendulum experiments were concerned, the second term of eq. (17) was negligible. Hence the electromagnetic impulse was taken to be

$$P_i = 10^{-7}kI_0^2(T/4)\,. \tag{18}$$

The magnitudes of I_0 and T were measured on the pulse current records.

The electric arcs bridging the 1 mm long gaps between current rails and pendulum legs produced a small amount of molten copper which streamed away from the gaps, in both directions, along the copper strips. The evidence of droplet streaming became unmistakable after the heaviest discharge currents. The arc pressure must have been responsible for generating some of the pendulum momentum, over and above the contribution made by the Lorentz force. As the arc pressure must have been symmetrical between rails and pendulum, it would not require to be balanced by a vacuum reaction force.

Approximately 100 discharge shots were carried out at various voltages and capacitance values. A typical set of results is reproduced in table I. This was obtained with the full 8 μF capacitance of the bank and, therefore, involved the largest currents and pendulum displacements. As long as the capacitance and circuit configuration were not changed between shots, the ringing frequency f, the circuit inductance L, the surge impedance $Z_0 = \sqrt{L/C}$, the time constant T and the effective damping resistance R all remained constant. Their magnitudes were derived from the current oscillograms. For the series of discharge pulses to which table I refers, these parameters came to

$$f = 15.7\ \text{kHz}, \qquad L = 12.8\ \mu\text{H}, \qquad Z_0 = 1.26\ \Omega,$$
$$T = 0.27\ \text{ms}, \qquad R = 94.8\ \text{m}\Omega.$$

The observed pendulum momenta listed in table I varied from just over 0.03 to 0.18 kg·m/s. The kinetic energies ($0.5mu^2$) associated with these momenta turn out to be less than one Joule, while the energy stored in the capacitors, U_c, was as high as 25.6 kJ. Hence very little of the spent energy is being converted to kinetic energy. When the Joule heat generated in the circuit is subtracted from the originally stored energy only up to 2.5 kJ remain. This immediately suggests that most of the field momentum change is associated with incoming energy being stopped and converted to Joule heat.

It must take a certain amount of energy to establish the three arcs at the switch and the two gaps E of fig. 2. This energy is sometimes called the latent heat of arc formation because it eventually appears as heat when the arc ions recombine. The latent heat of arc formation could conceivably account for several kJ, indicating that little may be available for radiation in the field.

It is instructive to evaluate the Lorentz force on the pendulum bridge and integrate it with respect to time to obtain the impulse which should be balanced by a change in field energy momentum. A suitable finite element method for computing the constant k in eq. (18) has been outlined in ref. ([5]). There an identical copper strip bent into a rectangular circuit of the same dimensions as the pendulum was resolved into ten parallel filaments of square cross-section. Each filament was then subdivided into cubic current elements. The computer solution gave $k = 9.27\ N/A^2$. This applied to uniform current distribution over the strip cross-section. At the frequency of 15.7 kHz, to which the results of table I refer, the current distribution will have been very nonuniform, with strong concentrations near the strip edges. If it is assumed that all the current flows in the two edge filaments, the constant in eq. (18) comes to $k = 11.0\ N/A^2$. Therefore, the Lorentz force appears to increase with frequency. A lower limit of this force is obtained by taking the 9.27 figure. Impulse values for this lower limit are listed in table I.

TABLE I.

$T = 0.27$ ms, $f = 15.7$ kHz	Symbol	Unit	40 kV	50 kV	60 kV	70 kV	80 kV
maximum pulse current	I_0	A	28 500	35 250	45 000	52 500	60000
pendulum displacement	s	cm	1.93	3.50	7.75	7.75	10.95
maximum pendulum momentum	mu	kg·m/s	0.0308	0.0558	0.0885	0.1236	0.1747
Joule heat dissipated	U_h	J	5 200	7 950	12 960	17 640	23 040
energy stored in capacitors	U_c	J	6 400	10 000	14 400	19 600	25 600
	$U_c - U_h$	J	1 200	2 050	1 440	1 960	2 560
electrodynamic impulse (Lorentz and Ampère)	P_i	N·s	0.0508	0.0778	0.1267	0.1725	0.2253
minimum required field energy	U_t	MJ	15.24	23.34	38.01	51.75	67.56
	U_t/U_c		1438	1 670	1 840	1 893	2 047
maximum Lorentz force (= Ampère force)	$F_{L,max}$	N	753	1 152	1 877	2 555	3 337
maximum pendulum velocity	u	cm/s	3.78	6.85	10.86	15.17	21.44

It will be noted that the calculated electrodynamic impulses P_t are greater than the measured linear pendulum momenta. The ratio of the two quantities is approximately 1.4. No field-theoretic explanation of this discrepancy can be offered. In determining the field energy U_f listed in table I, the measured *mu*-momenta were substituted into eq. (6). Table I also contains the energies U_c stored in the capacitors. The ratio U_f/U_c will be seen to vary between 1400 and 2000. The shortfall in available energy to satisfy field momentum conservation far outweighs all possible experimental errors. It supports the claim by PAPPAS that the Lorentz force is not balanced by an equal and opposite reaction force in the vacuum field.

To obtain an idea of the magnitude of the Lorentz force on the pendulum in the direction of its swing, we recognize that it would attain the maximum value of

$$F_{\mathrm{L,max}} = 9.27\cdot 10^{-7} I_0^2 \tag{19}$$

at the peak of the first half-cycle, but for the small exponential decrement due to damping. $F_{\mathrm{L,max}}$ is also listed in table I. The figures make it clear that, if the Lorentz force is indeed the motive force, the pendulum should feel a very sharp tug at its leading edge. The arc pressure is expected to be small compared to this sudden jerk.

3. – Momentum conservation with longitudinal Ampère forces.

Satisfactory Lorentz force explanations of certain experiments concerning wire fragmentation (6,7) and liquid metal jets (3) have not been forthcoming. The electrodynamic impulse pendulum is an addition to this list. Whether or not field energy momentum is a physical reality has direct technological impact on the developments of railguns (5). In all the cases in which the Lorentz force has been found inadequate, the original Ampère force law correctly predicts the relevant experimental effects. This is hardly surprising because AMPÈRE deduced his law from experiments with metallic circuits and, so long as it is applied to metallic circuits, it should be infallible.

The mutual Ampère repulsion (positive) or attraction (negative) force $\Delta F_{m,n}$ between a pair of current elements $i_m\,\mathrm{d}m$ and $i_n\,\mathrm{d}n$, separated by the distance $r_{m,n}$, may be written as

$$\Delta F_{m,n} = -\,(\mu_0/4\pi)\, i_m i_n (\mathrm{d}m\cdot \mathrm{d}n/r_{m,n}^2)(2\cos\varepsilon - 3\cos\alpha\cos\beta)\,, \tag{20}$$

where ε is the angle of inclination between the elements, and α and β are the angles which the elements make with the distance vector $r_{m,n}$.

When the two elements lie in the same straight line, point in the same direction and carry the same current i, eq. (20) reduces to

$$\Delta F_{m,n} = (\mu_0/4\pi)\, i^2(\mathrm{d}m\cdot \mathrm{d}n/r^2_{m,n})\,. \tag{21}$$

This produces repulsion between each current rail and the portion of the pendulum aligned with the rail. It is a strong force which in the Ampère electrodynamics provides the major impetus of propelling the pendulum. Equation (20) adds to this a small amount for the interaction of the rails with the pedulum bridge, and subtracts a little due to the attraction of each rail and the pendulum leg on the other side. On account of the mechanical decoupling by the arc gaps, the total Ampère repulsion between rails and pendulum can be obtained by integration of eq. (20). The finite element method with ten parallel filaments of cubic elements will yield virtually the same result. The numerical technique gave the value of $k = 9.24\ \mathrm{N/A^2}$. This applies to uniform current distribution over the conductor cross-section. It is remarkably similar to the corresponding $k = 9.27\ \mathrm{N/A^2}$ obtained with the Lorentz-force formula.

It has been known for many years that the calculated Lorentz and Ampère forces on the sides of a *rectangular* circuit are almost identical ([8]). They differ, however, with regard to how and where they act on the conductor material. Lorentz forces are always transverse to the current stream lines and would invariably be in conflict with Newton's third law unless there exist balancing vacuum reaction forces. Ampère forces produce a strong tensile component along current stream lines and have all their reaction forces in the conductor material. For transfer of the electronic Lorentz force to the metal ions one has to rely on the work function at the conductor surface. This mechanism cannot explain Ampère tension ([7]). Ampère forces seem to act directly on the metal lattice. Furthermore, in the case of an unsymmetrical circuit, say a right-angled 3:4:5 triangle, the two formulae do not agree on the force exerted on any of the sides. Even more perplexing is the fact that the Lorentz force on the triangular circuit as a whole then no longer comes to zero ([9]).

Since the Ampère force interaction of every pair of material current elements obeys Newton's third law, the equality of action and reaction between any two parts of a metallic circuit follows automatically. It merely reaffirms that the Ampère electrodynamics is a Newtonian theory which ascribes no physical properties to empty space.

It will be appreciated that Ampère's law claims the pendulum is being pushed from the rear rather than being pulled from the front. Hence the hairpin legs may buckle and bend. They are expected to store some elastic energy which results in vibrations rather than linear mechanical momentum. More elastic energy is likely to be stored in the current rails F of fig. 2. Hence in the Ampère electrodynamics it seems quite normal that not all of the impulse P_i should be converted to momentum mu, unless the apparatus had been made infinitely rigid. In the initial experiments neither the pendulum nor the rails were reinforced with dielectric structures. This resulted in non-reproducible pendulum swings. Although the reinforcement had a considerable stabilizing effect, the equipment was by no means infinitely rigid. This is how the Ampère electrodynamics explains why mu was smaller than P_i.

In conclusion, it was found that Ampère's force law makes the electrodynamic impulse applied to the hairpin pendulum compatible with momentum conservation and it allows for some of the impulse to be converted to vibrations rather than linear momentum.

4. – Discussion.

The Lorentz force was specifically designed to account for the dynamics of electrons and ions in vacuum. It became a cornerstone of the special theory of relativity and with it achieved great triumphs in the field of particle acceleration. The Ampère formula, on the other hand, was found to be incapable of dealing with ionic motions outside the metal lattice. Since the Lorentz force correctly predicted the reaction forces between separate closed metallic circuits, it was thought to incorporate the Ampère electrodynamics and the latter topic virtually disappeared from modern writings on electromagnetism.

Curiously, the success of the Lorentz force with metallic circuits was due to a component of it which is also contained in the Ampère law. This can be shown as follows. The Lorentz interaction between two current elements is given by eq. (7) and (8). It is neither a repulsion nor an attraction and, therefore, has to be expressed by two equations. The triple vector product of these equations may be split into two parts:

$$\Delta \boldsymbol{F}_m = (\mu_0/4\pi)(i_m i_n/r^2_{m,n})(\mathrm{d}\boldsymbol{n}\,\mathrm{d}m \cos\alpha_m - \mathbf{1}_r\,\mathrm{d}m\cdot\mathrm{d}n\cos\varepsilon)\,, \tag{22}$$

$$\Delta \boldsymbol{F}_n = (\mu_0/4\pi)(i_m i_n/r^2_{m,n})(\mathrm{d}\boldsymbol{m}\,\mathrm{d}n \cos\alpha_n - \mathbf{1}_r\,\mathrm{d}m\cdot\mathrm{d}n\cos\varepsilon)\,. \tag{23}$$

Equations (22) and (23) have a common second term which stands for repulsion or attraction. WHITTAKER [10] and others have shown that Ampère law and eq. (22) and (23) give the same force exerted on a current element by a *separate* closed current and that this common force is

$$-\,(\mu_0/4\pi)\,i_m\,i_n\int_n(\cos\varepsilon/r_{m,n})\,\mathrm{d}m\cdot\mathrm{d}n\,.$$

[10] E. WHITTAKER: *A History of the Theories of the Aether and Electricity*, Vol. 1 (Nelson, London, 1951), p. 84.

It proves the first term of eqs. (22) and (23) vanishes under the integration around a closed circuit. Precisely this vanishing term is responsible for the disagreement of the Lorentz force with Newton's third law. Therefore, when computing only the reaction forces between closed metallic circuits, the Lorentz force theory sheds its relativistic trimmings and becomes a Newtonian theory. This does, however, not apply to the forces generated inside an isolated circuit, as, for example, the impulse pendulum circuit. Then only Ampère law remains compatible with Newton's third law and momentum conservation.

These findings have important practical consequences. The acceleration force of a railgun [5], for instance, can amount to many tons. If the recoil force resides in the rails, as required by Ampère law, then the rails must be designed to have the appropriate buckling strength. Should the recoil be felt by the field, they would never be subjected to buckling. An even more tantalizing situation arises when the total Lorentz force on an unsymmetrical isolated circuit, due to its own current, remains finite. Would a circuit of this kind be able to act as a space engine, using electricity from a solar battery and ejecting weightless electromagnetic mass from the spaceship?

● RIASSUNTO (*)

Esperimenti ampiamente quantitativi di Pappas hanno indicato che l'impulso assegnato ad un pendolo ad impulso elettrodinamico non è bilanciato da un cambio d'impulso uguale e opposto dell'energia del campo come richiesto dalla teoria speciale della relatività. Gli autori ahnno ripetuto l'esperimento di Pappas usando correnti di scarico da una riserva di capacitori che contiene una quantità nota di energia immagazzinata. Risulta che, per la conservazione del momento, l'energia del campo magnetico richiesta sarebbe da 1000 a 2000 volte grande come l'energia che è in realtà immagazzinata nei capacitori. Nella seconda parte del lavoro gli esperimenti ccl pendolo sono interpretati sulle basi delle leggi di forza di Ampère. Si mostra che la forza di Ampère esercitata sul pendolo è quasi esattamente uguale alla forza di Lorentz, ma essa si verifica in parti differenti del conduttore del pendolo. Inoltre, la forza di reazione di Ampère non sta nel campo, ma nella parte stazionaria del circuito che fornisce corrente al pendolo. Quindi nell'elettrodinamica di Ampère l'impulso è conservato in maniera definita. I risultati sperimentali e dell'analisi confermano il lavoro di Pappas. Da questo studio è emerso un nuovo ed importante fatto sperimentale. Il momento impartito al pendolo appare essere specificativamente piú piccolo dell'impulso meccanico calcolato fornito dalle leggi di forza di Lorentz ed Ampère. La distribuzione di forze di Ampère offre una spiegazione di questa osservazione in termini di distorsione elastica della struttura del pendolo. La distribuzione di forze di Lorentz potrebbe non produrre questa distorsione.

(*) *Traduzione a cura della Redazione.*

Электромагнитный импульсный маятник и сохранение импульса.

Резюме (*). — Количественные эксперименты Паппаса показывают, что импульс, переданный электромагнитному импульсному маятнику, не компенсируется равным и противоположным изменением импульса поля, как это следует из специальной теории относительности. Авторы повторили эксперимент Паппаса, используя разряд батареи конденсаторов, которая содержит известное количество запасенной энергии. Оказывается, что для сохранения импульса требуемая энергия магнитного поля должна в 1000-2000 раз превосходить энергию, фактически запасенную в конденсаторах. Во второй части статьи эксперименты с маятником интерпретируются в терминах закона для силы Ампера. Показывается, что сила Ампера, действующая на маятник, оказывается точно такой же, как сила Лоренца, но возникает в разных частях проводника маятника. Кроме того, сила реакции Ампера принадлежит не полю, а стационарной части контура, который подводит ток к маятнику. Следовательно, электродинамический импульс Ампера сохраняется. Экспериментальные и теоретические результаты подтверждают работу Паппаса. Из проведенных исследований получен новый и важный экспериментальный результат. Обнаружено, что импульс, переданный маятнику, оказывается значительно меньше, чем вычесленный механический импульс, определяемый законами для сил Ампера и Лоренца. Распределение силы Ампера предлагает объяснение этого факта с помощью упругой деформации структуры маятника. Распределение силы Лоренца не может создавать такой деформации.

(*) *Переведено редакцией.*