Fall 2022 Math 1496 - Sample Test 2

1. Find the derivative of the following

$$\begin{array}{ll} (i) & y = x\sin^{-1}x \\ (ii) & y = \ln(x^2+e^x) \\ (iii) & x^2 - xy + y^4 = x - y \end{array}$$

2. Find the absolute minimium and maximum of the following on the given interval

$$\begin{array}{ll} (i) & f(x) = 2x^2 - 12x + 1 \quad \text{on} \ [0,6] \\ (ii) & f(x) = 2x^3 - 15x^2 + 24x \quad \text{on} \ [0,3] \end{array}$$

3. State Rolles Theorem. Verify Rolles Theorem for the following:

$$\begin{array}{ll} (i) & f(x) = x^2 - 2x + 3 \quad \text{on} \ [0,2] \\ (ii) & f(x) = x^4 - 2x^2 + 1 \quad \text{on} \ [-2,2] \end{array}$$

4. State the Mean Value Theorem. Verify the Mean Value Theorem for the following:

$$\begin{array}{ll} (i) & f(x) = x^3 - x \quad \text{on} \ [0,2] \\ (ii) & f(x) = \dfrac{x}{x+2} \quad \text{on} \ [1,10] \end{array}$$

5. If $y = x(x-4)^3$ calculate the following

(i) The critical numbers

(ii) When y is increasing and decreasing.

(iii) Determine whether any of the critical numbers are minimum or maximum.

(iv) When y is concave up and down and determine the points of inflection.

(v) Then sketch the curve.

6. A ladder 13 feet long is resting against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 ft/sec. At rate is the tip of the ladder moving down the wall when the base of the ladder is 5 ft away from the wall?

7. A paper cup in the shape of an inverted cone with height 10 cm and a base of radius 3 cm, is being filled at a rate of 2 cm^3/min. Find the rate of change in the height of the water when the height of the water is 5 cm.