

# Effects of friction between the ball and strings in tennis

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## Abstract

When a tennis ball is incident at an oblique angle on a tennis racket, the ball slides or rolls along the strings before it rebounds. The dynamics of this interaction in a direction perpendicular to the string plane are determined by the coefficient of restitution (COR). In a direction parallel to the string plane, the dynamics depend on the coefficients of sliding ( $\mu_S$ ) and rolling ( $\mu_R$ ) friction, and also depend on the COR. For example, if  $\mu_S = 0$ , and if the ball impacts in the middle of the strings, then the ball will rebound with no change in its spin or parallel speed. Spaghetti strings, with a high value of  $\mu_S$ , are banned from competitive tennis since they can be used to impart excessive spin to the ball. It is shown that the most useful strings are those with  $\mu_S > 0.3$  and that the performance of the strings deteriorates sharply if  $\mu_S$  drops below about 0.3.

*Keywords:* tennis, strings, coefficient of friction

## Nomenclature

$D$	Distance to line of action of $N$ (Fig. 2)
$e$	Coefficient of restitution
$e_A$	Apparent coefficient of restitution
$F$	Friction force
$I$	Moment of inertia of ball about its CM ( $2mR^2/3$ )
$m$	Mass of ball (57 g)
$M$	Mass of racket
$N$	Normal reaction force
$R$	Radius of ball (32.5 mm)
$v_1$	Incident velocity of ball in racket frame (Fig. 1)
$v_2$	Rebound velocity of ball in racket frame (Fig. 1)
$v_{in}$	Incident velocity of ball in court frame (Fig. 1)
$v_{out}$	Rebound velocity of ball in court frame (Fig. 1)
$V_2$	Rebound velocity of racket head
$V_R$	Incident velocity of racket head (see Fig. 1)
$\beta$	Head tilt (see Fig. 1)
$\theta_1$	Angle between incident ball and string plane (Fig. 1)
$\theta_2$	Angle between rebounding ball and string plane (Fig. 1)

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$\theta_{\text{in}}$	Angle between incident ball and horizontal (Fig. 1)
$\theta_{\text{out}}$	Angle between rebounding ball and horizontal (Fig. 1)
$\theta_{\text{R}}$	Approach angle of racket head (see Fig. 1)
$\mu_{\text{R}}$	Coefficient of rolling friction
$\mu_{\text{S}}$	Coefficient of sliding friction
$\omega_1$	Angular velocity of incident ball
$\omega_2$	Angular velocity of rebounding ball

## Introduction

The impact of a tennis ball with the strings of a tennis racket is most commonly studied for cases where the ball is incident normally on the strings. It appears that only two previous theoretical studies have been made under conditions where the ball is incident at other angles (Groppe *et al.* 1983; Brody 1987). However, these authors did not consider the effects of varying the coefficient of friction between the ball and the strings. This is an important parameter since it determines the dynamics of the collision in a direction parallel to the string plane and it determines the amount of spin that can be imparted to the ball. It also appears that only one previous measurement has been made of the coefficient of friction between the ball and the strings of a tennis racket. Putnam & Baker (1984) found that  $\mu_{\text{S}} = 0.30$  for a  $45^\circ$  impact on a clamped racket, based on the uncertain assumption that the ball was sliding throughout impact. To the author's knowledge, no systematic measurements have been made of the rebound angle, spin or speed of a ball incident at various angles on the string plane. Bower & Sinclair (1999) also studied the rebound speed and angle for a  $45^\circ$  impact on a clamped racket, but did not measure the coefficient of friction.

In this paper, a simple theoretical model is used to investigate effects of varying the coefficient of friction between the ball and the strings. The motivation for this work was to determine whether such effects might explain some of the subtle differences between different strings reported by elite players. For example, players describe new strings as having a 'crisper' feel than old strings, and they describe old strings as being dead or

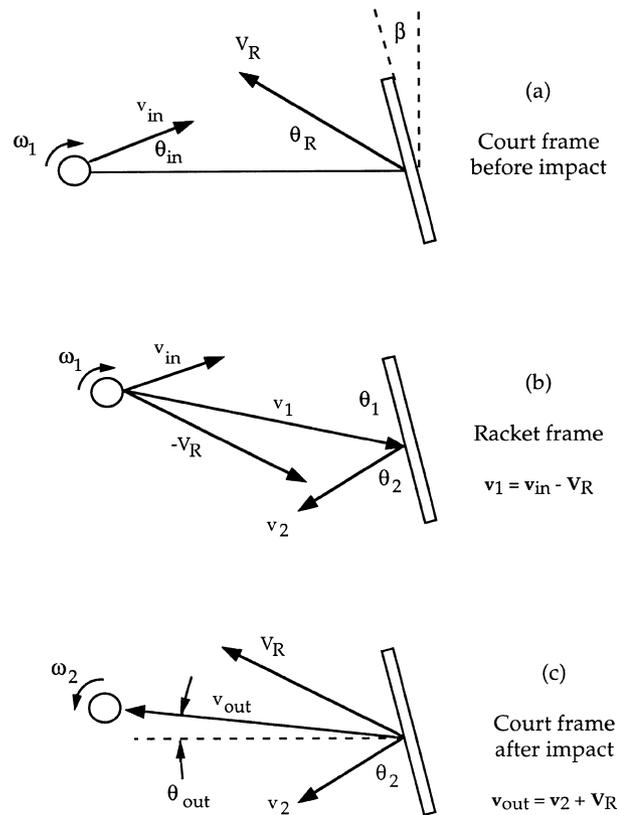
lifeless or lacking the power of new strings. These observations are not consistent with laboratory measurements of the rebound of a ball at normal incidence. If one drops a steel ball vertically onto the strings of a horizontal, head-clamped racket, there is almost no measurable difference in bounce height between new and old strings, at a 1% level, even if the racket contains strings that are 10 years old. Similarly, there is no obvious difference in bounce height between different types of string or different string tensions (Cross 2000a). These results show that the energy lost in the strings during an impact is negligible.

In the case of an impact with a tennis ball, racket power can be increased by decreasing the string tension so that the ball is not as severely deformed, in which case the energy loss in the ball is reduced. The question is, by how much? Detailed calculations for normal incidence show that the rebound speed of the ball increases by a negligible amount for the range of string tensions commonly used in tennis rackets. For a tennis ball incident on a head-clamped racket, the normal component of the rebound speed of the ball can be increased by about 7% when the string tension is halved (Bower & Sinclair 1999; Cross 2000b). However, the speed of a ball struck by a moving racket depends mainly on the speed of the racket head, and depends only weakly on string tension. For example, if the string tension is reduced from 60 lb (27.2 kg) to 50 lb (22.7 kg), the serve speed increases typically by only 0.7% and the rebound speed for a groundstroke increases typically by only 1.1% (Cross 2000b). Consequently, any differences between strings are more likely to be associated with differences in the transverse motion of the ball, as determined by the coefficient of friction between the ball and the strings.

## Theoretical model

The geometry of the problem addressed in this paper is shown in Fig. 1. A tennis ball of mass  $m$  is incident at velocity  $v_{in}$ , in the court frame of reference, on a racket of mass  $M$ . The ball is assumed to be moving upwards at an angle  $\theta_{in}$  to the horizontal when it strikes the racket. It is also assumed to be spinning, at angular velocity  $\omega_1$ , about a horizontal axis orientated transverse to the path of the ball. The point of impact on the racket head approaches the ball at velocity  $V_R$ , rising upwards at an angle  $\theta_R$  to the horizontal. The head of the racket is tilted forwards at an angle  $\beta$  to the vertical. It is assumed that the ball impacts at the centre of the strings, or at some other point along the long axis through the handle, so that there is no rotation about this axis as a result of the impact. In fact, the ball will tend to slide some distance along the strings before rebounding, but one can assume that there is no rotation about the long axis if the average position of the ball coincides with a point on the long axis. It is also assumed that  $v_{in}$  and  $V_R$  are in the same vertical plane, so that the rebound path of the ball is in the same vertical plane as the the incident path.

It is convenient to analyse the bounce off the strings in a reference frame where the racket is initially at rest. In the racket frame of reference, the ball is incident at velocity  $v_1 = v_{in} - V_R$ , at an angle  $\theta_1$  to the string plane, and it rebounds at velocity  $v_2$  and at an angle  $\theta_2$  to the string plane. The components of  $v_1$  are easily determined vectorially, from the diagram shown in Fig. 1(b). The heart of the problem is to calculate  $v_2$  and the rebound spin of the ball. One can then transform back to the court reference frame to obtain the rebound velocity  $v_{out} = v_2 + V_R$ , as shown in Fig. 1(c). The ball usually bounces off the court with topspin, but it is usually incident on the strings as backspin, as shown in Fig. 1(b), in which case  $\omega_1$  is taken to be negative. It is assumed that the ball rebounds at angular velocity  $\omega_2$ . The sign of  $\omega_2$  is taken as positive if the spin is reversed during the collision so that the ball rebounds with topspin. In fact, a ball bouncing off the court with topspin could be



**Figure 1** The impact of a tennis ball with a tennis racket (a) in the court reference frame, (b) transformed to the racket reference frame and (c) transformed back to the court frame.

incident on the strings with topspin, especially if the ball is rising sharply, the racket is moving more or less horizontally and the racket face is approximately vertical. This case requires a separate analysis, similar to that described below, but it will not be considered further in this paper since it is not as commonly encountered and it is not relevant to the calculations presented below.

In the racket frame of reference, the  $y$ -axis is taken normal to the strings and the  $x$ -axis is parallel to the string plane. The  $y$  and  $x$  axes both lie in the vertical plane. If the racket rebounds at velocity  $V_2$ , then the coefficient of restitution,  $e$ , for the collision is given by

$$e = \frac{v_{2y} + V_{2y}}{v_{1y}} \quad (1)$$

where the additional  $y$  subscripts denote velocity components in the  $y$  direction. The sign convention adopted in Eqn (1) and all following equations is to take all velocity components as positive, assuming that a ball incident in the negative  $y$  direction will rebound in the positive  $y$  direction, and that the racket will rebound in the negative  $y$  direction. Similarly, if a ball is incident in the positive  $x$  direction, then the ball and the racket are both assumed to rebound in the positive  $x$  direction.

The apparent coefficient of restitution,  $e_A$ , is defined by

$$e_A = v_{2y}/v_{1y} \quad (2)$$

Conservation of linear momentum in the  $y$  direction indicates that

$$e_A = (eM - m)/(m + M) \quad (3)$$

Conservation of linear momentum in the  $x$  direction gives

$$v_{2x} = v_{1x} - MV_{2x}/m \quad (4)$$

The rebound speed of the ball is completely determined if one can specify  $e_A$  and  $V_{2x}$ . The parameter  $e_A$  is easily measured from rebound experiments at normal incidence.  $e$  is more easily measured if the racket head is clamped, but it will differ from the result for a free racket if there is any energy loss due to frame vibrations. However,  $V_{2x}$ , as well as  $\omega_2$ , depends on the coefficient of friction,  $\mu$ , which has not previously been measured apart from the one uncertain value reported by Putnam & Baker (1984).

A trivial result is obtained if  $\mu = 0$ , since there is then no transverse force acting on  $M$  so  $V_x$  remains zero. There is also no change in the ball spin. Another simple result is obtained if  $F = \mu_S N$ , where  $N$  is the normal reaction force between the ball and the strings,  $F$  is the friction force in the  $x$  direction and  $\mu_S$  is the coefficient of sliding friction. In this case, the collision dynamics are described by

$$F = M dV_x/dt = -m dv_x/dt \quad (5)$$

$$N = M dV_y/dt = m dv_y/dt \quad (6)$$

and

$$FR = I d\omega/dt \quad (7)$$

where  $I = 2mR^2/3$  is the moment of inertia of the ball, taken as a thin spherical shell of radius  $R$ . Equations (5)–(7) can be integrated over the period of the impact to show that

$$V_{2x} = \mu_S V_{2y} \quad (8)$$

$$v_{2x} = v_{1x} - \mu_S(1 + e_A)v_{1y} \quad (9)$$

and

$$\omega_2 = \omega_1 + 1.5(v_{1x} - v_{2x})/R \quad (10)$$

Equation (9) shows that the change in the transverse speed of the ball depends on both  $\mu_S$  and  $e_A$ . This is because the friction force is proportional to both  $\mu_S$  and the normal reaction  $N$ . The change in the perpendicular speed of the ball depends only on  $e_A$ , from Eqn (2). Equations (7)–(10) are applicable only if the ball slides along the strings without rolling. This occurs in situations where  $\mu_S$  or  $\theta_1$  are relatively small. If  $\mu_S$  or  $\theta_1$  are sufficiently large, then  $v_x$  will decrease during the impact and  $\omega$  will increase up to a point where the ball will start to roll along the strings. If a player wishes to impart maximum spin to the ball, then the ball needs to go into a rolling mode, as described in the following Section. If  $\mu_S$  is not large enough for the particular stroke or grip the player chooses, then the ball will not go into a rolling mode and the spin imparted to the ball will be diminished. Consequently, it is in the player's interest to select strings with a large coefficient of friction. Given that the coefficient of friction of different strings has never been measured, it is not yet possible to specify whether this requires thin strings or thick strings, low tension or high tension, or any other variation in string properties or string pattern. It is known that  $\mu_S$  can be increased by using spaghetti strings (a second layer of strings threaded through the string plane to allow for a better grip on the ball), but this is now banned by the International Tennis Federation. Apart from that, and the fact that no 'protuberances' are allowed on the strings other than a vibration dampener located near the frame, there are no rules in tennis to limit the maximum value of  $\mu_S$ .

## Rolling friction

If a ball rolls horizontally at speed  $v_x$ , and the surface on which it rolls moves horizontally at speed  $V_x$ , then the condition for rolling is that

$$v_x - V_x = R\omega \quad (11)$$

The point or points of contact of the ball on the string are then motionless with respect to the string. The interaction with the strings can be modelled using a similar approach to that adopted by Brody (1984) to determine the rebound speed and angle for a bounce off the court surface. However, the ball and the strings are both deformed significantly during an impact with the strings, in which case the coefficient of rolling friction is not entirely negligible as assumed by Brody. When a rolling ball decelerates as a result of surface friction, the normal reaction force,  $N$ , on the ball acts at a distance  $D$  ahead of the centre of mass, as shown in Fig. 2. When  $D$  is finite,  $N$  provides a torque that opposes the torque due to  $F$ , in which case  $v_x$  and  $\omega$  can both decrease at a rate that maintains the rolling condition. The resulting torque on the ball about its centre of mass is then

$$FR - ND = I d\omega/dt \quad (12)$$

where  $F = \mu_R N$  is the horizontal friction force on the ball and  $\mu_R$  is the coefficient of rolling friction. Differentiating Eq (11), and using Eqs (5) and (12), one finds that

$$D = R\mu_R \left[ 1 + \frac{2}{3} \left( 1 + \frac{m}{M} \right) \right] \quad (13)$$

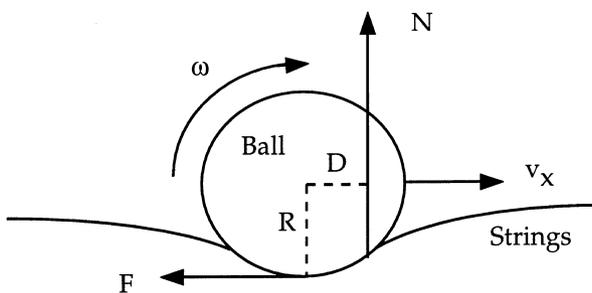


Figure 2 Forces acting on a rolling ball.

$\mu_R$  cannot be larger than about 0.6, otherwise  $D$  would be larger than  $R$ . As described below,  $\mu_R$  is typically about 0.05 for small deformations of the ball and string plane.

A ball that is incident on the strings with no spin or with backspin initially slides along the strings. It will commence to roll if  $v_x$  decreases to a value  $v_{x0}$  given, from Eqs (5), (7) and (11) by

$$v_{x0} = \frac{R\omega_1 + (1.5 + m/M)v_{1x}}{2.5 + m/M} \quad (14)$$

The ball will therefore roll if  $v_{2x} < v_{x0}$  where  $v_{2x}$  is given by Eq (9). If this condition is satisfied, then Eq (9) is invalid. In order to determine  $v_{2x}$  correctly, the differential equations describing the motion of the ball and racket must be integrated over two separate periods, to account for sliding while  $v_x > v_{x0}$  and then rolling when  $v_x$  decreases to  $v_{x0}$ . In this case, it can be shown that

$$v_{2x} = v_{1x} - [(1 - \mu_R/\mu_S)(v_{1x} - v_{x0}) + \mu_R(1 + e_A)v_{1y}] \quad (15)$$

Equations (14) and (15) reduce to those given by Brody (1984) when  $M = \infty$ ,  $\omega_1 = 0$  and  $\mu_R = 0$ . Then  $v_{2x} = v_{x0} = 0.6v_{1x}$ . Equation (15) reduces to Eq (9) if  $\mu_R = \mu_S$ . If a ball starts rolling before it rebounds, then

$$\omega_2 = \frac{v_{2x} - V_{2x}}{R} = \frac{v_{2x}}{R} - \frac{m(v_{1x} - v_{2x})}{MR} \quad (16)$$

If  $\mu_R$  is large enough, Eq (15) indicates that  $v_{2x}$  can be negative when the ball is incident with positive  $v_{1x}$ . When a rolling ball is slowed by friction, it normally comes to rest without reversing its motion. Consequently, it is assumed that  $v_{2x}$  cannot be negative, and that the friction force drops to zero if  $v_x = V_x$ . Under these conditions,  $\omega_2 = 0$  and

$$v_{2x} = \frac{v_{1x}}{1 + M/m} \quad (17)$$

## Experimental data

Several experiments were undertaken to determine the coefficients of sliding and rolling friction, at relatively low values of the normal reaction force. Using masses up to 10 kg held lightly on top of a

new tennis ball to prevent the mass toppling, and a spring balance to measure the force needed to drag the ball at constant speed across the strings of a racket, it was found that  $\mu_S$  varied from 0.27 to 0.42 for five different rackets. By comparison, it was found that  $\mu_S = 0.8$  for a tennis ball sliding on rubber, 0.54 when sliding on the Sydney Olympic Games tennis courts, 0.25 when sliding on a low pile carpet and 0.13 when sliding on a smooth wood surface.

A separate experiment was set up to measure  $\mu_R$ . A horizontal platform containing masses up to 74 kg was placed on top of four balls resting on the strings, and a horizontal force was applied to the platform so that the balls could roll on the strings underneath the platform. The 74 kg load, shared equally by the four balls, represents about a quarter of the force on a single ball for a typical ground-stroke. This gave  $\mu_R \approx 0.035$  for loads less than 10 kg and  $\mu_R = 0.05 \pm 0.01$  for loads between 20 and 74 kg. The ball diameter decreased by up to 14 mm in the rolling experiments, indicating that  $\mu_R$  is not a strong function of ball deformation. The coefficients obtained therefore provide a useful guide for calculation purposes.

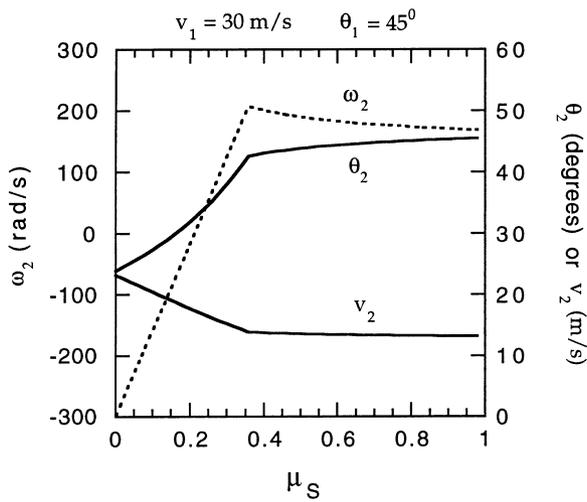
Other parameters used in the calculations presented below are as follows:  $e = 0.9$ ,  $e_A = 0.443$ ,  $M = 0.18$  kg,  $m = 57$  g,  $R = 32.5$  mm,  $\omega_1 = -300$  rad s<sup>-1</sup>,  $v_{in} = 10$  m s<sup>-1</sup> and  $\theta_{in} = 0$ . The value of  $M$  represents the effective mass of the racket at the impact point, which is typically about half the actual mass for an impact near the centre of the strings. The effective mass is equal to the actual mass only if the ball impacts at the centre of mass of the racket. At any other impact point, the effective mass is less than the actual mass due to rotation of the racket about an axis through the centre of mass. In the above analysis it was assumed that the effective mass is the same in the  $x$  and  $y$  directions. In fact,  $M$  is slightly different in the two directions since the moment of inertia of a racket about one axis is typically about 10% larger than that about the other axis. The added complexity of this effect was ignored as being of minor significance, particularly since it would alter terms such as  $2.5 + m/M$  in Eqs (14), (18) and (19) by a relatively small amount.

A value of  $\mu_R = 0.05$  was used in all calculations since it was found that the ball rebound properties are not particularly sensitive to  $\mu_R$ , at least when  $\mu_R < 0.1$ . The calculations are more sensitive to  $\omega_1$ ,  $v_{in}$  and  $\theta_{in}$ , but the essential features and main conclusions of this paper do not depend on the choice of these parameters. The latter parameters were chosen to be typical of those in a forehand or backhand stroke (Elliott & Christmass 1995; Takahashi *et al.* 1996). The parameters  $e$  and  $e_A$  are also typical for such an impact (Cross 2000b).

### Rebound results in the racket reference frame

Results of the model calculations, in the racket frame of reference, are shown in Figs 3 and 4. From a player's point of view, results in the court frame are more important, but results in the racket frame are easier to interpret. Furthermore, laboratory measurements of a rebounding ball would be best made in a reference frame where the racket is at rest with the head firmly clamped (in which case  $M = \infty$ ). As shown in Fig. 3,  $\omega_2$  increases and  $v_2$  decreases as  $\mu_S$  increases from zero, up to a point where the ball starts rolling before it rebounds. Up to this point, the effect of increasing  $\mu_S$  is to increase the friction force on the ball, thereby decreasing  $v_{2x}$  and increasing the torque on the ball. The normal component of the ball rebound velocity,  $v_{2y}$ , is independent of  $\mu_S$ . Consequently, the rebound angle,  $\theta_2 = \tan^{-1}(v_{2y}/v_{2x})$  increases as  $\mu_S$  increases. This result is analogous to the behaviour of the ball when it bounces on the court surface. On a fast grass surface, the ball skids and rebounds at a low angle. On clay, the ball kicks up. Once the ball starts to roll, there is a slight decrease in  $v_2$  and  $\omega_2$  as  $\mu_S$  increases, resulting from the relatively small coefficient of rolling friction.

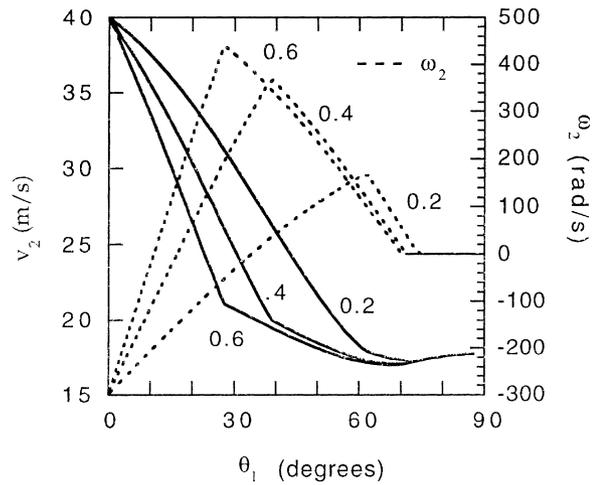
In terms of imparting topspin to the ball,  $\omega_2$  can always be increased by increasing  $v_1$ , for example by hitting the ball harder. If  $\mu_S$  is relatively large, then  $\omega_2$  can also be increased by decreasing  $\theta_1$ , which can be achieved by hitting the ball closer to glancing incidence. This strategy



**Figure 3** Model calculations, in the racket frame of reference, showing the rebound speed ( $v_2$ ), rebound angle ( $\theta_2$ ) and angular velocity ( $\omega_2$ ) vs.  $\mu_S$ . The ball is incident at speed  $v_1 = 30 \text{ m s}^{-1}$  at an angle  $\theta_1 = 45^\circ$  to the string plane and with initial spin  $\omega_1 = -300 \text{ rad s}^{-1}$ .

is often used in table tennis where  $\mu_S$  is probably quite high due to the dimpled rubber bat. However, if  $\mu_S$  is relatively small, this strategy will reduce  $\omega_2$ . For any given value of  $\mu_S$  there is an optimum value of  $\theta_1$  which will maximize  $\omega_2$ . This effect is shown in Fig. 4 for cases where  $v_1 = 40 \text{ m s}^{-1}$  and where  $\mu_S = 0.2, 0.4$  or  $0.6$ . Each of these curves is characterized by a region at low  $\theta_1$  where the ball slides across the strings without rolling, and a region with  $\theta_1 > 70^\circ$  where the ball rolls to a stop before it rebounds.

The  $\mu_S = 0.4$  curve in Fig. 4 shows that if a player wishes to maximize topspin, then the optimum angle of incidence is  $\theta_1 = 40^\circ$ , giving  $\omega_2 = 370 \text{ rad s}^{-1}$ . Similarly, if  $\mu_S$  is increased to say  $0.6$ , then the optimum angle of incidence is  $\theta_1 = 28^\circ$ , giving  $\omega_2 = 435 \text{ rad s}^{-1}$ . However, if  $\mu_S$  decreases to say  $0.2$ , and the angle of incidence is held at  $\theta_1 = 40^\circ$  then  $\omega_2 = 40 \text{ rad s}^{-1}$ . This is perhaps an extreme example, but it highlights a remarkable sensitivity of ball spin to the coefficient of friction. The outgoing ball speed,  $v_2$ , is also sensitive to  $\mu_S$ , but the effect would not be as noticeable to a player since the rebound speed in the court frame depends more on the racket speed than on  $v_2$ .



**Figure 4** Model calculations, in the racket frame of reference, showing the rebound speed ( $v_2$ , solid curves) and angular velocity ( $\omega_2$ , dashed curves) vs.  $\theta_1$  when  $\mu_S = 0.2, 0.4$  or  $0.6$  as labelled. The ball is incident at speed  $v_1 = 40 \text{ m s}^{-1}$  and with  $\omega_1 = -300 \text{ rad s}^{-1}$ .

The results in Figs 3 and 4 can be regarded as representative of conditions encountered for groundstrokes when  $\omega_1 = -300 \text{ rad s}^{-1}$ . The general behaviour does not change when  $\omega_1$  is varied, but the effects of changing  $\omega_1$  can be described analytically as follows. The maximum value of  $\omega_2$  is obtained when the ball starts to roll at the end of the impact period. Under these conditions,  $v_{2x}$  in Eq (9) is equal to  $v_{x0}$  as given by Eq (14). By equating these results, it is easy to show that  $\omega_2$  is maximized when

$$\mu_S = \frac{1 - R\omega_1/v_{1x}}{(1 + e_A)(2.5 + m/M)\tan \theta_1} \quad (18)$$

From Eqs (14) and (16), the maximum value of  $\omega_2$  is given by

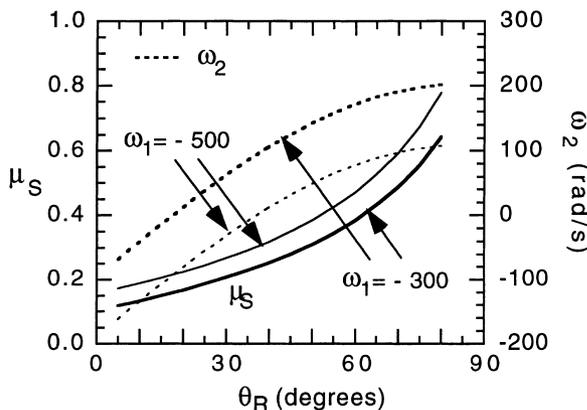
$$\omega_2 = \frac{(1 + m/M)\omega_1 + 1.5v_{1x}/R}{2.5 + m/M} \quad (19)$$

For example, in Fig. 3, where  $\omega_1 = -300 \text{ rad s}^{-1}$ ,  $\theta_1 = 45^\circ$  and  $v_{1x} = 21.21 \text{ m s}^{-1}$ ,  $\omega_2$  has a maximum value of  $207 \text{ rad s}^{-1}$  when  $\mu_S = 0.36$ . If  $\omega_1 = -500 \text{ rad s}^{-1}$ , then the maximum value of  $\omega_2$  is  $114 \text{ rad s}^{-1}$ , when  $\mu_S = 0.43$ .

**Rebound results in the court reference frame**

Calculations in the court reference frame, with  $V_R = 20 \text{ m s}^{-1}$ , are shown in Figs 5–7 to illustrate the effects of varying the orientation of the racket. Apart from varying the racket speed, the only other choices available to the player are to vary  $\theta_R$  or  $\beta$ . The player might also vary the return angle across court, but it is assumed in this paper that the incident and rebound paths of the ball are in the same vertical plane.

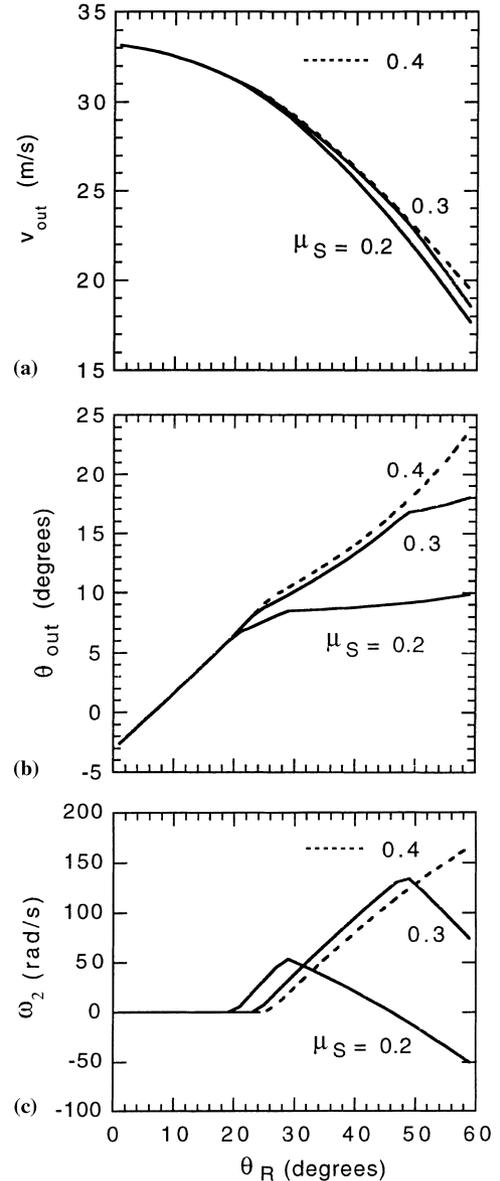
Figure 5 shows, for any given  $\theta_R$ , the value of  $\mu_S$  required for the ball to enter a rolling mode. The theoretical curves in Fig. 5 are solutions of Eqs (18) and (19) in the court reference frame, when  $v_{in} = 10 \text{ m s}^{-1}$ ,  $\theta_{in} = 0$ ,  $V_R = 20 \text{ m s}^{-1}$ ,  $\beta = 5^\circ$ , and when  $\omega_1 = -300$  or  $-500 \text{ rad s}^{-1}$ . These parameters are typical of a groundstroke. If  $\mu_S$  is equal to the value given by Eq (18) then the ball will rebound with maximum spin for that value of  $\theta_R$ . A larger value of  $\mu_S$  will cause the ball to rebound with marginally smaller spin due to the effect of rolling friction. A lower value of  $\mu_S$  means that the strings have insufficient friction for the ball to start rolling, and hence the ball will rebound with reduced spin. Provided that  $\mu_S$  is large enough, the player can increase the spin by swinging the racket at a larger angle upwards. However,  $\mu_S$  is typically about 0.3–0.4, in which



**Figure 5** Solutions of Eqs (18) and (19) in the court reference frame with  $v_{in} = 10 \text{ m s}^{-1}$ ,  $\theta_{in} = 0$ ,  $V_R = 20 \text{ m s}^{-1}$ ,  $\beta = 5^\circ$ , and with  $\omega_1 = -300$  or  $-500 \text{ rad s}^{-1}$ .

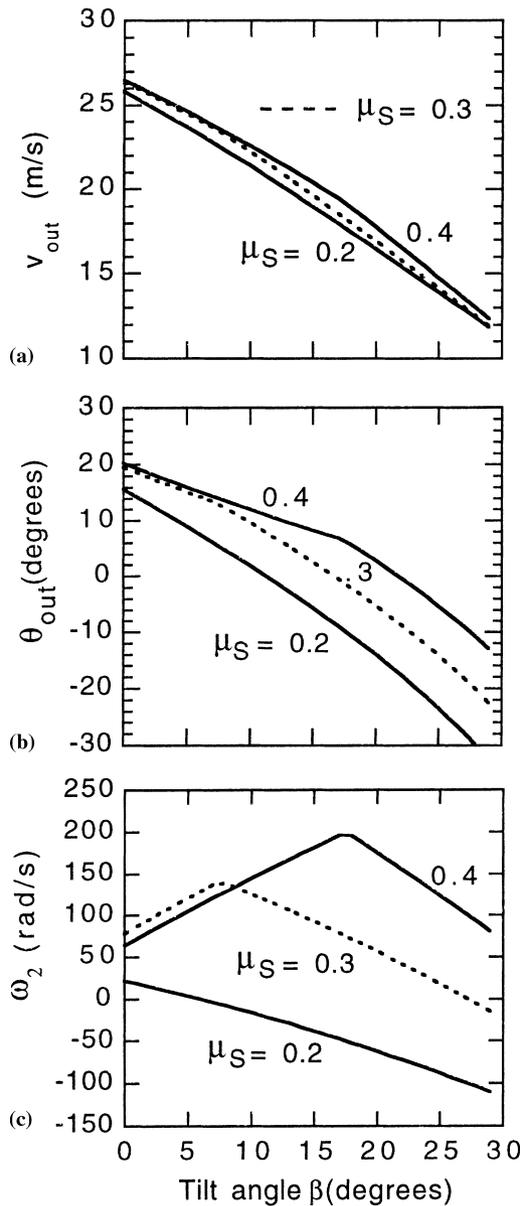
case this strategy will work only up to a certain limiting value of  $\theta_R$  as shown in Fig. 6(c).

Figure 6 shows court frame results where the head is tilted slightly forwards, at  $\beta = 5^\circ$ , while  $\theta_R$  is varied from  $0^\circ$  to  $60^\circ$  to examine the effects on  $v_{out}$  (Fig. 6a),  $\theta_{out}$  (Fig. 6b) and  $\omega_2$  (Fig. 6c). Each figure shows results with  $\mu_S = 0.2, 0.3$  and  $0.4$ . Figure 6(c) shows that  $\omega_2$  passes through a



**Figure 6** Court frame results showing (a)  $v_{out}$ , (b)  $\theta_{out}$  and (c)  $\omega_2$  vs.  $\theta_R$  when  $v_{in} = 10 \text{ m s}^{-1}$ ,  $\theta_{in} = 0$ ,  $V_R = 20 \text{ m s}^{-1}$  and  $\beta = 5^\circ$ .

maximum as  $\theta_R$  is increased and that the maximum increases as  $\mu_S$  increases. This is similar to the result in Fig. 4, but the figures are reversed in the sense that an increase in  $\theta_R$  corresponds to a decrease in  $\theta_1$ . For example, if  $\theta_R < 20^\circ$  then  $\theta_1 > 70^\circ$  and the ball rolls to a stop before it rebounds, so  $\omega_2 = 0$ .



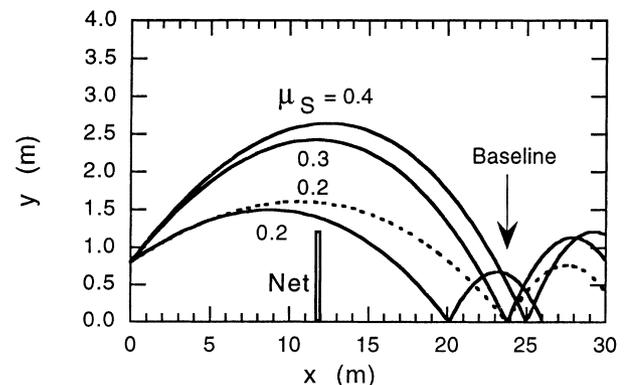
**Figure 7** Court frame results showing (a)  $v_{out}$ , (b)  $\theta_{out}$  and (c)  $\omega_2$  vs.  $\beta$  when  $v_{in} = 10 \text{ m s}^{-1}$ ,  $\theta_{in} = 0$ ,  $V_R = 20 \text{ m s}^{-1}$  and  $\theta_R = 45^\circ$ .

Figure 7 shows a similar set of results where the racket rises upwards at an angle  $\theta_R = 45^\circ$  and the head is tilted forwards by an angle  $\beta$  which is varied from 0 to  $30^\circ$  to examine the effects on  $v_{out}$  (Fig. 7a),  $\theta_{out}$  (Fig. 7b) and  $\omega_2$  (Fig. 7c). When  $\mu_S = 0.2$ , the ball slides across the strings without rolling, regardless of the angle  $\beta$ .

### Ball trajectories

The effects of friction between the ball and strings are best illustrated, from a practical point of view, by considering the effects on the ball trajectory. For this purpose one can consider a typical ground-stroke where a ball is returned from a point directly above the baseline so that it lands close to the opponent's baseline. The perpendicular distance between the two baselines is 78 feet (23.774 m). The net is located 39 feet (11.887 m) from each baseline and is 3 feet (0.914 m) high in the centre of the court. For purposes of illustration, it is assumed that the ball is incident at  $v_{in} = 10 \text{ m s}^{-1}$ , with  $\theta_{in} = 0$ ,  $\omega_1 = -300 \text{ rad s}^{-1}$  and it impacts the centre of the strings at a point 0.8 m above the baseline. The resulting trajectory of the ball depends on  $\mu_S$  and  $\mu_R$  as well as the player's choice of  $V_R$ ,  $\theta_R$  and  $\beta$ . Equations used to compute the trajectory are given in the Appendix.

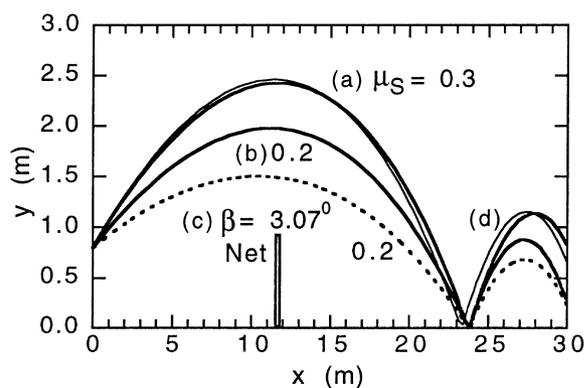
Typical ball trajectories are shown in Fig. 8. If  $\theta_R = 45^\circ$ ,  $\beta = 5^\circ$ ,  $\mu_S = 0.3$  and  $V_R = 24.15 \text{ m s}^{-1}$ ,



**Figure 8** Ball trajectories with  $V_R = 24.15 \text{ m s}^{-1}$  when  $\mu_S = 0.2, 0.3$  or  $0.4$  (solid lines). The dashed curve is for  $V_R = 32.3 \text{ m s}^{-1}$  and  $\mu_S = 0.2$ . The baseline is at  $x = 23.77 \text{ m}$ .

then the ball will land on the opponent's baseline. If  $\theta_R$ ,  $\beta$  and  $V_R$  are unaltered, but  $\mu_S = 0.4$ , then the ball will land 1.23 m beyond the baseline. If  $\mu_S$  drops to 0.2, then the ball will land 3.7 m short of the baseline since the ball rebounds from the racket at a relatively low value of  $\theta_{out}$ . In the latter case the player could correct the stroke, for example by increasing the speed of the racket to  $V_R = 32.3 \text{ m s}^{-1}$ , in order for the ball to land on the opponent's baseline, as shown by the dashed trajectory in Fig. 8. Alternatively, the player could correct the trajectory by maintaining the same racket speed (i.e.  $24.15 \text{ m s}^{-1}$ ) and either decreasing  $\theta_R$  to  $25.5^\circ$  or decreasing  $\beta$  to  $3.07^\circ$ . These corrected trajectories are shown in Fig. 9. The corrected trajectories are flatter, and the ball does not bounce as steeply off the court as when  $\mu_S = 0.3$ , since the ball rebounds off the strings with lower values of  $\theta_{out}$  and  $\omega_2$ . In Figs 8 and 9, the bounce off the court was determined using the same procedure as that used to determine the bounce off the strings, but the bounce parameters were changed to  $e = 0.745$ ,  $\mu_S = 0.6$ ,  $\mu_R = 0$  and  $M = \infty$ .

Figure 9(d) (thin line) shows the effect on curve (a) of reducing the coefficient of restitution,  $e$ , from 0.9 to 0.85, corresponding to a reduction in  $e_A$  from 0.443 to 0.405. All other results in this paper are computed with  $e = 0.9$ . A small reduction in  $e$  occurs when the string tension is increased so that



**Figure 9** Ball trajectories with  $V_R = 24.15 \text{ m s}^{-1}$  when (a)  $\mu_S = 0.3$ ,  $\theta_R = 45^\circ$ ,  $\beta = 5$ , (b)  $\mu_S = 0.2$ ,  $\theta_R = 25.5^\circ$ ,  $\beta = 5$  and (c)  $\mu_S = 0.2$ ,  $\theta_R = 45^\circ$ ,  $\beta = 3.07^\circ$ . The ball lands on the baseline at  $x = 23.77 \text{ m}$  in each case. The thin line curve (d) is for the same parameters as (a) but  $e$  is reduced to 0.85.

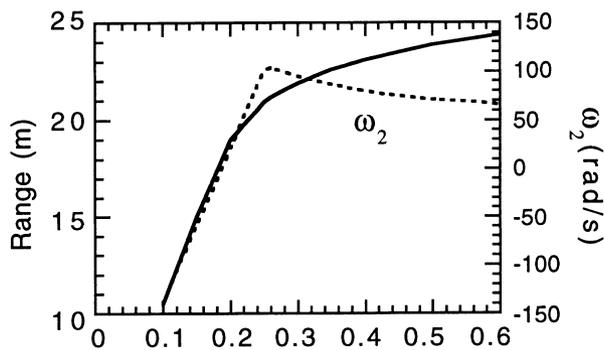
the strings store a smaller fraction of the impact energy and the ball dissipates a larger fraction. Despite the relatively large change in  $e_A$ , there is an almost negligible change in the ball trajectory since the outgoing speed of the ball is determined primarily by the racket speed rather than the incident speed of the ball.

There are several ways to interpret these results, but the player is likely to conclude, if  $\mu_S$  drops below 0.3, that the strings are not as responsive or that the strings have lost power. This conclusion will be reinforced if the player needs to hit the ball much harder to achieve the same range as strings with  $\mu_S = 0.3$ . In terms of the dynamics of the impact, Fig. 7(a) shows that the reduction in  $v_{out}$  is negligible when  $\mu_S$  decreases. The main effect is that both  $\theta_{out}$  and  $\omega_2$  decrease when  $\mu_S$  decreases.

Elite players have an ability to hit the ball accurately to any desired position of the court, and with any desired top or backspin, by changing the grip and racket trajectory as required. This is done on the basis of previous experience and familiarity with the racket and strings that they are using. Consequently, if a player were to find that the ball lands consistently short of the expected position, it is highly likely that the player would change rackets or strings rather than change the grip or racket trajectory. Changes in  $\mu_S$  result in relatively small changes in ball spin and trajectory when  $\mu_S$  is larger than about 0.3, in which case an elite player should be able to compensate for these changes with sufficient practice. However, when  $\mu_S$  drops below about 0.3, the changes in ball spin and trajectory are relatively large and are not within a player's normal expectations. These effects are shown more clearly in Fig. 10, where the range and spin of the ball is plotted as a function of  $\mu_S$  for similar initial conditions to those in Fig. 8. The range is defined as the distance travelled by the ball in the  $x$  direction between  $x = 0$  and the initial impact point on the court.

## Discussion

The results presented above show that the coefficient of sliding friction plays a dominant role in the



**Figure 10** Range and  $\omega_2$  vs.  $\mu_S$  for ball trajectories with  $v_{in} = 10 \text{ m s}^{-1}$ ,  $\theta_{in} = 0$ ,  $\omega_1 = -300 \text{ rad s}^{-1}$ ,  $V_R = 20 \text{ m s}^{-1}$ ,  $\theta_R = 40^\circ$  and  $\beta = 5^\circ$ . The racket strikes the ball 0.8 m above the court surface.

performance of tennis strings. A cursory experimental investigation of the variations in  $\mu_S$  between different rackets showed small differences, but this needs to be investigated further, especially since the measurements of  $\mu_S$  were made under conditions that are not representative of those normally encountered during a high speed impact. Ideally, the coefficients of friction should be measured under impact conditions where a ball is fired at high speed at various angles of incidence onto the string plane. One can speculate that  $\mu_S$  will depend on a number of factors such as string type, tension, diameter and number of strings. It may also depend on the extent of lateral movement of the strings during an impact, and this is likely to depend on string tension and on the coefficient of friction between the strings. Friction between the strings will depend on the amount of wear and tear, and on the extent to which the strings are 'bedded in' due to the formation of notches where the strings overlap. The condition of the ball may also have a significant effect on  $\mu_S$ .

Putnam & Baker (1984) found that  $\mu_S$  does not depend on the string pattern, but this result was based on the assumption that sliding occurred throughout the impact. The need for such an assumption highlights a difficulty with this type of measurement. If the conditions are such that the ball starts to roll before the end of the impact period, then it can be seen from Fig. 3 that measurements of  $\omega_2$ ,  $\theta_2$  and  $v_2$  are all insensitive

to  $\mu_S$  (in Fig. 3, rolling occurs when  $\mu_S > 0.36$ ). Inspection of the results shown in Fig. 4 shows that it is necessary to choose a low angle of incidence, where  $\theta_1 < 30^\circ$ , in order to obtain a reliable measurement of  $\mu_S$ . For the same reason, the ITF specifies an angle of incidence of  $16^\circ$  for impact tests of a ball on a court surface when measuring the pace of the court, where 'pace' is defined as  $1 - \mu_S$ . Since Putnam and Baker chose an incident angle  $\theta_1 = 45^\circ$ , it is possible to interpret their results to mean that conventional and diagonal strings both have a value of  $\mu_S$  that is 0.3 or larger.

An additional effect, that can be regarded as an effective change in  $\mu_S$ , arises when a ball impacts the strings off-centre. As described by Cross (2000a), the string plane will then deform asymmetrically, resulting in a transverse force acting on the ball towards the centre of the strings. Even if the ball impacts in the centre of the strings, it will slide or roll across the strings by a significant distance during the impact. For example, if the relative speed between the ball and strings is  $30 \text{ m s}^{-1}$  and the ball is incident at  $45^\circ$  to the string plane, then the ball has a velocity component of  $21 \text{ m s}^{-1}$  in a direction parallel to the strings. It would travel a distance of about 70 mm across the string plane during a typical 5 ms impact, rebounding with a parallel velocity component somewhat less than  $21 \text{ m s}^{-1}$ . For part of this time, there is a restoring force acting towards the centre of the strings, which could accelerate or decelerate the ball depending on the initial contact location. The magnitude of the transverse force is such that  $\mu_S$  is effectively increased or decreased by about 0.1 (Cross 2000a). However, this effect cannot be considered in isolation, since there will also be a tendency for the racket to rotate about its long axis whenever the ball impacts towards the top or bottom edge of the racket (Groppel *et al.* 1987; Brody 1997; Cross 2000b).

## Conclusion

Much has been written in popular tennis magazines about the properties of tennis strings and the significance of factors such as elasticity, string tension, tension loss with time, string diameter,

string type, string pattern, etc., but the significance of the coefficient of friction has not previously received much attention, apart from the study by Putnam & Baker (1984) and the fact that spaghetti strings are banned by the ITF. In this paper, it has been shown that the coefficient of friction plays a dominant role in determining the interaction between the ball and the strings. All tennis strings are able to store elastic energy efficiently, without significant energy loss, regardless of string tension or type or previous history. The coefficient of restitution can be increased slightly, with a consequent increase in racket power, by decreasing the string plane stiffness so that the impact is 'softer' and so that the ball dissipates a smaller fraction of the total energy. However, the resulting increase in ball speed is typically less than 1% and would not be apparent to most players.

It appears more likely that any change in string performance, related to a change in string type or tension or general wear and tear, will be associated with a change in the coefficient of friction between the ball and the strings. Such a direct connection was not established in this paper, since no experiments were undertaken to see if there is any correlation between string type or tension and the coefficient of friction. However, it was shown that a small decrease in the coefficient of sliding friction below about 0.3 results in a large change in the rebound angle of the ball. This is accompanied by a slight change in rebound speed of the ball, but players are more likely to perceive these effects as a decrease in racket power since the ball lands short of the target and since the amount of topspin imparted to the ball is reduced. If  $\mu_S$  remains larger than about 0.3, a small change in  $\mu_S$  results in a relatively small change in rebound angle and spin. However, preliminary measurements indicate that  $\mu_S$  is typically about 0.3, which is somewhat marginal in regard to string performance.

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## Appendix

The trajectory of a tennis ball through the air depends on the spin of the ball and can be calculated in terms of lift and drag coefficients, as described by Stepanek (1988), De Mestre (1990) or Mehta (1985). The drag force on a spherical ball is given by

$$F_D = C_D(\rho v^2 A/2) \quad (A1)$$

where  $\rho = 1.21 \text{ kg m}^{-3}$  is the density of air,  $A = \pi R^2$  is the cross-sectional area of a ball of radius  $R$ ,  $v$  is the relative speed of the ball and the air, and  $C_D$  is the drag coefficient. The trajectory of a spinning ball of mass  $m$  can be described by the relations

$$dv_x/dt = -kv(C_D v_x + C_L v_y) \quad (\text{A2})$$

$$dv_y/dt = -kv(C_D v_y - C_L v_x) - g \quad (\text{A3})$$

where  $x$  and  $y$  are the horizontal and vertical coordinates, respectively,  $v$  is the ball speed,  $v_x$  and  $v_y$  are the components of  $v$ ,  $C_L$  is the lift coefficient and

$$k = \pi \rho R^2 / (2m) \quad (\text{A4})$$

In Eqs (A2) and (A3) it is assumed that the lift is upwards or that the ball has backspin. The sign of  $C_L$  changes if the ball has topspin since the 'lift' or Magnus force is then directed towards the ground. The drag and lift coefficients of a spinning tennis ball have been measured by Stepanek (1988) at ball

speeds up to  $28 \text{ m s}^{-1}$  and at angular speeds up to  $340 \text{ rad s}^{-1}$ . Within this range, Stepanek found that

$$C_D = 0.508 + \frac{1}{[22.503 + 4.196(v_{\text{spin}}/v)^{-5/2}]^{2/5}} \quad (\text{A5})$$

$$C_L = \frac{1}{2.022 + 0.981(v/v_{\text{spin}})} \quad (\text{A6})$$

where  $v_{\text{spin}} = R\omega$  is the peripheral speed of the ball and  $\omega$  is the angular speed about a horizontal axis perpendicular to the path of the ball. Stepanek used a factor 2.202 in his Eq (10), but the correct factor is 2.022 as shown in his Fig. 4. It is assumed that the lift and drag coefficients are independent of the Reynold's number for all ball speeds of interest and that  $C_D = 0.508$  and  $C_L = 0$  when  $v_{\text{spin}} = 0$ . Recent measurements of  $C_D$  using wind tunnels, conducted independently by the author and by Haake *et al.* (2000), show that  $C_D$  remains constant even at ball speeds up to  $60 \text{ m s}^{-1}$ .