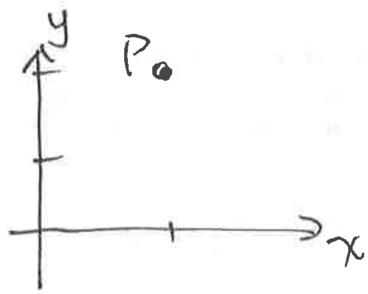
Math 1497 – Calc 2

Briggs Chapter 11 – Vector and Geometry in 3D

Consider $P(1,2)$ This is a point in 2-D



Suppose that I walked from the origin to the pt P I would walk in a certain direction and a certain amount.

This is the concept of a vector.

- magnitude & direction

To denote the particular vector we use angle brackets $\langle \cdot , \cdot \rangle$ with 2 components a movement in the x direction & y direction

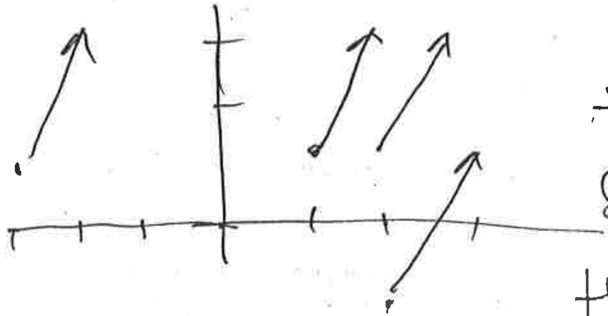
Above this would be

$\bar{u} = \langle 1, 2 \rangle$

1 unit x dir., 2 units y direction

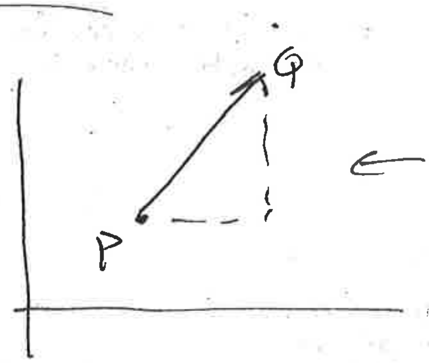
Consider starting at a different point

say $(1,1)$ $(2,1)$ $(-3,2)$ $(+2,-1)$



these all represent the same direction & have the same magnitude

Magnitude



$\overline{PQ} = \langle u_1, u_2 \rangle$

← right angled triangle

so $\|\overline{PQ}\| = \sqrt{u_1^2 + u_2^2}$

~~Cons~~ Consider

$P(1,1)$ $Q(3,2)$ $R(-2,-1)$ $S(0,2)$

$\overline{PQ} = \langle 3-1, 2-1 \rangle$
$= \langle 2, 1 \rangle$

$\overline{RS} = \langle 0-(-2), 2-(-1) \rangle$
$= \langle 2, 3 \rangle$

is $\overline{PQ} = \overline{RS}$

Yes!

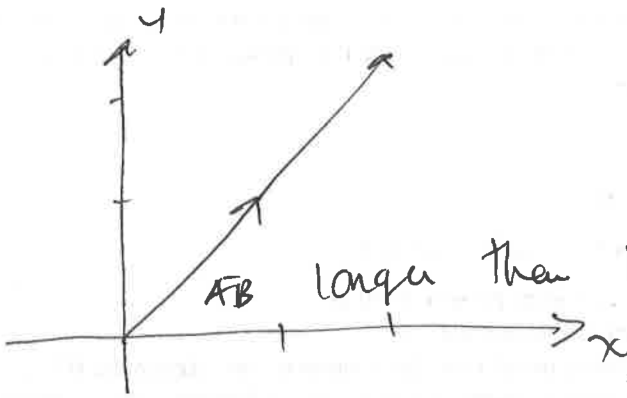
How about $\overline{PQ}$ with $M(-2,0)$ $N(-1,1)$

$$\overline{MN} = \langle -1-(-2),\ 1-0 \rangle$$
$$= \langle 1, 1 \rangle \quad \text{Yes}$$

same dir & magnitude

How about $\overline{PQ}$ & $A(2,3)$ $B(4,5)$

$$\overline{AB} = \langle 4-2,\ 5-3 \rangle = \langle 2, 2 \rangle$$
$$= 2\langle 1, 1 \rangle$$



longer than $\overline{PQ}$ same direction though

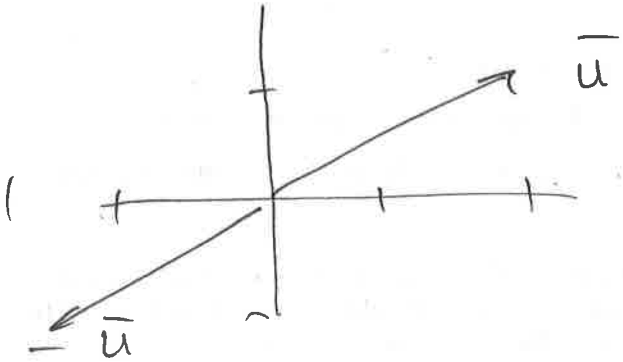
so $\overline{PQ} \neq \overline{AB}$

ZERO VECTOR $\quad \overline{u} = \langle 0, 0 \rangle$

Scalars

$$c\,\overline{u} = c\langle u_1, u_2 \rangle = \langle cu_1, cu_2 \rangle$$

if $\bar{u} = \langle 2,1\rangle$ then $-\bar{u} = -\langle 2,1\rangle$
$= \langle -2,-1\rangle$

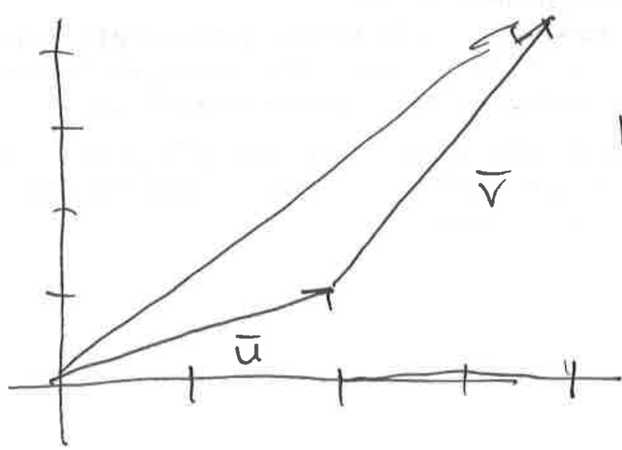


send the vector in the other direction

Vector Addition / subtraction

Consider

$\bar{u} = \langle 2,1\rangle$ $\quad \bar{v} = \langle 2,3\rangle$



looks like $\langle 4,4\rangle$

note $\bar{u}+\bar{v}$

$= \langle 2,1\rangle + \langle 2,3\rangle$

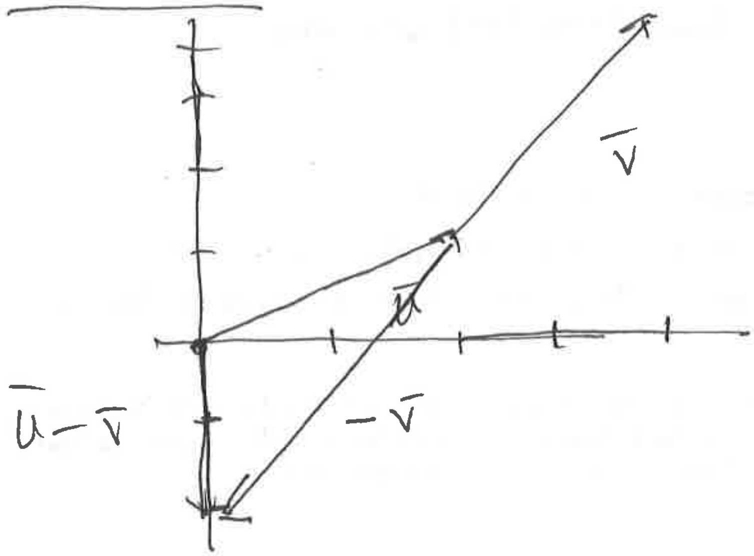
$= \langle 2+2, 1+3\rangle$

$= \langle 4,4\rangle$

so we just add 1st & 2nd components.

note: order doesn't matter $\bar{u}+\bar{v} = \bar{v}+\bar{u}$

subtraction



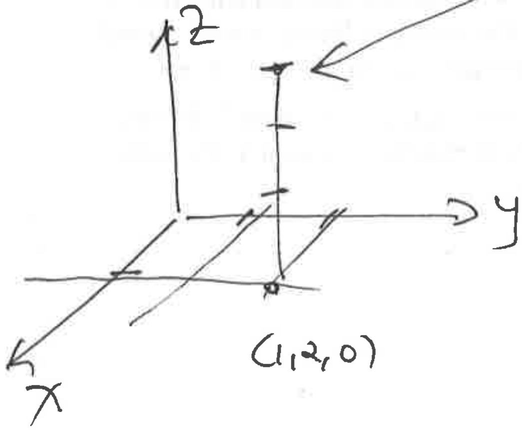
looks like $\langle 0,-2\rangle$

$$\bar{u}-\bar{v} = \langle 2,1\rangle - \langle 2,3\rangle$$
$$= \langle 2-2, 1-3\rangle$$
$$= \langle 0,-2\rangle \checkmark$$

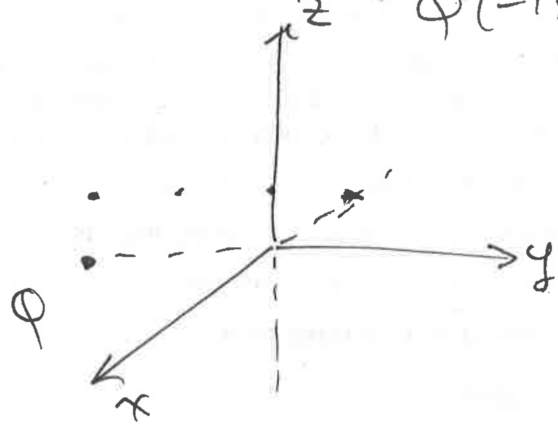
so with subtraction - subtract components

3-D Easily Extends

Points (x, y, z) Ex $P(1,2,3)$



$Q(-1,-3,-1)$

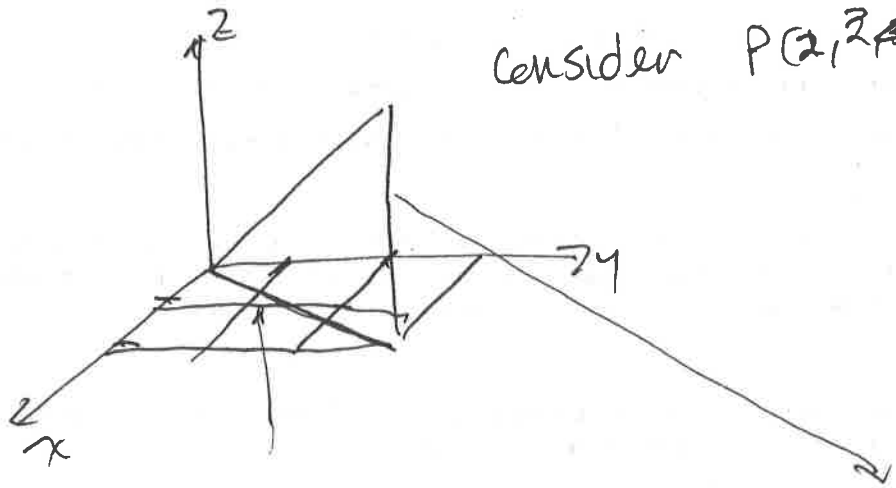


Distance Formula

Analogous to 2D $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

There is a formula for

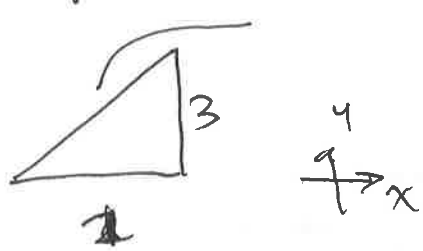
$P(x_1, y_1, z_1)$ $\quad$ $Q(x_2, y_2, z_2)$



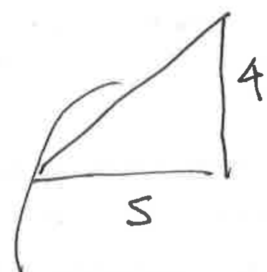
consider $P(2,3,4)$

and here

triangle here



$s = \sqrt{2^2+3^2}$



distance $\sqrt{s^2+4^2} = \sqrt{2^2+3^2+4^2}$

in general $\|\overline{PQ}\| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Unit Vectors

Vectors of unit length

Ex. $\bar{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$ $\quad$ $\|\bar{u}\| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{1} = 1$ ✓

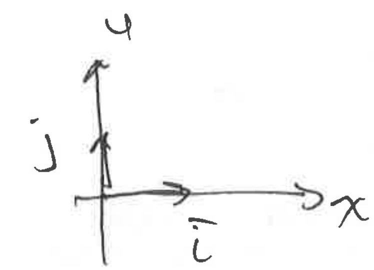
If a vector is not of unit length, divide by magnitude to make it unit

Ex $\bar{u} = \langle 3, 4, 12 \rangle$

$$\|\bar{u}\| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

unit vector $\dfrac{\bar{u}}{\|\bar{u}\|} = \dfrac{1}{13} \langle 3, 4, 12 \rangle = \left\langle \dfrac{3}{13}, \dfrac{4}{13}, \dfrac{12}{13} \right\rangle$

Standard Vectors (Base Vectors)



$\bar{i} = \langle 1, 0 \rangle$, $\bar{j} = \langle 0, 1 \rangle$

$\bar{i} = \langle 1, 0, 0 \rangle$ $\bar{j} = \langle 0, 1, 0 \rangle$ $\bar{k} = \langle 0, 0, 1 \rangle$

All vectors are linear combinations of these

Eg. $\langle 1, 2, 3 \rangle = \langle 1, 0, 0 \rangle + \langle 0, 2, 0 \rangle + \langle 0, 0, 3 \rangle$

$= \langle 1, 0, 0 \rangle + 2 \langle 0, 1, 0 \rangle + 3 \langle 0, 0, 1 \rangle$

$= \vec{i} + 2\vec{j} + 3\bar{k}$