Exercise 6A

SOLUTIONS 1

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 8ab \\ -5ab \\ 3ab \\ -ab \\ \hline 5ab \end{array}$$

SOLUTIONS 2

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 7x \\ -3x \\ 5x \\ -x \\ -2x \\ \hline 6x \end{array}$$

SOLUTIONS 3

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{l} 3a - 4b + 4c \\ 2a + 3b - 8c \\ a - 6b + c \\ \hline 6a - 7b - 3c \end{array}$$

SOLUTIONS 4

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 5x-8y+2z \\ -2x-4y+3z \\ -x+6y-z \\ 3x-3y-2z \\ \hline 5x-9y+2z \end{array}$$

SOLUTIONS 5

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 6ax-2by+3cz \\ -11ax+6by-cz \\ -2ax-3by+10cz \\ \hline -7ax+by+12cz \end{array}$$

SOLUTIONS 6

Answer :

On arranging the terms of the given expressions in the descending powers of x and adding column-wise:

$$\begin{array}{r} 2x^3-9x^2+0x+8 \\ 0x^3+3x^2-6x-5 \\ 7x^3+0x^2-10x+1 \\ -4x^3-5x^2+2x+3 \\ \hline 5x^3-11x^2-14x+7 \end{array}$$

SOLUTIONS 7

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise:

$$\begin{array}{l} 6p+\ 4q\ -\ r+3 \\ -5p+\ 0q+2r-6 \\ -7p+11q+2r-1 \\ 0p+\ 2q-3r+4 \\ \hline -6p+17q+0r+0 \\ =-6p+17q \end{array}$$

SOLUTIONS 8

Answer :

On arranging the terms of the given expressions in the descending powers of x and adding column-wise:

$$\begin{array}{l} 4x^2+4y^2-7xy-3 \\ x^2+\ 6y^2-8xy+0 \\ 2x^2-5y^2-2xy+6 \\ \hline 7x^2+5y^2-17xy+3 \end{array}$$

SOLUTIONS 9

Answer :

On arranging the terms of the given expressions in the descending powers of x and subtracting:

$$\begin{array}{l} -5a^2b \\ \ \ 3a^2b \\ - \\ \hline -8a^2b \end{array}$$

SOLUTIONS 10

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} 6pq \\ -8pq \\ + \\ \hline 14pq \end{array}$$

SOLUTIONS 11

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} -8abc \\ -2abc \\ + \\ \hline -6abc \end{array}$$

SOLUTIONS 12

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} -11p \\ -16p \\ + \\ \hline 5p \end{array}$$

SOLUTIONS 13

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{cccc} 3a & -4b & -c & +6 \\ 2a & -5b & +2c & -9 \\ - & + & - & + \\ \hline a & +b & -3c & +15 \end{array}$$

SOLUTIONS 14

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} p - 2q - 5r - 8 \\ -6p + \ \ q + 3r + 8 \\ + \quad - \quad - \quad - \\ \hline 7p - 3q - 8r - 16 \\ \hline \end{array}$$

SOLUTIONS 15

Answer :

On arranging the terms of the given expressions in the descending powers of x and subtracting column-wise:

$$\begin{array}{r} 3x^3 - x^2 + 2x - 4 \\ x^3 + 3x^2 - 5x + 4 \\ - \quad - \quad + \quad - \\ \hline 2x^3 - 4x^2 + 7x - 8 \\ \hline \end{array}$$

SOLUTIONS 16

Answer :

Arranging the terms of the given expressions in the descending powers of x and subtracting column-wise:

$$\begin{array}{r} 4y^4 - 2y^3 - 6y^2 - y + 5 \\ 5y^4 - 3y^3 + 2y^2 + y - 1 \\ - \quad + \quad - \quad - \quad + \\ \hline -y^4 + y^3 - 8y^2 - 2y + 6 \\ \hline \end{array}$$

SOLUTIONS 17

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} 3p^2 - 4q^2 - 5r^2 - 6 \\ 4p^2 + 5q^2 - 6r^2 + 7 \\ - \quad - \quad + \quad - \\ \hline -p^2 - 9q^2 + r^2 - 13 \end{array}$$

SOLUTIONS 18

Answer :

Let the required number be x.

$$\left(3a^2 - 6ab - 3b^2 - 1\right) - x = 4a^2 - 7ab - 4b^2 + 1$$

$$\left(3a^2 - 6ab - 3b^2 - 1\right) - \left(4a^2 - 7ab - 4b^2 + 1\right) = x$$

$$\begin{array}{r} 3a^2 - 6ab - 3b^2 - 1 \\ 4a^2 - 7ab - 4b^2 + 1 \\ - \quad + \quad + \quad - \\ \hline -a^2 + ab + b^2 - 2 \end{array}$$

∴ Required number = $-a^2 + ab + b^2 - 2$

SOLUTIONS 19

Answer :

Sides of the rectangle are l and b.

$l = 5x^2 - 3y^2$

$b = x^2 + 2xy$

Perimeter of the rectangle is $(2l + 2b)$.

$$\text{Perimeter} = 2\left(5x^2 - 3y^2\right) + 2\left(x^2 + 2xy\right)$$

$$= 10x^2 - 6y^2 + 2x^2 + 4xy$$

$$\begin{array}{l} 10x^2 - 6y^2 \\ \underline{2x^2 \quad\quad + 4xy} \\ 12x^2 - 6y^2 + 4xy \end{array}$$

Hence, the perimeter of the rectangle is $12x^2 - 6y^2 + 4xy$.

SOLUTIONS 20

Answer :

Let $a,\ b$ *and* c be the three sides of the triangle.

$\therefore$ Perimeter of the triangle $= (a + b + c)$

Given perimeter of the triangle = $6p^2 - 4p + 9$

One side (a) = $p^2 - 2p + 1$

Other side (b) = $3p^2 - 5p + 3$

Perimeter = $(a + b + c)$

$(6p^2 - 4p + 9) = (p^2 - 2p + 1) + (3p^2 - 5p + 3) + c$

$6p^2 - 4p + 9 - p^2 + 2p - 1 - 3p^2 + 5p - 3 = c$

$(6p^2 - p^2 - 3p^2) + (-4p + 2p + 5p) + (9 - 1 - 3) = c$

$2p^2 + 3p + 5 = c$

Thus, the third side is $2p^2 + 3p + 5$.

Exercise 6B

SOLUTIONS 1

Answer :

By horizontal method:

$(5x+7)\times(3x+4)$
$=5x(3x+4)+7(3x+4)$
$=15x^2+20x+21x+28$
$=15x^2+41x+28$

SOLUTIONS 2

Answer :

By horizontal method:

$(4x+9)\times(x-6)$
$=4x(x-6)+9(x-6)$
$=4x^2-24x+9x-54$
$=4x^2-15x-54$

SOLUTIONS 3

Answer :

By horizontal method:

$(2x+5)\times(4x-3)$
$=2x(4x-3)+5(4x-3)$
$=8x^2-6x+20x-15$
$=8x^2+14x-15$

SOLUTIONS 4

Answer :

By horizontal method:

$(3y-8)\times(5y-1)$
$=3y(5y-1)-8(5y-1)$
$=15y^2-3y-40y+8$
$=15y^2-43y+8$

SOLUTIONS 5

Answer :

By horizontal method:

$$
\begin{aligned}
&(7x+2y)\times(x+4y)\\
&=7x(x+4y)+2y(x+4y)\\
&=7x^2+28xy+2xy+8y^2\\
&=7x^2+30xy+8y^2
\end{aligned}
$$

SOLUTIONS 6

Answer :

By horizontal method:

$$
\begin{aligned}
&(9x+5y)\times(4x+3y)\\
&9x(4x+3y)+5y(4x+3y)\\
&=36x^2+27xy+20xy+15y^2\\
&=36x^2+47xy+15y^2
\end{aligned}
$$

SOLUTIONS 7

Answer :

By horizontal method:

$$
\begin{aligned}
&(3m-4n)\times(2m-3n)\\
&=3m(2m-3n)-4n(2m-3n)\\
&=6m^2-9mn-8mn+12n^2\\
&=6m^2-17mn+12n^2
\end{aligned}
$$

SOLUTIONS 8

Answer :

By horizontal method:

$$
\begin{aligned}
&(x^2-a^2)\times(x-a)\\
&=x^2(x-a)-a^2(x-a)\\
&=x^3-ax^2-a^2x+a^3
\end{aligned}
$$

i.e $(x^3+a^3)-ax(x-a)$

SOLUTIONS 9

Answer :

By horizontal method:

$$\left(x^2-y^2\right)\times\left(x+2y\right)$$
$$=x^2\left(x+2y\right)-y^2\left(x+2y\right)$$
$$=x^3+2x^2y-xy^2-2y^3$$
$$i.e\left(x^3-2y^3\right)+xy\left(2x-y\right)$$

SOLUTIONS 10

Answer :

By horizontal method:

$$\left(3p^2+q^2\right)\times\left(2p^2-3q^2\right)$$
$$=3p^2\left(2p^2-3q^2\right)+q^2\left(2p^2-3q^2\right)$$
$$=6p^4-9p^2q^2+2p^2q^2-3q^4$$
$$i.e6p^4-7p^2q^2-3q^4$$

SOLUTIONS 11

Answer :

By horizontal method:

$$\left(2x^2-5y^2\right)\times\left(x^2+3y^2\right)$$
$$=2x^2\left(x^2+3y^2\right)-5y^2\left(x^2+3y^2\right)$$
$$=2x^4+6x^2y^2-5x^2y^2-15y^4$$
$$=2x^4+x^2y^2-15y^4$$

SOLUTIONS 12

Answer :

By horizontal method:

$$\begin{aligned}&\left(x^3-y^3\right)\times\left(x^2+y^2\right)\\&=x^3\left(x^2+y^2\right)-y^3\left(x^2+y^2\right)\\&=x^5+x^3y^2-x^2y^3-y^5\\&=\left(x^5-y^5\right)+x^2y^2(x-y)\end{aligned}$$

SOLUTIONS 13

Answer :

By horizontal method:

$$\begin{aligned}&\left(x^4+y^4\right)\times\left(x^2-y^2\right)\\&=x^4\left(x^2-y^2\right)+y^4\left(x^2-y^2\right)\\&=x^6-x^4y^2+y^4x^2-y^6\\&=\left(x^6-y^6\right)-x^2y^2\left(x^2-y^2\right)\end{aligned}$$

SOLUTIONS 14

Answer :

By horizontal method:

$$\begin{aligned}&\left(x^4+\frac{1}{x^4}\right)\times\left(x+\tfrac{1}{x}\right)\\&=x^4\left(x+\tfrac{1}{x}\right)+\tfrac{1}{x^4}\left(x+\tfrac{1}{x}\right)\\&=x^5+x^3+\tfrac{1}{x^3}+\tfrac{1}{x^5}\\&i.e\; x^3\left(x^2+1\right)+\tfrac{1}{x^3}\left(1+\frac{1}{x^2}\right)\end{aligned}$$

SOLUTIONS 15

Answer :

By horizontal method:

$$\begin{aligned}&\left(x^2-3x+7\right)\times\left(2x+3\right)\\&=2x\left(x^2-3x+7\right)+3\left(x^2-3x+7\right)\\&=2x^3-6x^2+14x+3x^2-9x+21\\&=2x^3-3x^2+5x+21\end{aligned}$$

SOLUTIONS 16

Answer :

By horizontal method:

$$(3x^2+5x-9)\times(3x-5)$$
$$=3x(3x^2+5x-9)-5(3x^2+5x-9)$$
$$=9x^3+15x^2-27x-15x^2-25x+45$$
$$=9x^3-52x+45$$

SOLUTIONS 17

Answer :

By horizontal method:

$$(x^2-xy+y^2)\times(x+y)$$
$$=x(x^2-xy+y^2)+y(x^2-xy+y^2)$$
$$=x^3-x^2y+y^2x+x^2y-xy^2+y^3$$
$$=x^3+y^3$$

SOLUTIONS 18

Answer :

By horizontal method:

$$(x^2+xy+y^2)\times(x-y)$$
$$x(x^2+xy+y^2)-y(x^2+xy+y^2)$$
$$=x^3+x^2y+xy^2-x^2y-xy^2-y^3$$
$$=x^3-y^3$$

SOLUTIONS 19

Answer :

By horizontal method:

$$(x^3-2x^2+5)\times(4x-1)$$
$$=4x(x^3-2x^2+5)-1(x^3-2x^2+5)$$
$$=4x^4-8x^3+20x-x^3+2x^2-5$$
$$=4x^4-9x^3+2x^2+20x-5$$

SOLUTIONS 20

Answer :

By horizontal method:

$$\left(9x^2 - x + 15\right) \times \left(x^2 - 3\right)$$
$$= x^2\left(9x^2 - x + 15\right) - 3\left(9x^2 - x + 15\right)$$
$$= 9x^4 - x^3 + 15x^2 - 27x^2 + 3x - 45$$
$$= 9x^4 - x^3 - 12x^2 + 3x - 45$$

SOLUTIONS 21

Answer :

By horizontal method:

$$\left(x^2 - 5x + 8\right) \times \left(x^2 + 2\right)$$
$$= x^2\left(x^2 - 5x + 8\right) + 2\left(x^2 - 5x + 8\right)$$
$$= x^4 - 5x^3 + 8x^2 + 2x^2 - 10x + 16$$
$$= x^4 - 5x^3 + 10x^2 - 10x + 16$$

SOLUTIONS 22

Answer :

By horizontal method:

$$\left(x^3 - 5x^2 + 3x + 1\right) \times \left(x^2 - 3\right)$$
$$= x^2\left(x^3 - 5x^2 + 3x + 1\right) - 3\left(x^3 - 5x^2 + 3x + 1\right)$$
$$= x^5 - 5x^4 + 3x^3 + x^2 - 3x^3 + 15x^2 - 9x - 3$$
$$= x^5 - 5x^4 + 16x^2 - 9x - 3$$

SOLUTIONS 23

Answer :

By horizontal method:

$$\left(3x + 2y - 4\right) \times \left(x - y + 2\right)$$
$$x\left(3x + 2y - 4\right) - y\left(3x + 2y - 4\right) + 2\left(3x + 2y - 4\right)$$
$$= 3x^2 + 2xy - 4x - 3xy - 2y^2 + 4y + 6x + 4y - 8$$
$$= 3x^2 - 2y^2 - xy + 2x + 8y - 8$$

SOLUTIONS 24

Answer :

By horizontal method:

$$\left(x^2-5x+8\right)\times\left(x^2+2x-3\right)$$
$$=x^2\left(x^2-5x+8\right)+2x\left(x^2-5x+8\right)-3\left(x^2-5x+8\right)$$
$$=x^4-5x^3+8x^2+2x^3-10x^2+16x-3x^2+15x-24$$
$$=x^4-3x^3-5x^2+31x-24$$

SOLUTIONS 25

Answer :

By horizontal method:

$$\left(2x^2+3x-7\right)\times\left(3x^2-5x+4\right)$$
$$=2x^2\left(3x^2-5x+4\right)+3x\left(3x^2-5x+4\right)-7\left(3x^2-5x+4\right)$$
$$=6x^4-10x^3+8x^2+9x^3-15x^2+12x-21x^2+35x-28$$
$$=6x^4-x^3-28x^2+47x-28$$

SOLUTIONS 26

Answer :

By horizontal method:

$$\left(9x^2-x+15\right)\times\left(x^2-x-1\right)$$
$$=x^2\left(9x^2-x+15\right)-x\left(9x^2-x+15\right)-1\left(9x^2-x+15\right)$$
$$=9x^4-x^3+15x^2-9x^3+x^2-15x-9x^2+x-15$$
$$=9x^4-10x^3+7x^2-14x-15$$

Exercise 6C
SOLUTIONS 1

Answer :

(i) $24x^2y^3$ by $3xy$

$$\frac{24x^2y^3}{3xy}$$
$$\Rightarrow \left(\frac{24}{3}\right)\left(x^{2-1}\right)\left(y^{3-1}\right)$$
$$\Rightarrow 8xy^2.$$

Therefore, the quotient is $8xy^2$.

(ii) $36xyz^2$ by $-9xz$

$$\frac{36xyz^2}{-9xz}$$
$$\Rightarrow \left(\frac{36}{-9}\right)\left(x^{1-1}\right)\left(y^{1-0}\right)\left(z^{2-1}\right)$$
$$\Rightarrow -4yz$$

Therefore, the quotient is $-4yz$.

(iii)

$-72x^2y^2z \text{ by } -12xyz$

$$\frac{-72x^2y^2z}{-12xyz}$$
$$\Rightarrow\left(\frac{-72}{-12}\right)\left(x^{2-1}\right)\left(y^{2-1}\right)\left(z^{1-1}\right)$$
$$\Rightarrow 6xy$$

Therefore, the quotient is $6xy$.

(iv) $-56mnp^2$ by $7mnp$

$$\frac{-56mnp^2}{7mnp}$$
$$\Rightarrow\left(\frac{-56}{7}\right)\left(m^{1-1}\right)\left(n^{1-1}\right)\left(p^{2-1}\right)$$
$$\Rightarrow -8p$$

Therefore, the quotient is $-8p$.

SOLUTIONS 2

Answer :

(i) $5m^3 - 30m^2 + 45m$ by $5m$

$$\left(5m^3 - 30m^2 + 45m\right) \div 5m$$
$$\Rightarrow \frac{5m^3}{5m} - \frac{30m^2}{5m} + \frac{45m}{5m}$$
$$\Rightarrow m^2 - 6m + 9$$

Therefore, the quotient is $m^2 - 6m + 9$.

(ii) $8x^2y^2 - 6xy^2 + 10x^2y^3$ by $2xy$

$$\left(8x^2y^2 - 6xy^2 + 10x^2y^3\right) \div 2xy$$
$$\Rightarrow \frac{8x^2y^2}{2xy} - \frac{6xy^2}{2xy} + \frac{10x^2y^3}{2xy}$$
$$\Rightarrow 4xy - 3y + 5xy^2$$

Therefore, the quotient is $4xy - 3y + 5xy^2$.

(iii) $9x^2y - 6xy + 12xy^2$ by $-3xy$

$$\left(9x^2y \;-\; 6xy \;+\; 12xy^2\right) \div \;-3xy$$
$$\Rightarrow \frac{9x^2y}{-3xy} - \frac{6xy}{-3xy} + \frac{12xy^2}{-3xy}$$
$$\Rightarrow -3x \;+\; 2 \;-\; 4y$$

Therefore, the quotient is $-3x + 2 - 4y$.

(iv) $12x^4 + 8x^3 - 6x^2$ by $-2x^2$

$$\left(12x^4 \;+\; 8x^3 \;-\; 6x^2\right) \div \;-2x^2 \;\; {}^2\text{-}4x+3^2$$
$$\Rightarrow \frac{12x^4}{-2x^2} + \frac{8x^3}{-2x^2} - \frac{6x^2}{-2x^2}$$
$$\Rightarrow -6x$$

Therefore the quotient is $-6x^2 - 4x + 3$.

SOLUTIONS 3

Answer :

$$(x^2 - 4x + 4) \div (x - 2)$$

$$\begin{array}{r|l} x-2 \; & \; x^2 - 4x + 4 \;\big(\; x - 2 \\ & \; x^2 - 2x \\ & \; \underline{-\quad +} \\ & \quad -2x + 4 \\ & \quad -2x + 4 \\ & \quad \underline{+\quad -} \\ & \quad\quad \times \end{array}$$

Therefore, the quotient is $(x - 2)$ and the remainder is 0.

SOLUTIONS 4

Answer :

$$
\begin{array}{r|l}
x+2 & x^2 - 4 \quad (x-2 \\
 & x^2 \quad + 2x \\
 & \underline{- \qquad -} \\
 & -2x - 4 \\
 & -2x - 4 \\
 & \underline{+ \quad +} \\
 & \underline{\times}
\end{array}
$$

Therefore, the quotient is x−2 and the remainder is 0.

SOLUTIONS 5

Answer :

(x^2 + 12x + 35) by (x + 7)

$$
\begin{array}{r|l}
x+7 & x^2 + 12x + 35 \quad (x+5 \\
 & x^2 + \;\; 7x \\
 & \underline{- \quad -} \\
 & 5x + 35 \\
 & 5x + 35 \\
 & \underline{- \quad -} \\
 & \underline{\times}
\end{array}
$$

Therefore, the quotient is $(x + 5)$ and the remainder is 0.

SOLUTIONS 6

Answer :

$$
\begin{array}{r|l}
3x+2 & 15x^2 + \quad x - 6 \quad (5x-3 \\
 & 15x^2 + 10x \\
 & \underline{- \qquad -} \\
 & -9x - 6 \\
 & -9x - 6 \\
 & \underline{+ \quad +} \\
 & \underline{\times}
\end{array}
$$

Therefore, the quotient is $(5x - 3)$ and the remainder is 0.

SOLUTIONS 7

Answer :

$$
\begin{array}{r|l}
7x-9 & 14x^2-53x+45 \;(2x-5 \\
 & 14x^2-18x \\
 & -\quad + \\
\hline
 & -35x+45 \\
 & -35x+45 \\
 & +\quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

Therefore, the quotient is $(2x - 5)$ and the remainder is 0.

SOLUTIONS 8

Answer :

$$
\begin{array}{r|l}
2x-5 & 6x^2-31x+47 \;(3x-8 \\
 & 6x^2-15x \\
 & -\quad + \\
\hline
 & -16x+47 \\
 & -16x+40 \\
 & +\quad - \\
\hline
 & 7 \\
\hline
\end{array}
$$

Therefore, the quotient is $(3x - 8)$ and the remainder is 7.

SOLUTIONS 9

Answer :

$$
\begin{array}{r|l}
2x+3 & 2x^3+x^2-5x-2 \;(x^2-x-1 \\
 & 2x^3+3x^2 \\
 & -\quad - \\
\hline
 & -2x^2-5x \\
 & -2x^2-5x \\
 & +\quad + \\
\hline
 & -2x-2 \\
 & -2x-3 \\
 & +\quad + \\
\hline
 & 1 \\
\hline
\end{array}
$$

Therefore, the quotient is $(x^2 - x - 1)$ and the remainder is 1.

SOLUTIONS 10

Answer :

$$
\begin{array}{r|l}
x+1 & x^3+1 \quad (x^2-x+1 \\
 & \underline{\overset{-}{x^3} \quad \overset{-}{+}\, x^2} \\
 & -x^2+1 \\
 & \underline{\overset{+}{-x^2} \quad \overset{+}{-x}} \\
 & x+1 \\
 & \underline{\overset{-}{x}\, \overset{-}{+}\, 1} \\
 & \underline{0}
\end{array}
$$

Therefore, the quotient is x^2-x+1 and the remainder is 0.

SOLUTIONS 11

Answer :

$$
\begin{array}{r|l}
x^2+x+1 & x^4-2x^3+2x^2+x+4 \quad (x^2-3x+4 \\
 & \underline{\overset{-}{x^4} \overset{-}{+} x^3 \overset{-}{+} x^2} \\
 & -3x^3+x^2+x \\
 & \underline{\overset{+}{-}3x^3 \overset{+}{-} 3x^2 \overset{+}{-} 3x} \\
 & 4x^2+4x+4 \\
 & \underline{\overset{-}{4x^2} \overset{-}{+} 4x \overset{-}{+} 4} \\
 & \underline{\times}
\end{array}
$$

Therefore, the quotient is (x^2 - 3x + 4) and remainder is 0.

SOLUTIONS 12

Answer :

$$
\begin{array}{r|l}
x^2-5x+6 & x^3-6x^2+11x-6 \quad (x-1 \\
 & \underline{\overset{-}{x^3} \overset{+}{-} 5x^2 \overset{-}{+} 6x} \\
 & -1x^2+5x-6 \\
 & \underline{\overset{+}{-}1x^2 \overset{-}{+} 5x \overset{+}{-} 6} \\
 & \underline{\times}
\end{array}
$$

Therefore, the quotient is (x-1) and the remainder is 0.

SOLUTIONS 13

Answer :

$$x^2 - 3x + 4 \overline{\big) \; 5x^3 - 12x^2 + 12x + 13 \; \big(} \; 5x + 3$$
$$x^3 - 15x^2 + 20x$$
$$- \quad + \quad -$$
$$3x^2 - 8x + 13$$
$$3x^2 - 9x + 12$$
$$- \quad + \quad -$$
$$x + 1$$

Therefore, the quotient is ($5x$+ 3) and the remainder is (x + 1).

SOLUTIONS 14

Answer :

$$2x^2 - 3x + 5 \overline{\big) \; 2x^3 - 5x^2 + 8x - 5 \; \big(} \; x - 1$$
$$2x^3 - 3x^2 + 5x$$
$$- \quad + \quad -$$
$$-2x^2 + 3x - 5$$
$$-2x^2 + 3x - 5$$
$$+ \quad - \quad +$$
$$\times$$

Therefore, the quotient is (x-1) and the remainder is 0.

SOLUTIONS 15

Answer :

$$2x^2 + x - 1 \overline{\big) \; 8x^4 + 10x^3 - 5x^2 - 4x + 1 \big(} \; 4x^2 + 3x - 2$$
$$8x^4 + 4x^3 - 4x^2$$
$$- \quad - \quad +$$
$$6x^3 - x^2 - 4x + 1$$
$$6x^3 + 3x^2 - 3x$$
$$- \quad - \quad +$$
$$-4x^2 - x + 1$$
$$-4x^2 - 2x + 2$$
$$+ \quad + \quad -$$
$$x - 1$$

Therefore, the quotient is ($4x^2$+ $3x$ -2) and the remainder is (x-1).

Exercise 6D

SOLUTIONS 1

Answer :

(i) We have:

$$\left(x+6\right)\left(x+6\right)$$
$$=\left(x+6\right)^2$$
$$=x^2+6^2+2\times x\times 6 \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=x^2+36+12x$$

(ii) We have:

$$\left(4x+5y\right)\left(4x+5y\right)$$
$$=\left(4x+5y\right)^2$$
$$=(4x)^2+(5y)^2+2\times 4x\times 5y \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=16x^2+25y^2+40xy$$

(iii) We have:

$$\left(7a+9b\right)\left(7a+9b\right)$$

$$=\left(7a+9b\right)^2$$

$$=(7a)^2+(9b)^2+2\times 7a\times 9b \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$

$$=49a^2+81b^2+126ab$$

(iv) We have:

$$\left(\frac{2}{3}x+\frac{4}{5}y\right)\left(\frac{2}{3}x+\frac{4}{5}y\right)$$

$$=\left(\frac{2}{3}x+\frac{4}{5}y\right)^2$$

$$=\left(\frac{2}{3}x\right)^2+\left(\frac{4}{5}y\right)^2+2\times\frac{2}{3}x\times\frac{4}{5}y \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$

$$=\frac{4}{9}x^2+\frac{16}{25}y^2+\frac{16}{15}xy$$

(v) We have:

$$\left(x^2+7\right)\left(x^2+7\right)$$

$$=\left(x^2+7\right)^2$$

$$=(x^2)^2+7^2+2\times x^2\times 7 \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$

$$=x^4+49+14x^2$$

(vi) We have:

$$\left(\frac{5}{6}a^2+2\right)\left(\frac{5}{6}a^2+2\right)$$

$$=\left(\frac{5}{6}a^2+2\right)^2$$

$$=\left(\frac{5}{6}a^2\right)^2+(2)^2+2\times\frac{5}{6}a^2\times 2 \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$

$$=\frac{25}{36}a^4+4+\frac{10}{3}a^2$$

SOLUTIONS 2

Answer :

(i) We have:

$\left(x-4\right)\left(x-4\right)$

$=\left(x-4\right)^2$

$=x^2-2\times x\times 4+4^2$ $\quad\left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$

$=x^2-8x+16$

(ii) We have:

$\left(2x-3y\right)\left(2x-3y\right)$

$=\left(2x-3y\right)^2$

$=(2x)^2-2\times 2x\times 3y+(3y)^2$ $\quad\left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$

$=4x^2-12xy+9y^2$

(iii) We have:

$\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$

$=\left(\frac{3}{4}x-\frac{5}{6}y\right)^2$

$= \left(\frac{3}{4}x\right)^2 - 2 \times \frac{3}{4}x \times \frac{5}{6}y + \left(\frac{5}{6}y\right)^2 \qquad \left[\text{using } \left(a-b\right)^2 = a^2 - 2ab + b^2\right]$

$= \frac{9}{16}x^2 - \frac{15}{12}xy + \frac{25}{36}y^2$

(iv) We have:

$\left(x - \frac{3}{x}\right)\left(x - \frac{3}{x}\right)$

$= \left(x - \frac{3}{x}\right)^2$

$= (x)^2 - 2 \times x \times \frac{3}{x} + \left(\frac{3}{x}\right)^2 \qquad \left[\text{using } \left(a-b\right)^2 = a^2 - 2ab + b^2\right]$

$= x^2 - 6 + \frac{9}{x^2}$

(v) We have:

$\left(\frac{1}{3}x^2 - 9\right)\left(\frac{1}{3}x^2 - 9\right)$

$= \left(\frac{1}{3}x^2 - 9\right)^2$

$= \left(\frac{1}{3}x^2\right)^2 - 2 \times \frac{1}{3}x^2 \times 9 + (9)^2 \qquad \left[\text{using } \left(a-b\right)^2 = a^2 - 2ab + b^2\right]$

$= \frac{1}{9}x^4 - 6x^2 + 81$

(vi) We have:

$\left(\frac{1}{2}y^2 - \frac{1}{3}y\right)\left(\frac{1}{2}y^2 - \frac{1}{3}y\right)$

$= \left(\frac{1}{2}y^2 - \frac{1}{3}y\right)^2$

$= \left(\frac{1}{2}y^2\right)^2 - 2 \times \frac{1}{2}y^2 \times \frac{1}{3}y + \left(\frac{1}{3}y\right)^2 \qquad \left[\text{using } \left(a-b\right)^2 = a^2 - 2ab + b^2\right]$

$= \frac{1}{4}y^4 - \frac{1}{3}y^3 + \frac{1}{9}y^2$

SOLUTIONS 3

Answer :

We shall use the identities $(a+b)^2 = a^2 + b^2 + 2ab$ and $(a-b)^2 = a^2 + b^2 - 2ab$.

(i) We have:

$$\left(8a+3b\right)^2$$
$$=(8a)^2+2\times 8a\times 3b+(3b)^2$$
$$=64a^2+48ab+9b^2$$

(ii)We have:

$$\left(7x+2y\right)^2$$
$$=(7x)^2+2\times 7x\times 2y+(2y)^2$$
$$=49x^2+28xy+4y^2$$

(iii) We have :

$$\left(5x+11\right)^2$$
$$=(5x)^2+2\times 5x\times 11+(11)^2$$
$$=25x^2+110x+121$$

(iv) We have:

$$\left(\frac{a}{2}+\frac{2}{a}\right)^2$$
$$=\left(\frac{a}{2}\right)^2+2\times\frac{a}{2}\times\frac{2}{a}+\left(\frac{2}{a}\right)^2$$
$$=\frac{a^2}{4}+2+\frac{4}{a^2}$$

(v) We have:

$$\left(\frac{3x}{4}+\frac{2y}{9}\right)^2$$
$$=\left(\frac{3x}{4}\right)^2+2\times\frac{3x}{4}\times\frac{2y}{9}+\left(\frac{2y}{9}\right)^2$$
$$=\frac{9x^2}{16}+\frac{1}{3}xy+\frac{4y^2}{81}$$

(vi) We have:

$$\left(9x-10\right)^2$$
$$(9x)^2-2\times 9x\times 10+(10)^2$$
$$=81x^2-180x+100$$

(vii) We have:

$$\left(x^2y - yz^2\right)^2$$

$$\left(x^2y\right)^2 - 2\times x^2y\times yz^2 + \left(yz^2\right)^2$$

$$= x^4y^2 - 2x^2y^2z^2 + y^2z^4$$

(viii) We have:

$$\left(\frac{x}{y} - \frac{y}{x}\right)^2$$

$$= \left(\frac{x}{y}\right)^2 - 2\times\frac{x}{y}\times\frac{y}{x} + \left(\frac{y}{x}\right)^2$$

$$= \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$$

(ix) We have:

$$\left(3m - \frac{4}{5}n\right)^2$$

$$= (3m)^2 - 2\times 3m\times\frac{4}{5}n + \left(\frac{4}{5}n\right)^2$$

$$= 9m^2 - \frac{24mn}{5} + \frac{16}{25}n^2$$

SOLUTIONS 4

Answer :

(i) We have:

$$\left(x+3\right)\left(x-3\right)$$

$$= x^2 - 9 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(ii) We have:

$$\left(2x+5\right)\left(2x-5\right)$$

$$= 4x^2 - 25 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(iii) We have:

$$\left(8+x\right)\left(8-x\right)$$

$$= 64 - x^2 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(iv) We have:

$$\left(7x+11y\right)\left(7x-11y\right)$$
$$=49x^2-121y^2 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

(v) We have:

$$\left(5x^2+\frac{3}{4}y^2\right)\left(5x^2-\frac{3}{4}y^2\right)$$
$$=25x^4-\frac{9}{16}y^4 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

(vi) We have:

$$\left(\frac{4x}{5}-\frac{5y}{3}\right)\left(\frac{4x}{5}+\frac{5y}{3}\right)$$
$$=\frac{16x^2}{25}-\frac{25y^2}{9} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right)\right]$$

(vii) We have:

$$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$
$$=x^2-\frac{1}{x^2} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

(viii) We have:

$$\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)$$
$$=\frac{1}{x^2}-\frac{1}{y^2} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

(ix) We have:

$$\left(2a+\frac{3}{b}\right)\left(2a-\frac{3}{b}\right)$$
$$=4a^2-\frac{9}{b^2} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

SOLUTIONS 5

Answer :

We shall use the identity $(a+b)^2 = a^2 + b^2 + 2ab$.

(i)

$(54)^2$
$= \left(50+4\right)^2$
$= (50)^2 + 2 \times 50 \times 4 + (4)^2$
$= 2500 + 400 + 16$
$= 2916$

(ii)

$(82)^2$
$= \left(80+2\right)^2$
$= (80)^2 + 2 \times 80 \times 2 + (2)^2$
$= 6400 + 320 + 4$
$= 6724$

(iii)

$(103)^2$
$= \left(100+3\right)^2$
$= (100)^2 + 2 \times 100 \times 3 + (3)^2$
$= 10000 + 600 + 9$
$= 10609$

(iv)

$(704)^2$
$= \left(700+4\right)^2$
$= (700)^2 + 2 \times 700 \times 4 + (4)^2$
$= 490000 + 5600 + 16$
$= 495616$

SOLUTIONS 6

Answer :

We shall use the identity $(a-b)^2 = a^2 + b^2 - 2ab$.

(i)

$$(69)^2$$
$$= \left(70 - 1\right)^2$$
$$= (70)^2 - 2 \times 70 \times 1 + 1$$
$$= 4900 - 140 + 1$$
$$= 4761$$

(ii)

$$(78)^2$$
$$= \left(80 - 2\right)^2$$
$$= (80)^2 - 2 \times 80 \times 2 + 4$$
$$= 6400 - 320 + 4$$
$$= 6084$$

(iii)

$$(197)^2$$
$$= \left(200 - 3\right)^2$$
$$= (200)^2 - 2 \times 200 \times 3 + 9$$
$$= 40000 - 1200 + 9$$
$$= 38809$$

(iv)

$$(999)^2$$
$$= \left(1000 - 1\right)^2$$
$$= (1000)^2 - 2 \times 1000 \times 1 + 1$$
$$= 1000000 - 2000 + 1$$
$$= 998001$$

SOLUTIONS 7

Answer :

We shall use the identity $(a-b)(a+b)=a^2 - b^2$.

(i)

$$(82)^2 - (18)^2$$
$$= (82-18)(82+18)$$
$$= (64)(100)$$
$$= 6400$$

(ii)

$$(128)^2 - (72)^2$$
$$= (128-72)(128+72)$$
$$= (56)(200)$$
$$= 11200$$

(iii)

$$197 \times 203$$
$$= \left(200-3\right)\left(200+3\right)$$
$$= (200)^2 - (3)^2$$
$$= 40000 - 9$$
$$= 39991$$

(iv)

$$\frac{198\times198-102\times102}{96}$$
$$=\frac{(198)^2-(102)^2}{96}$$
$$=\frac{(198-102)(198+102)}{96}$$
$$=\frac{(96)(300)}{96}$$
$$=300$$

(v)

$$(14.7\times15.3)$$
$$=(15-0.3)\times(15+0.3)$$
$$=(15)^2-(0.3)^2$$
$$=225-0.09$$
$$=224.91$$

(vi)

$$(8.63)^2-(1.37)^2$$
$$=(8.63-1.37)(8.63+1.37)$$
$$=(7.26)(10)$$
$$=72.6$$

SOLUTIONS 8

Answer :

$$(9x^2+24x+16)$$
$$Given,\ x=12$$
$$\Rightarrow(3x)^2+2(3x)(4)+(4)^2$$
$$\Rightarrow(3x+4)^2$$
$$\Rightarrow(3(12)+4)^2$$
$$\Rightarrow(36+4)^2$$
$$\Rightarrow(40)^2=1600$$

Therefore, the value of the expression ($9x^2 + 24x + 16$), when $x = 12$, is 1600.

SOLUTIONS 9

Answer :

$\left(64x^2+81y^2+144xy\right)$

$Given:$

$x=11$

$y=\frac{4}{3}$

$\Rightarrow(8x)^2+(9y)^2+2(8x)(9y)$

$\Rightarrow(8x+9y)^2$

$\Rightarrow\left(8\left(11\right)+9\left(\frac{4}{3}\right)\right)^2$

$\Rightarrow(88+12)^2$

$\Rightarrow(100)^2$

$\Rightarrow10000$

Therefore, the value of the expression $(64x^2 + 81y^2 + 144xy)$, when $x = 11$ and $y = \frac{4}{3}$, is 10000.

SOLUTIONS 10

Answer :

$\left(36x^2+25y^2-60xy\right)$

$\Rightarrow x=\frac{2}{3}, y=\frac{1}{5}$

$=(6x)^2+(5y)^2-2(6x)(5y)$

$=(6x-5y)^2$

$=\left(6\left(\frac{2}{3}\right)-5\left(\frac{1}{5}\right)\right)^2$

$=(4-1)^2$

$=(3)^2$

$\Rightarrow9$

SOLUTIONS 11

Answer :

$\left(i\right)\ \left(x+\frac{1}{x}\right)=4$

Squaring both the sides :

$\Rightarrow\left(x+\frac{1}{x}\right)^2=(4)^2$

$\Rightarrow\left(x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)\right)=16$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)+2=16$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)=16-2$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)=14$

Therefore, the value of $x^2+\frac{1}{x^2}$ is 14.

$\left(x^2+\frac{1}{x^2}\right)=14$

Squaring both the sides :

$\Rightarrow\left(x^4+\frac{1}{x^4}+2(x^2)\left(\frac{1}{x^2}\right)\right)=(14)^2$

$\Rightarrow\left(x^4+\frac{1}{x^4}\right)+2=196$

$\Rightarrow\left(x^4+\frac{1}{x^4}\right)=196-2$

$\Rightarrow\left(x^4+\frac{1}{x^4}\right)=194$

Therefore, the value of $x^4+\frac{1}{x^4}$ is 194.

SOLUTIONS 12

Answer :

$\left(i\right)\ \left(x-\frac{1}{x}\right)=5$

$\Rightarrow$ Squaring both the sides :

$\Rightarrow\left(x-\frac{1}{x}\right)^2=(5)^2$

$\Rightarrow\left(x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)\right)=25$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)-2=25$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)=25+2$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)=27$

Therefore, the value of $\left(x^2+\frac{1}{x^2}\right)$ is 27.

$\left(x^2+\frac{1}{x^2}\right)=27$

$\Rightarrow$ Squaring both the sides :

$\Rightarrow\left(x^4+\frac{1}{x^4}-2(x^2)\left(\frac{1}{x^2}\right)\right)=(27)^2$

$\Rightarrow\left(x^4+\frac{1}{x^4}\right)-2=729$

$\Rightarrow\left(x^4+\frac{1}{x^4}\right)=729+2$

$\Rightarrow\left(x^4+\frac{1}{x^4}\right)=731$

Therefore, the value of $\left(x^4+\frac{1}{x^4}\right)$ is 731.

SOLUTIONS 13

Answer :

(i) $(x+1)(x-1)(x^2+1)$
$\Rightarrow (x^2-x+x-1)(x^2+1)$
$\Rightarrow (x^2-1)(x^2+1)$
$\Rightarrow (x^2)^2-(1^2)^2$ $\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow x^4-1.$
Therefore, the product of $(x+1)(x-1)(x^2+1)$ is x^4-1.

(ii) $(x-3)(x+3)(x^2+9)$
$\Rightarrow \left((x)^2-(3)^2\right)(x^2+9)$ $\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow (x^2-9)(x^2+9)$
$\Rightarrow (x^2)^2-(9)^2$ $\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow x^4-81$
Therefore, the product of $(x-3)(x+3)(x^2+9)$ is x^4-81.

(iii) $(3x-2y)(3x+2y)(9x^2+4y^2)$
$\Rightarrow \left((3x)^2-(2y)^2\right)(9x^2+4y^2)$
$\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow (9x^2-4y^2)(9x^2+4y^2)$
$\Rightarrow (9x^2)^2-(4y^2)^2$ $\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow 81x^4-16y^4.$
Therefore, the product of $(3x-2y)(3x+2y)(9x^2+4y^2)$ is $81x^4-16y^4$.

(iv) $(2p+3)(2p-3)(4p^2+9)$
$\Rightarrow \left((2p)^2-(3)^2\right)(4p^2+9)$ $\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow (4p^2-9)(4p^2+9)$
$\Rightarrow (4p^2)^2-(9)^2$ $\left[\text{according to the formula } a^2-b^2=(a+b)(a-b)\right]$
$\Rightarrow 16p^4-81.$
Therefore, the product of $(2p+3)(2p-3)(4p^2+9)$ is $16p^4-81$.

Answer :

$x + y = 12$

On squaring both the sides :

$\Rightarrow (x+y)^2 = (12)^2$

$\Rightarrow x^2 + y^2 + 2xy = 144$

$\Rightarrow x^2 + y^2 = 144 - 2xy$

Given :

$xy = 14$

$\Rightarrow x^2 + y^2 = 144 - 2(14)$

$\Rightarrow x^2 + y^2 = 144 - 28$

$\Rightarrow x^2 + y^2 = 116$

Therefore, the value of $x^2 + y^2$ is 116.

SOLUTIONS 15

Answer :

$x - y = 7$

$\Rightarrow$ On squaring both the sides :

$\Rightarrow (x-y)^2 = (7)^2$

$\Rightarrow x^2 + y^2 - 2xy = 49$

$\Rightarrow x^2 + y^2 = 49 + 2xy$

Given :

$xy = 9$

$\Rightarrow x^2 + y^2 = 49 + 2(9)$

$\Rightarrow x^2 + y^2 = 49 + 18$

$\Rightarrow x^2 + y^2 = 67.$

Therefore, the value of $x^2 + y^2\ is\ 67.$