A New Approach for Evaluation of Fuzzy Reliability of Fault-Tolerant Hypercube Parallel Computer Interconnection Network

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Abstract- When fault arises, a hypercube may operate in a gracefully degraded manner by supporting parallel algorithms in smaller fault-free subcubes. This paper proposes a new method to identify all the maximal incomplete sub cubes present in a faulty cube taking maximum fault tolerance level equal to the system dimension. The procedure is a distributed one, as every healthy node next to a failed one performs the same procedure independently and concurrently. A new and simple algorithm has been proposed to evaluate the fuzzy reliability of the cube based topology. The system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. The proposed method is well illustrated through an example. Fuzzy reliability of fault-tolerant Hypercube parallel computer interconnection networks has been evaluated by the proposed method.

Keywords- Hypercube, Maximal Incomplete sub cube, discarded region. Reliability, Fuzzy set, Interconnection network

Notations

IC_n	*n-cube Interconnection n/w*
HC_n	*Hypercube Interconnection n/w*
s	*Source node*
t	*Destination node*
$\otimes$	*Discarding operation*
n	*System dimension*
u,v,w	*Adjacent nodes of source node*
$\bar{v}$, $\bar{w}$	*Antipodal nodes of v, w*
X	*a set containing a space of points in the probability domain*
x	*an element of X*
p_i	*fuzzy probability of an event i*
$\bar{p}_i$	*complement of fuzzy probability of an event i*
$\mu_{p_i}(p)$	*membership function of fuzzy probability p_i*
N	*number of nodes of the cube*
R	*fuzzy reliability of cube*
G	*reliability logic graph*
V	*vertex set*
E	*edge set*
S	*system success containing all paths between the source node(s) to destination node (t)*
P_i	*path at the i^{th} step*
$W, \overline{W}$	*indicator variables*

Assumptions:-

1. Node failures are statistically independent of each other.
2. The link failure and link success probability is assumed to be fuzzy numbers
3. Repair facility is not available.

I. INTRODUCTION

As parallel computer communication systems are very much popular and commercially widely used in real time applications, therefore considerable interest and increasing efforts have been made to develop such large communication systems. A major part of it is a parallel computer interconnection network, which is used to interconnect a large number of standalone processors. Therefore a wide variety of interconnection networks have been proposed, one of the widely used topology is the hypercube [1]. Due to attractive properties like regularity, symmetry, small diameter, strong connectivity, recursive construction and partition ability the hypercube topology has enjoyed the largest popularity [2].

The probability of fault in a larger system is uncertain and given due importance. Whenever a fault arises, an hypercube may operate in a gracefully degradable manner due to the execution of parallel algorithms in smaller fault free sub cubes[3], which are comprises of healthy nodes. In order to maintain hypercube topology in the presence of faults, researchers have proposed addition of spare nodes thereby replacing the failed components with spares. This results in a much larger system than what is attained by any conventional reconfiguration scheme which identifies only complete sub cube [5]. Also fault tolerance can be achieved by reconfiguring the larger system to smaller sized system after the occurrence of fault [4]. Unlike a complete one, an incomplete cube can be of any arbitrary size, i.e. can be used to interconnect systems with any numbers of processors, making it possible to finish a given batch of jobs faster than it's complete counterpart alone by supporting simultaneous execution of multiple jobs of different sizes by assigning more nodes to execute the job cooperatively. Similar research work can be found in literature [6] and [7].Thus reconfiguring a faulty hypercube in to a maximal incomplete cube tends to lower potential performance degradation .

With the increase in size, the complexity of the interconnection network increases there by corresponding increase in computational power to maintain acceptable performance under reliable conditions [8] and [9]. For this the reliability prediction of a multi computer hypercube network is quite essential. However, there lies a large degree of uncertainty in system failure and therefore, the conventional methods [11] and [12] of reliability evaluation for large parallel computer system may not be appropriate to get a realistic value. Under such condition, one of the tools to cope with imprecision of available information in reliability analysis is *fuzzy set theory* [10]

Tanaka et al [13] and Misra and Weber [14] showed how fuzzification can be carried out for the quantitative analysis of fault tree. Chowdhury and Mishra [15] evaluated the reliability of a non-series parallel network. Patra et al [16] presents a method for evaluating fuzzy reliability of a communication network with fuzzy element capacities and probabilities. Tripathy et al [17] have proposed a method to evaluate fuzzy reliability of MINs. But none of methods discussed above considers the cube based interconnection networks and suggests a general method of evaluating fuzzy reliability of the said networks where there lies a large degree of uncertainty in system failure. So, there is always a need to search for a general and efficient method to evaluate the fuzzy reliability of such systems.

In this paper, a general and efficient method has been proposed to find an expression of fuzzy system reliability of hypercube parallel systems taking in to consideration the special requirements of fuzzy sets. This method is supported by an efficient algorithm and well illustrated through a 3-dimensinal hypercube.

II. METHODOLOGY FOR FINDING SUB CUBES

Let IC_n denotes an n-dimensional interconnection network i.e. n-dimensional hypercube. Each node in IC_n is labeled by a n-bit string. For a given source node s, there exists a numbers of adjacent nodes, out of which at least one node is faulty. Otherwise it will destroy the regularity property of IC_n. The addresses of the adjacent nodes are differ exactly one bit. Assume 'u' be the faulty node, 'v' and 'w' be the non-faulty nodes. Where 'u', 'v' , 'w' are represented as binary strings [3]. $\bar{v}$ and $\bar{w}$ be the antipodal nodes of v and w. Taking bit operation $u \otimes \bar{v}$ and $u \otimes \bar{w}$ results n discarded regions. This leads to formation of an incomplete interconnection network I_{n-1}^{m}.m numbers of nodes in fault free incomplete cube with dimension of n-1.

III. PROPOSED METHOD FOR FUZZY RELIABILITY EVALUATION

Fuzzy Probability

Fuzzy probability represents a fuzzy number between zero and one, assigned to the probability of an event. One can chose different types of membership functions for fuzzy probability. For instance, a fuzzy probability may have a trapezoidal membership function. The fuzzy probabilities of an event i can then be denoted by a four parameter function i.e.

$$p_i = (\alpha_{i1}, \alpha_{i2}, \beta_{i2}, \beta_{i1}) \quad (1)$$

The membership function is given by

$$\mu_{p_i}(p) = \begin{cases} 0, & 0 \le p \le \alpha_{i1} \\ 1 - \dfrac{\alpha_{i2} - p}{\alpha_{i2} - \alpha_{i1}} & \alpha_{i1} \le p \le \alpha_{i2} \\ 1 & \alpha_{i2} \le p \le \beta_{i2} \\ 1 - \dfrac{p - \beta_{i2}}{\beta_{i1} - \beta_{i2}} & \beta_{i2} \le p \le \beta_{i1} \\ 0 & \beta_{i1} \le p \le 1 \end{cases} \quad (2)$$

Operation used in computing fuzzy reliability

Let p_i and p_j be two fuzzy sets that have membership functions given by $\mu(p_i) \,\&\, \mu(p_j)$, respectively. The operations used in fuzzy reliability evaluation, i.e. multiplication and complementation can be defined as follows:

1. *Multiplication-*

$p_i.p_j$ = product of p_i and p_j

$$= \mu_{p_i p_j}(p) = \mu_{p_i}(p).\mu_{p_j}(p) \quad (3)$$

How ever, Tanka et al [13] provided an approximation of the multiplication procedure by defining

$$p_{ij} = p_i.p_j = (\alpha_{i1}.\alpha_{j2}, \alpha_{i2}.\alpha_{j2}, \beta_{i2}.\beta_{j2}, \beta_{i1}\beta_{j1}) \quad (4)$$

2. *Complementation*

The complementation of any fuzzy set p_i will be given by

$$\bar{p}_i = 1 - \mu_{p_i} \quad (5)$$

for example, in case of trapezoidal membership function, one could obtain

$$\bar{p}_i = (1-\alpha_{i1}, 1-\alpha_{i2}, 1-\beta_{i2}, 1-\beta_{i1}) \quad (6)$$

Proposed Approach

Mathematical Basis:-

Let S_k denote the kth minimal path set and p_k be the fuzzy probability associated with S_k. Also let R_k be the fuzzy reliability at *kth* step of the sum of fuzzy path probabilities. The reliability expression can be found out by a recursive formula:

$$R_k = R_{k-1} + \Pr\left\{ S_k \cap \overline{\left(\bigcup_{i}^{k-1} S_i \right)} \right\}$$
$$= R_{k-1} + \Pr\{ S_k \cap \bar{S}_1 \cap \bar{S}_2 \cap \cdots \cap \bar{S}_{k-1} \} \quad (7)$$

Proposed Algorithm

1. Convert distributed network in to a probabilistic graph (G(N, E))
2. Find the maximal incomplete cube of G using the proposed method

3. Generate the fuzzy probabilities of links $e \in E'$ using Eq. 2
4. Enumerate all the minimal path sets from the source to destination node.
5. Rearrange the total number of path sets (h) according to increasing order of cardinality.
6. Set k=1 and $R_0=0$
7. Repeat 6-7 for k=1 to h
8. The reliability expression can be given as

$$R_k = (R_{k-1} + S_k)_{dis}$$

9. $p_k = R_k - R_{k-1}$
10. Compute the fuzzy reliability of the system as

$$R = 1 - \prod_{k=1}^{h} \bar{p}_k$$

Efficiency:
Finding the minimal paths using the above proposed algorithm requires $O(N^3)$ operations. So, the running time of proposed algorithm is $O(N^3)$ which is polynomial.

IV. ILLUSTRATION

The proposed method is illustrated through the following example.

Example:
Using the proposed method, the hypercube interconnection system of Fig.1 (a) is viewed as a connected undirected graph G (N, E) where N is the set of nodes (processing elements) and E is the set of edges (links).

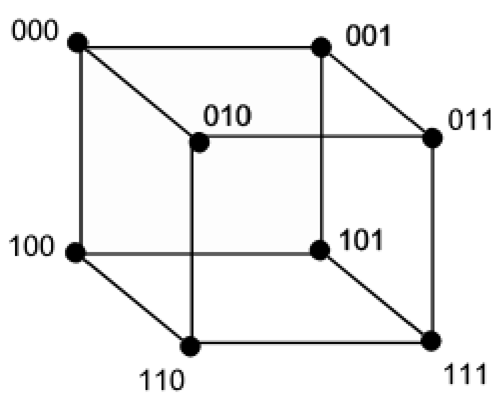

Fig.1(a): Hypercube (n=3).

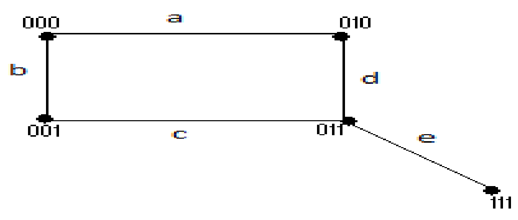

Fig.1(b): Maximal incomplete subcube

Assuming the trapezoidal membership function as given in Equation 2, the fuzzy probabilities of links *a*, *b*, *c*, *d*, *e* are given as follows
$p_a=(0.2,0.8, 0.95,0.99)$, $p_b=(0.1, 0.7, 0.85,0.96)$, $p_c=(0.2, 0.37, 0.6,0.79)$, $p_d=(0.1,0.6,0.85,0.96)$, $p_e=(0.17, 0.2, 0.3,0.6)$,

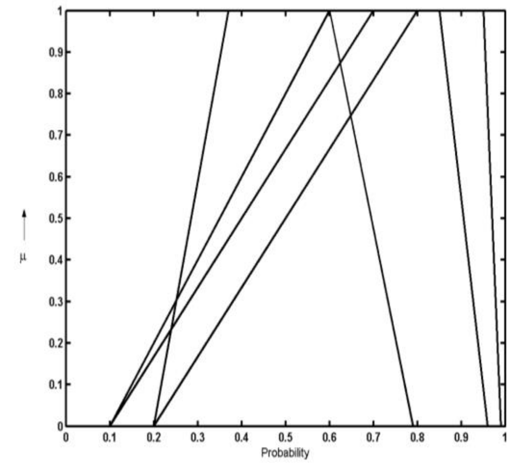

Fig.2(a): Fuzzy probabilities of links *a*, *b*, *c*, *d*

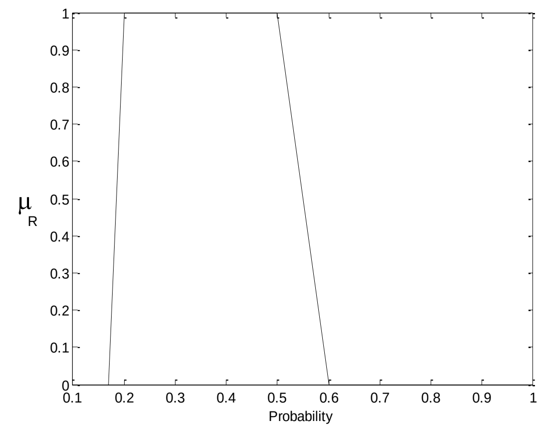

Fig.2(b): Fuzzy probability of link e

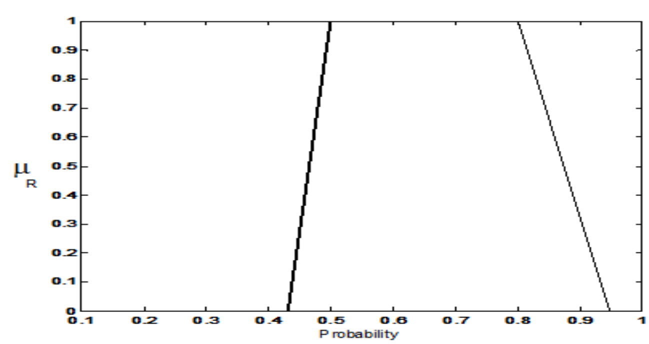

Fig.3: Fuzzy reliability of Hypercube

The path set is given by

$P = \{ade, bce\}$

Using the proposed algorithm, the membership function of the fuzzy reliability of the Hypercube network is given by

$\mu_R = \{0.4317, 0.5, 0.8, 0.9461\}$

V. CONCLUSION

This paper proposed two new approaches for fuzzy reliability evaluation. The first one is meant for finding fault-free cube from a faulty one and the second one is for evaluating the fuzzy reliability. Basically, the proposed methods use the path enumeration technique in evaluating fuzzy reliability. The algorithm enumerates all the path sets from the source node to destination node. Then the system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. Using the proposed techniques the fuzzy reliability of Hypercube parallel computer interconnection network was evaluated. The methods proposed here can also be extended further to evaluate all categories of parallel computer networks.

VI. REFERENCES

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