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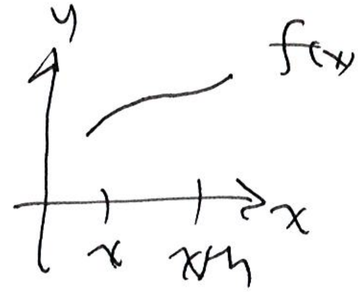
Math 1496 Calc I

1-5

Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Ex $f(x) = 2x$

$$\lim_{h \to 0} \frac{2(x+h) - 2x}{h} = \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{2x}}{h}$$

$$\lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = 2$$

so $f'(x) = 2$

Notation $\frac{d}{dx}(2x) = 2$

Some more derivative

$f(x) = e^x$ $\quad \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} e^x \cdot \frac{e^h - 1}{h}$

$$e^x \lim_{h \to 0} \frac{e^h - 1}{h} = e^x \cdot (1)$$

$$\boxed{\text{so } \frac{d e^x}{dx} = e^x}$$

$f(x) = \sin x$

$$f' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h \to 0} \sin x \left(\frac{\cos h - 1}{h} \right) + \frac{\sin h}{h} \cdot \cos x$$

$$= \sin x \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin x (0) + \cos x (1)$$

so $\boxed{\frac{d}{dx} \sin x = \cos x}$

$f(x) = \cos x$

$$f' = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

Ex $\frac{d}{dx}(e^x \sin x)$

$f = e^x \quad g = \sin x$

$f' = e^x \quad g' = \cos x$

$f'g + fg' = e^x \sin x + e^x \cos x$

on the run

$\frac{d}{dx}(x^3 e^x)$

$3x^2 \cdot e^x + x^3 e^x$

more terms

$\frac{d}{dx}(x e^x \sin x)$

$= 1(e^x \sin x) + x(e^x)\sin x + x \cdot e^x \cdot (\cos x)$

$$= \lim_{h \to 0} \cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h}$$

$$= \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \cos x \, (0) - \sin x \, (1)$$

So $\boxed{\frac{d}{dx}(\cos x) = -\sin x}$

Rules

(i) $\frac{dc}{dx} = 0$ (ii) $\frac{dx^n}{dx} = n x^{n-1}$

(iii) $\frac{d(cf)}{dx} = c\frac{df}{dx}$ (iv) $\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$

Ex $f(x) = 4x^3 + 2x + 5$

$f'(x) = 4 \cdot 3x^2 + 2$

$= 12x^2 + 2.$

What about products

$$\frac{d}{dx}\left(f(x)\,g(x)\right) ? \quad \text{is it } \frac{df}{dx}\cdot\frac{dg}{dx}$$

ex $\dfrac{d}{dx} x^2 \cdot x^3 = \dfrac{d}{dx} x^5 = 5x^4$

$$\frac{dx^2}{dx}\cdot\frac{dx^3}{dx} = 2x \cdot 3x^2 = 6x^3 \neq 5x^4$$ so no!

Rule

$$\frac{d(fg)}{dx} = f'g + fg'$$

check $f = x^2 \quad g = x^3$

$f' = 2x \quad g' = 3x^2$

$$f'g + fg' = 2x \cdot x^3 + x^2 \cdot 3x^2$$
$$= 2x^4 + 3x^4$$
$$= 5x^4 \checkmark$$