Design of Experiments
Part 2

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Steps for Designing and Conducting DOE

1. **Look at historical data and/or collect data to determine current process capability.**
2. **Determine the objective of the experiment.**
3. **Determine what to measure as the output of the experiment.**
4. **Identify factors (control factors and noise factors) that could affect the output.**
5. **Determine the number of levels for each factor, and their actual values.**

What we did in the previous module!

Steps for Designing and Conducting DOE

6. **Select an experimental layout that will accommodate the selected factors and levels and decide number of repetitions or replications.**
7. **Verify all measurement systems (Remember: garbage in, garbage out!).**
8. **Plan and prepare the resources (people, materials, etc.) for conducting the experiment. Should we randomize the runs? Create a test plan.**
9. **Conduct the experiment. Where appropriate, assure that each unit is labeled according to the experimental condition by which it was produced.**
10. **Measure the experimental units.**
11. **Analyze the data and identify strong factors for mean and for variation.**

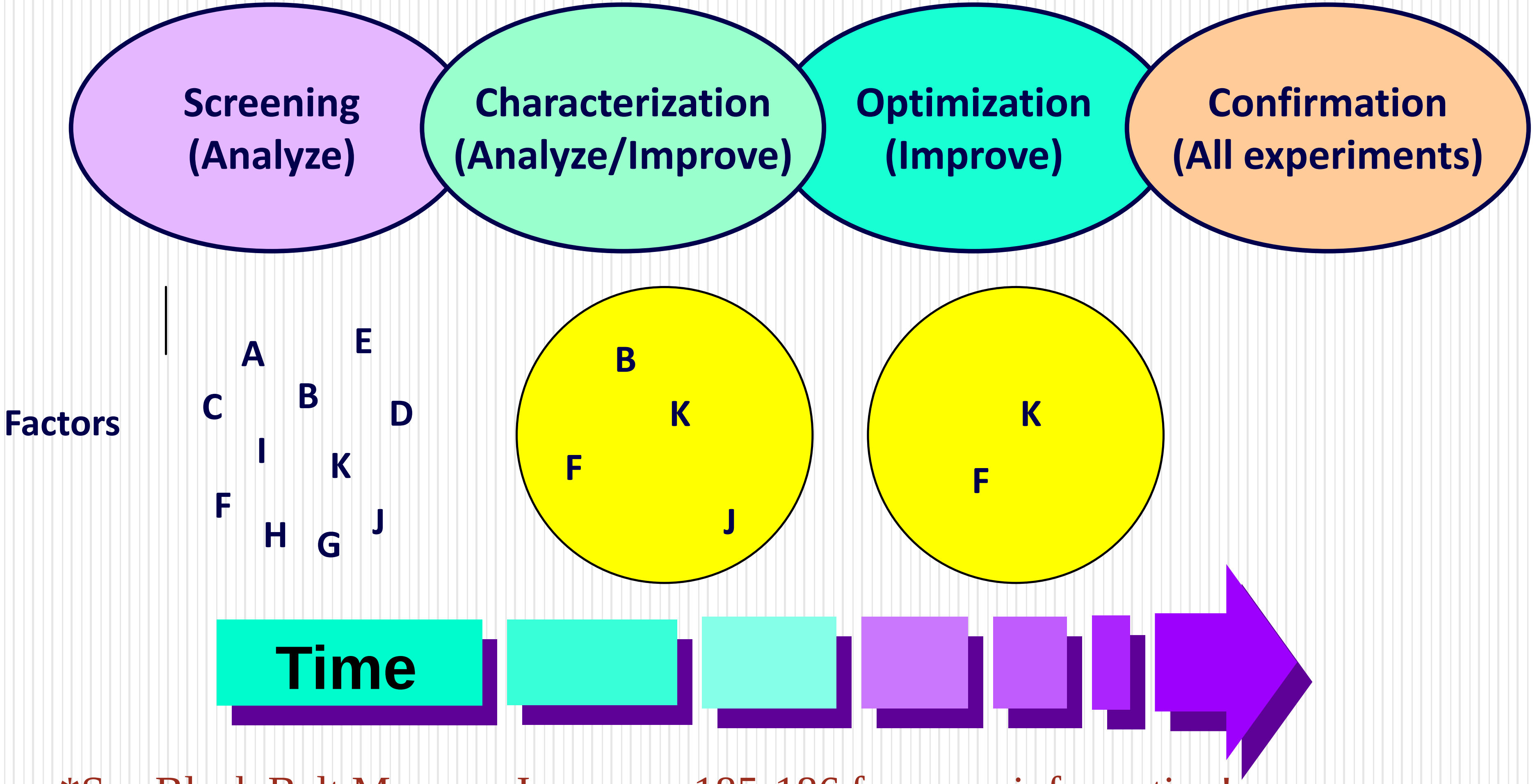
Selecting the Type of Experiment

6. Select an experimental layout.

Many types of experiments exist.

- Selecting the type of experiment best suited for a particular application depends on the….
  - Objective of the study
    (what questions do we want answered)
  - Number of factors and levels
    we wish to investigate
  - Cost of each experimental trial

Design of Experiments – Stages



*See Black Belt Memory Jogger, p. 185-186 for more information!

Types of Experiments

Common Types of Experiments	Objectives	Typical Number of Controllable Factors
1. Full Factorial (all combinations of factors and levels)	• To find the factor levels that provide the best results. • To build a math model (evaluates all interactions)	4 or fewer
2. Fractional Factorial (subset of total number of combinations)	• To find the factor levels that provide the best results • To build a math model (evaluates some interactions)	5 or more
3. Screening	• To test a large number of factors to find the vital few. (evaluates no interactions)	7 or more

Types of Experiments (Cont.)

Common Types of Experiments	Objectives	Typical Number of Controllable Factors
4. Central Composite Design (CCD), or Box-Behnken	• Optimization • To build a math model when non-linear effects are present (Response Surface Methodology is often used)	3 or more
5. Robust Design	• Optimization • To find the factor levels that minimize variation in the presense of changing noise factors	5 or more
6. Taguchi's Dynamic Robust Design (Ideal Function)	• Optimization • To optimize the function of a product or manufacturing process • To minimize sensitivity to noise and maximize sensitivity to input signal.	7 or more

Over-achievers' page. This material falls beyond the scope of Black Belt training. See examples & references at the end of this module for further insight.

Additional Approaches to Optimizing Processes

- Traditional Approach
- If there are a large number of factors, run a screening experiment to identify the vital few.
- Thoroughly study the vital few factors using a factorial, central composite or Box-Behken design.

Dr. Taguchi's Robust Design

- **Determine the Ideal Function of the product or manufacturing process.**
- **Study a large number of control factors in one dynamic experiment to discover the combination that minimizes the deviation from ideal function.**

Note

Over-achievers' page. We will use the traditional approach. Dr. Taguchi's Robust Design is beyond the scope of this course.

Sample Size

- Sample size (the number of data values at each test combination) impacts experimental error, power, and reliability of results.
- Generally, the more data (the more degrees of freedom), the better the estimate.
- However, practical considerations (time, cost, etc.) must be weighed against statistical considerations.
- While exceptions may exist, a good rule of thumb is to collect a minimum of 3 data values for each test combination (unlike the preceding example!).

Note: Degrees of Freedom and Experimental Error are important statistical concepts that exceed the scope of our practitioner-based training.

Repetition, Replication

- Repetition means that all of the data for a test combination is collected without resetting the run (eg. Set up the statapult and shoot three times).
- Replication means that each data value is collected after resetting the test combination (eg. Statapult reset to different configuration after each shot).
- Questions
  1. How does the method of collecting data (repetition vs. replication) affect the results of your study?
  2. Which method tends to be more accurate by randomizing noise?
  3. Which method is more costly?
  4. Which method do you think is used more often in industrial experiments?

These two terms are often (improperly) used interchangeably!

Fractional Factorial Designs

Fractional Factorial Designs

- Contents
  - What is Fractional Design
  - Why use a Fractional Design
  - Confounding and Aliasing
  - Design Resolution
  - 2 - Level Fractional Factorials
  - Selection Strategy
  - DOE Planning
  - DOE for Variation

Fractional Factorial Designs – Learning Objectives

After completing this module, you will be able to.....

1. Design, conduct and analyze fractional factorial experiments.
2. Select an experimental design that allows clear evaluation of factor effects and interaction effects (especially 2-factor interactions) thought to be strong.
3. Perform DOE for Variation

Fractional Factorials

- In fractional factorial designs, a fraction of the total number of combinations are run.

Full Factorial

Run \ Factor	A	B	C
1	-1	-1	-1
2	-1	-1	+1
3	-1	+1	-1
4	-1	+1	+1
5	+1	-1	-1
6	+1	-1	+1
7	+1	+1	-1
8	+1	+1	+1

Fractional Factorial

Run \ Factor	A	B	C
1	-1	-1	+1
2	+1	-1	-1
3	-1	+1	-1
4	+1	+1	+1

These 4 runs represent half of the total combinations that exist for 3 factors at 2 levels.

Why Use Fractional Factorials

- Often, experimenters have a large number of factors to investigate.
- The size of full factorials limits their use to only a few factors.

Example

The full factorial for 1 2-level factor and 7 3-level factors requires…

2^1 x 3^7 = 4374 test combinations.

These same factors can be investigated using only 18 combinations, but *what do we lose?*

Obtaining the Critical Information

When all test combinations are not run, information will be lost.

- If the test combinations (fraction) to be run are carefully selected, the information we care about the most can be obtained:

  Main effects
  2 - factor interaction effects

- This approach has been standardized with the development of numerous fractional factorial designs.

Translation: YOU don't have to invent the DOE designs!

Fractional Factorials - Confounding

A major concern with fractional factorials is that effects are confounded (confused) with each other. To illustrate this consider the standard 1/2 fraction of a 23 design.

	Main Effects			2-Way Interactions			3-Way Interactions	3 Replications of each run		
Factor / Run	A	B	C	AB	AC	BC	ABC	Data		
1	-1	-1	+1	+1	-1	-1	+1	X_1	X_2	X_3
2	+1	-1	-1	-1	-1	+1	+1	X_4	X_5	X_6
3	-1	+1	-1	-1	+1	-1	+1	X_7	X_8	X_9
4	+1	+1	+1	+1	+1	+1	+1	X_{10}	X_{11}	X_{12}

Note: The interaction columns are generated by multiplying appropriate columns, and are used by Minitab in DOE analysis.

Confounding (continued)

Notice the ABC column of all +1's; the ABC effect can not be evaluated.

Notice the A and BC columns are identical as are B and AC, C and AB.

	Main Effects			2-Way Interactions			3-Way Interactions	3 Replications of each run		
Factor / Run	A	B	C	AB	AC	BC	ABC	Data		
1	-1	-1	+1	+1	-1	-1	+1	X_1	X_2	X_3
2	+1	-1	-1	-1	-1	+1	+1	X_4	X_5	X_6
3	-1	+1	-1	-1	+1	-1	+1	X_7	X_8	X_9
4	+1	+1	+1	+1	+1	+1	+1	X_{10}	X_{11}	X_{12}

This means that when effects are computed the analysis will give:

A effect = BC effect
B effect = AC effect
C effect = AB effect

Confounding (continued)

Factor / Run	Main Effects			2-Way Interactions			3-Way Interactions	3 Replications of each run		
	A	B	C	AB	AC	BC	ABC	Data		
1	-1	-1	+1	+1	-1	-1	+1	X_1	X_2	X_3
2	+1	-1	-1	-1	-1	+1	+1	X_4	X_5	X_6
3	-1	+1	-1	-1	+1	-1	+1	X_7	X_8	X_9
4	+1	+1	+1	+1	+1	+1	+1	X_{10}	X_{11}	X_{12}

This does *not* mean that the effects are equal, it means the design cannot separate them.

What is being measured in the Minitab analysis is

A + BC, B + AC, and C + AB, the *combined* effects.

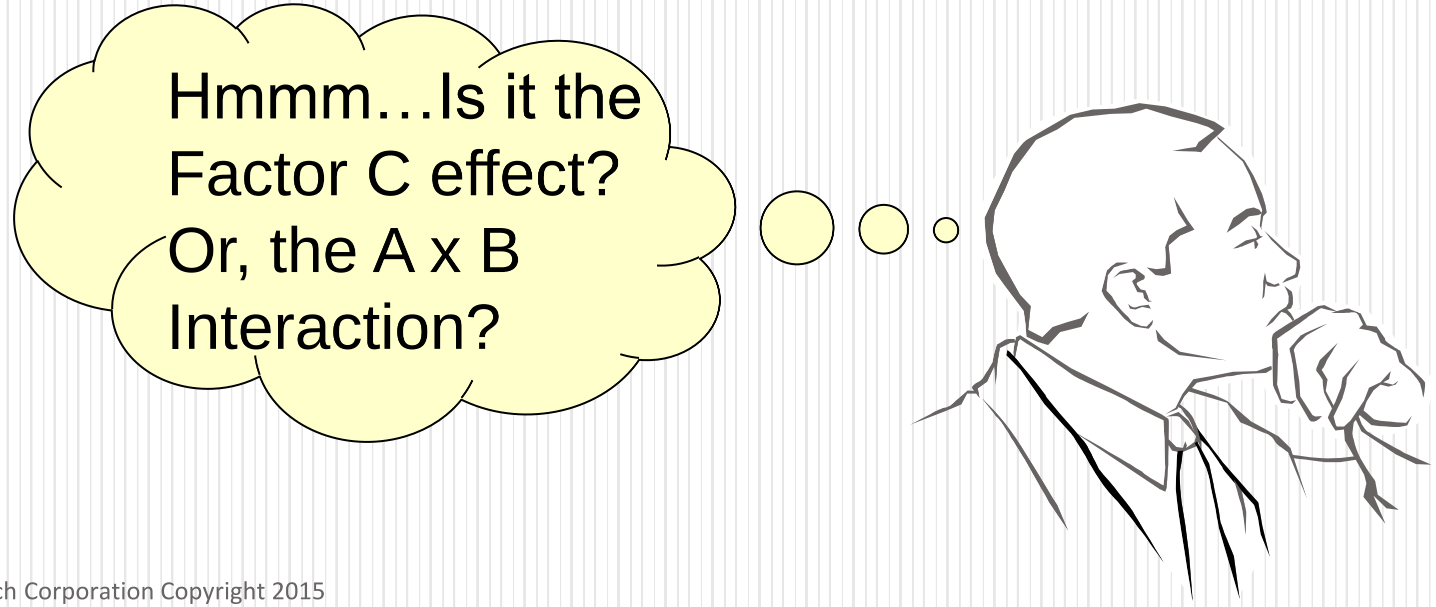
We say A is **confounded** with BC and write it as:

$A \equiv BC$, $B \equiv AC$, $C \equiv AB$.

The *actual* effects being measured is called the **Alias pattern**.

Caution When Selecting Type of Experiment

- Whenever all possible combinations of factors and levels are NOT run, certain factor and interaction effects will be confused together (confounded).
- Care should be taken to select an experimental design that allows clear evaluation of main effects and interaction effects (especially 2-factor interactions) thought to be strong.



Design Resolution

In the 23 half fraction design, factor effects are confounded with 2 - factor interactions (2-f.i.).

We say this is a Resolution III design.

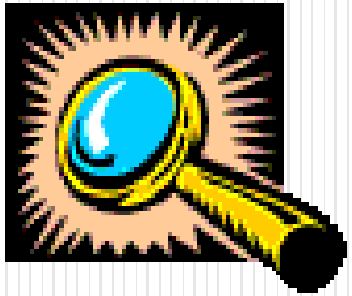
The general relationship is:

Resolution III: Factor $\equiv$ 2 f.i.

Resolution IV : Factor $\equiv$ 3 f.i., 2 f.i. $\equiv$ 2 f.i.

Resolution V: Factor $\equiv$ 4 f.i., 2 f.i. $\equiv$ 3 f.i.

Think of your digital camera. *Higher* Resolution = *Better!*



Note: Minitab shows DOE resolution along with red/yellow/green warning when you select your design.

Selecting Design Resolution

Guidelines for selecting which resolution design to use is:

Resolution III if team believes no strong 2 factor interactions (f.i.'s) are present (many factor screening DOE).

Resolution IV if suspect 2 f.i.'s, so don't want main effects confused with 2 f.i. Not interested in evaluating 2 f.i.'s. (more runs required than Resolution III DOE).

Resolution V if need to evaluate main effects *and* 2 f.i.'s.

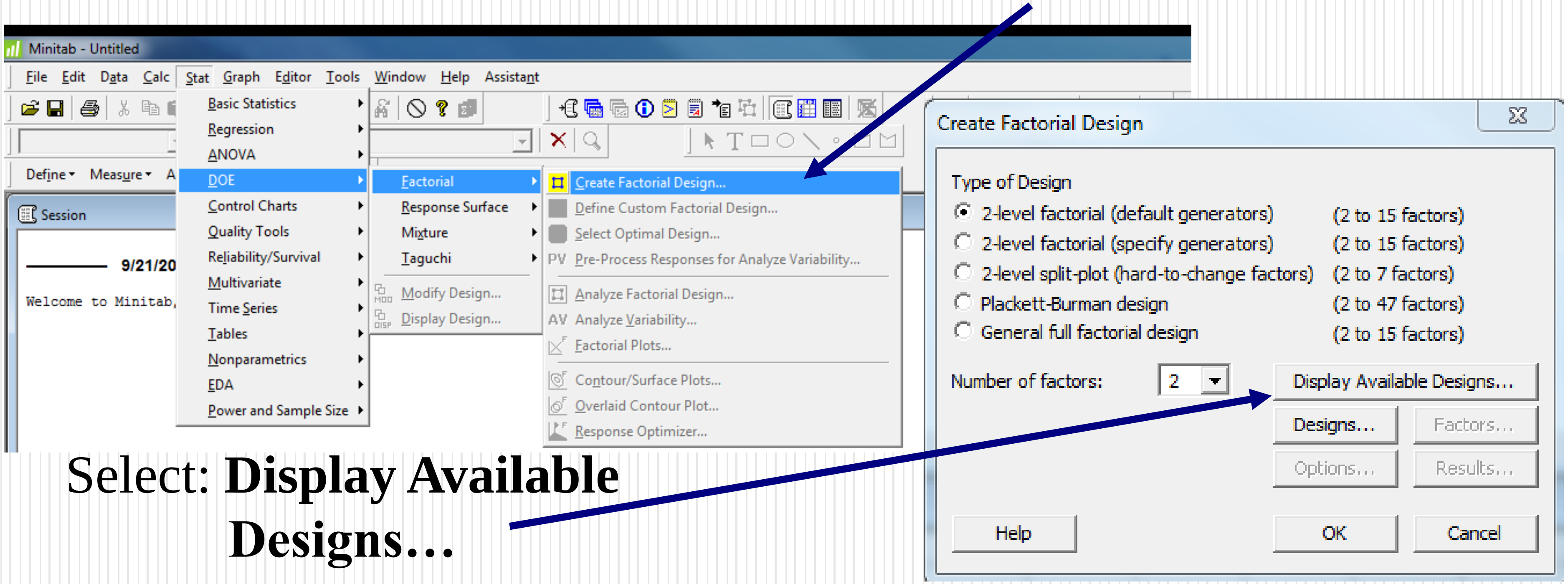
(More runs than Resolution IV DOE).

Note the problem: For a fixed number of factors, higher resolution = more data = more runs!

Minitab & Confounding

Minitab will generate the 1/2 fraction, and produce the alias structure.

Select: Stat > DOE > Create factorial design



Select: **Display Available Designs…**

Minitab & Confounding

Create Factorial Design - Display Available Designs

Available Factorial Designs (with Resolution)

	Factors													
Run	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	Full	III												
8		Full	IV	III	III	III								
16			Full	V	IV	IV	IV	III	III	III	III	III	III	III
32				Full	VI	IV	IV	IV	IV	IV	IV	IV	IV	IV
64					Full	VII	V	IV	IV	IV	IV	IV	IV	IV
128						Full	VIII	VI	V	V	IV	IV	IV	IV

Available Resolution III Plackett-Burman Designs

Factors	Runs	Factors	Runs	Factors	Runs
2-7	12,20,24,28,...,48	20-23	24,28,32,36,...,48	36-39	40,44,48
8-11	12,20,24,28,...,48	24-27	28,32,36,40,44,48	40-43	44,48
12-15	20,24,28,36,...,48	28-31	32,36,40,44,48	44-47	48
16-19	20,24,28,32,...,48	32-35	36,40,44,48		

Help OK

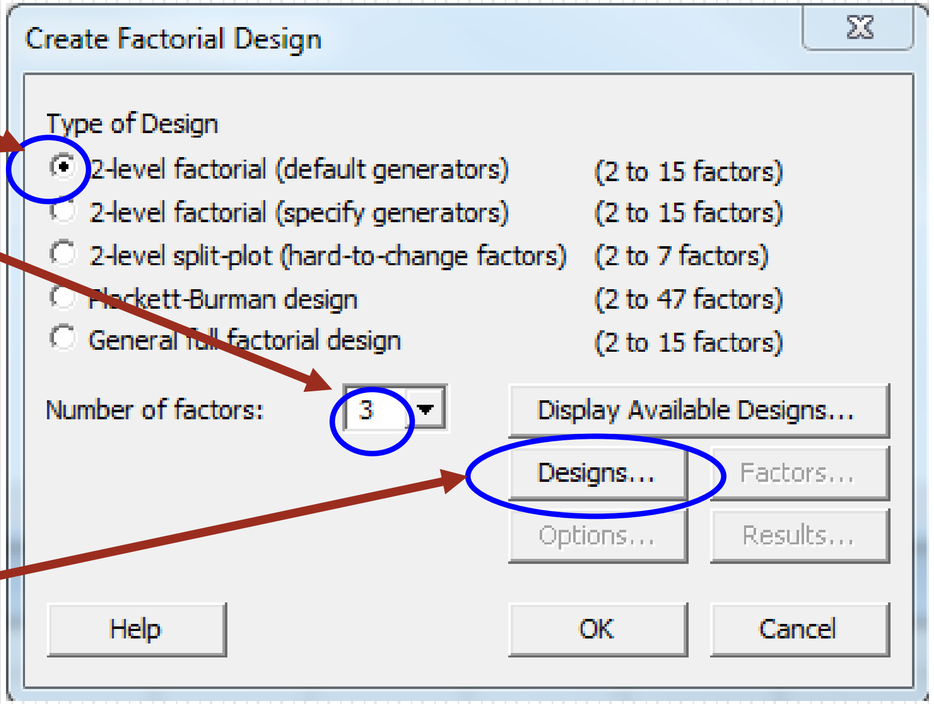
If we want to evaluate 3 Factors, each at 2 levels, how many runs will it take with a Full Factorial design?

If we cut the total runs in half (use a ½ fraction DOE design), what will the resolution of our DOE be? Is this acceptable?

Minitab & Confounding

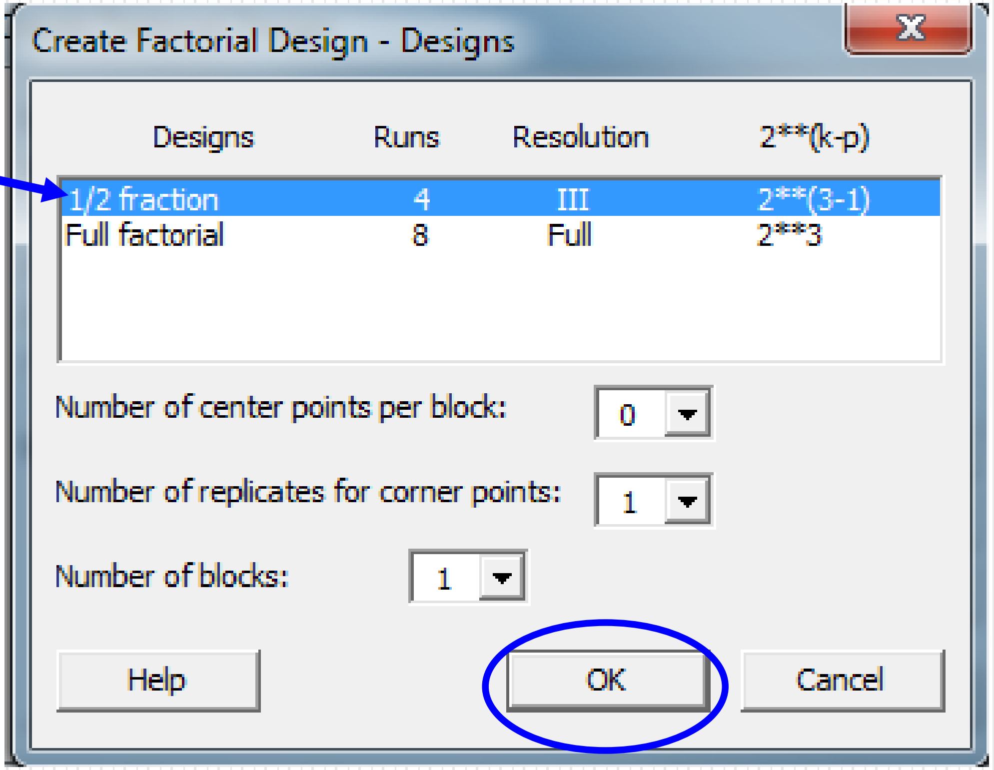
Minitab will generate the 1/2 fraction DOE design, and automatically produce the alias structure

- Close the Display Available Designs window by clicking OK.
- Click on 2 - level factorial (default generators)
- Set Number of factors = 3
- Click on ***Designs***



Minitab & Confounding

- Select **1/2 fraction**
- Click OK then OK again.



- The design will be in columns 5, 6, 7 of the worksheet.
- The alias pattern will be in the session window.

Minitab Output – Alias Structure

Fractional Factorial Design

```
Factors:   3   Base Design:          3, 4   Resolution:  III
Runs:      4   Replicates:              1   Fraction:    1/2
Blocks:    1   Center pts (total):      0

* NOTE * Some main effects are confounded with two-way
interactions.

Design Generators: C = AB

Alias Structure

I + ABC

A + BC
B + AC
C + AB
```

Minitab Session Window

Example – Injection Molding

Tired of the trade imbalance from overseas, you purchase an injection molding machine and begin making cheap plastic toys in your garage to sell at the local Stuff Mart.

Stronger parts mean fewer returns. You decide to run an experiment to maximize part strength.

Factors	Levels -1	Levels +1
A: Die Temp (°F)	130°	170°
B: Nozzle Temp (°F)	350°	375°
C: Shot Size (grams)	6.7	10
D: Injection Press.(PSI)	700	900

Your team (you, your spouse, the neighbor’s kid, and a stray dog named Sparky) strongly suspect an AxB, and AxD interaction.

No other interactions are judged likely to exist.

How many runs are required for a full factorial DOE?

Is this necessary? How do I know? (See slide 48!)

Questions for Injection Molding Experiment

Objective: To achieve *uniform part dimension* (minimum variation) at a particular *target value*.

Factors	**Output**
Cycle Time	Part Dimension
Molding Temp.	
Holding Time	
Material Type	

First ask,

- which factors affect variation in part dimension?

Then ask,

- which factors affect the average dimension?

Time Out!: What Did We Learn About the Output Metrics?

- In this molding experiment, your team decided to measure part strength (a "Maximum Value is Best" characteristic).
- Remember: As a BB, you should know the output measures that provide the greatest optimization power are those that relate to the basic function of the manufacturing process. (These are usually "Target Value is Best" characteristics. See slide 23-28).
- Is the function of the molding process to create part strength ? NO!
- The function of the process is to create parts of specific dimensions. Factor combinations that minimize variation in dimensions, are combinations that most uniformly distribute particles and stress.

What happens to "part strength" when stresses are uniformly distributed throughout the part?

Remember

Often, the best way to improve strength and other problem-related measures is not to measure the problem directly. Instead, measure and optimize the process' intended function.

Injection Molding – Selecting the Design

- A Full Factorial design will allow evaluation of the four factors and all interactions, at a cost of 16 runs.
- If the team is correct in their judgement that only two interactions are likely to exist, then a Resolution IV will work, needing only 8 runs:

In Mintab, use **Stat> DOE> Factorial> Analyze Factorial Design**

Fractional Factorial Design

```
Factors:    4    Base Design:         4, 8
               Resolution:    IV
Runs:      24    Replicates:             3
               Fraction:     1/2
Blocks:     1    Center pts (total):     0
          Design Generators: D = ABC

               Alias Structure
 I + ABCD          A + BCD          AB + CD
                   B + ACD          AC + BD
                   C + ABD          AD + BC
                                     D + ABC
```

Injection Molding – Layout and Data

- Three replications were run for each test combination as shown below:

Run \ Factor	Die Temp	Nozzle Temp	Shot Size	Inject Temp	Data Replication: Repl. 1	Repl. 2	Repl. 3
1	-1	-1	-1	-1	63	59	61
2	+1	-1	-1	+1	60	63	65
3	-1	+1	-1	+1	85	81	77
4	+1	+1	-1	-1	62	60	61
5	-1	-1	+1	+1	70	69	68
6	+1	-1	+1	-1	35	39	37
7	-1	+1	+1	-1	36	35	35
8	+1	+1	+1	+1	46	47	45

Data is Part Strength in Newtons (no, not figs!)

Injection Molding – Significant Effects

- From the MiniTab output we see that factors Die Temp, Shot Size and
- Injection Pressure and both interactions AxB, AxD are significant.

Fractional Factorial Fit: Data versus Die Temp, Nozzle Temp, ...

Estimated Effects and Coefficients for Data (coded units)

Term	Effect	Coef	Coef SE	T	P
Constant		56.625	0.7235	78.27	0.000
Die Temp	-10.000	-5.000	0.7235	-6.91	0.000
Nozzle T	-1.500	-0.750	0.7235	-1.04	0.330
Shot Siz	-19.500	-9.750	0.7235	-13.48	0.000
Inj.Pre	16.000	8.000	0.7235	11.06	0.000
Die Temp*Nozzle T	5.250	2.625	0.7235	3.63	0.007
Die Temp*Shot Siz	-0.750	-0.375	0.7235	-0.52	0.618
Die Temp*Inj.Pre	10.750	5.375	0.7235	-7.43	0.000

Injection Molding – Response Table

- Minitab also gives the average output at each level of each factor and interaction:

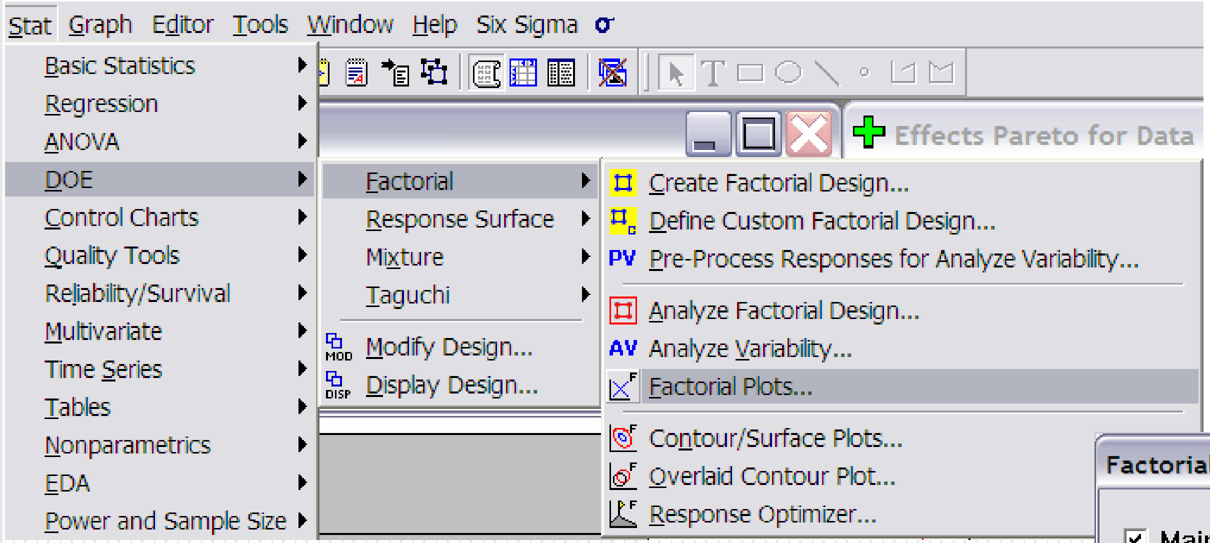
Least Squares Means for Data

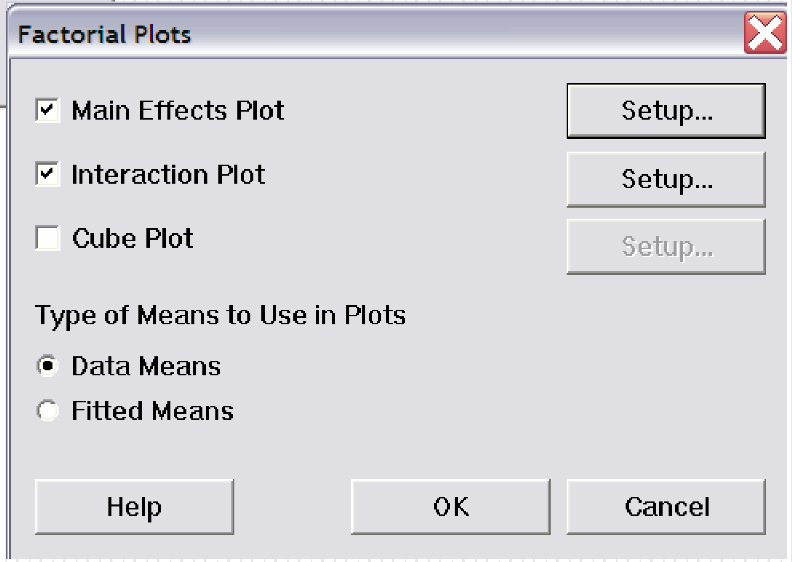
Die Temp (A)	Mean	SE Mean
-1	61.63	1.023
1	51.63	1.023
Nozzle T (B)		
-1	57.38	1.023
1	55.88	1.023
Shot Siz (C)		
-1	66.38	1.023
1	46.88	1.023
Inj. Pre (D)		
-1	48.63	1.023
1	64.63	1.023

Die Temp*Nozzle T		Mean	SE Mean
-1	-1	65.00	1.447
1	-1	49.75	1.447
-1	1	58.25	1.447
1	1	53.50	1.447
Die Temp*Shot Siz			
-1	-1	71.00	1.447
1	-1	61.75	1.447
-1	1	52.25	1.447
1	1	41.50	1.447
Die Temp*Inj. Pre			
-1	-1	48.25	1.447
1	-1	49.00	1.447
-1	1	75.00	1.447
1	1	54.25	1.447

Injection Molding – Response Table

- To convert the Session Window output into a pretty graph, we do the following:





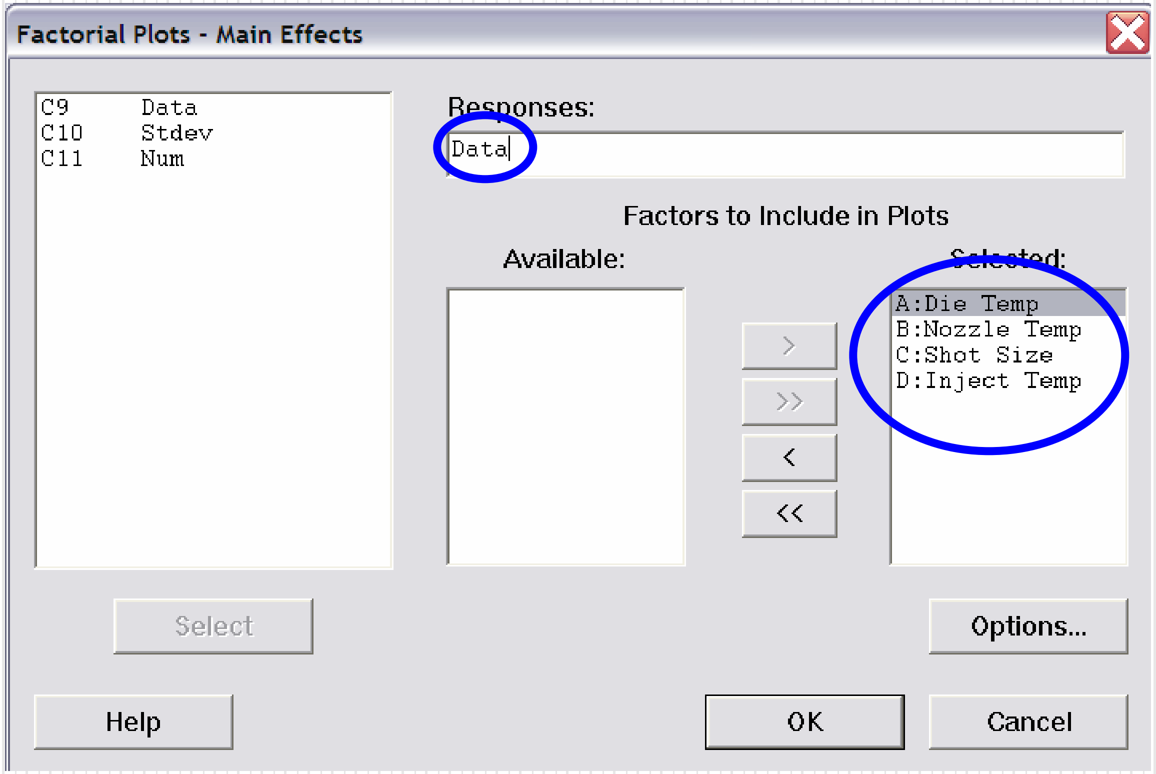
Stat>DOE>Factorial>Factorial Plots…

Select Main Effects Plot & Interaction Plot

Select Setup…

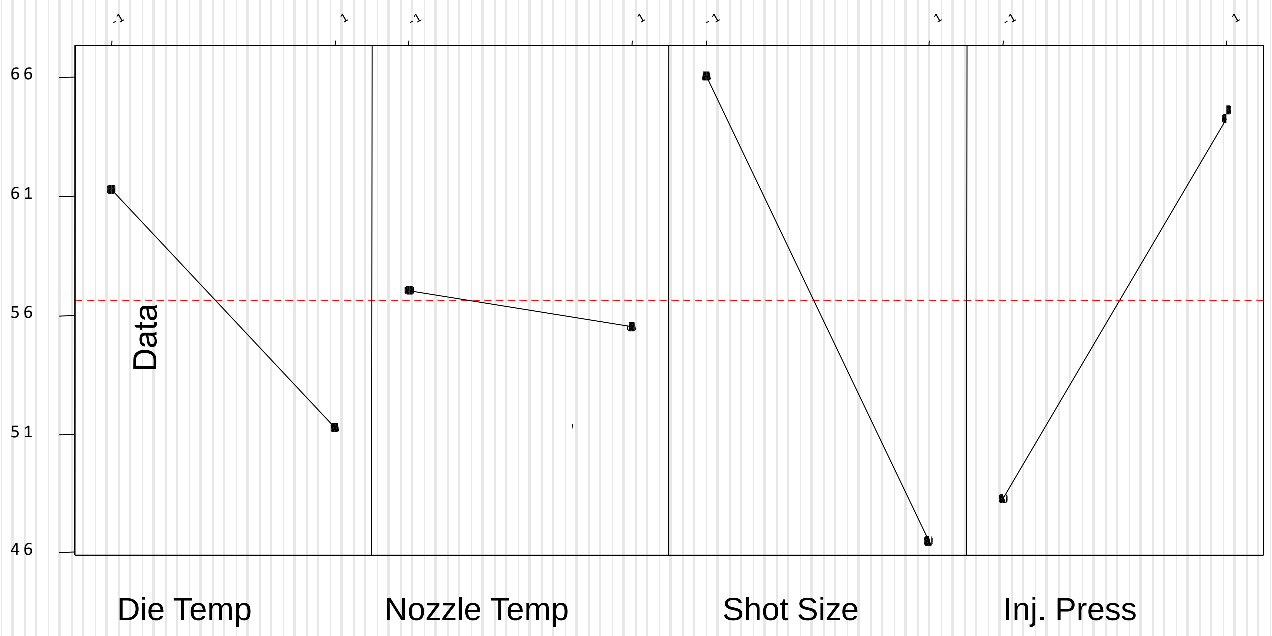
Injection Molding – Response Table

- Select the output column desired, then choose the factors you would like to evaluate (With many factors, you may choose to de-select factors that are known to be insignificant (p>1.0)
- Select OK and do the same for Interactions.



Main Effects Graphs

Main Effects Plot (data means) for Data



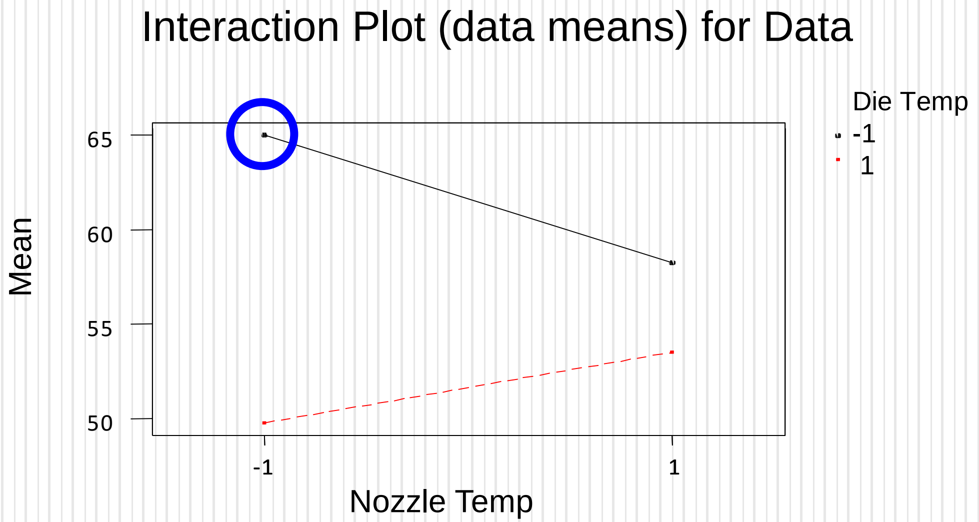
Reminder:

Does a steeper line indicate more significance or less significance?

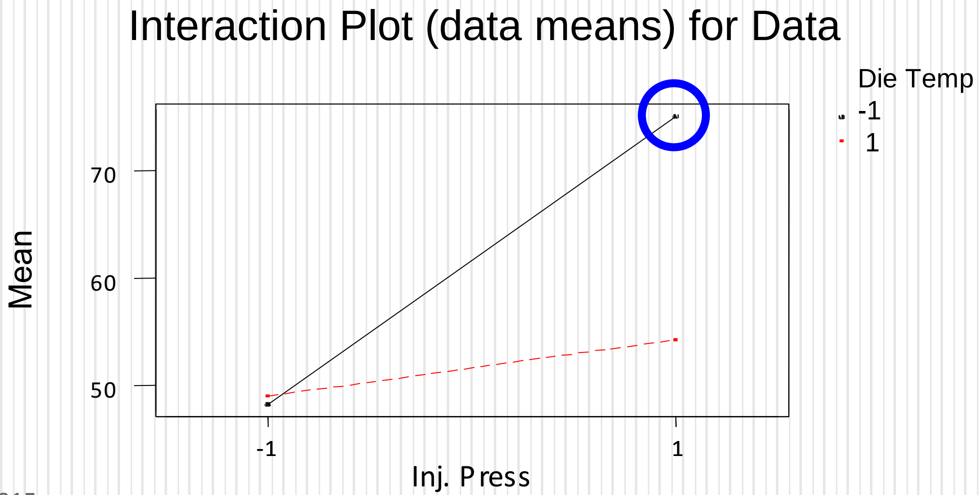
Why?

Interaction Graphs

A, B



A, D



Do more perpendicular lines indicate more significance or less significance?

Which interaction is "best" for our team?

Best Levels

To maximize strength the levels to be used are:

A_{-1} B_{-1} :highest from the AxB graph

A_{-1} D_{+1} :highest from the AxD graph

C_{-1} :highest from the C graph

The analysis picks A_{-1} B_{-1} C_{-1} D_{+1} to be the combination to maximize strength. Notice, it was not one of the tested combinations.

Prediction

We will use A (Die Temp), C (Shot Size), D (Inj. Press.), AxB, AxD as the terms in our prediction (regression) equation.

Regression Analysis: Data versus Die Temp, Shot Size, ...

```
The regression equation is
Data = 56.6 - 5.00 Die Temp - 9.75 Shot Size + 8.00 Inj. Press
          + 2.62 DTemp*NTemp - 5.38 DTemp*IPress
```

Predicted Values for New Observations

New Obs	Fit	SE Fit	95.0% CI	95.0% PI
1	87.375	1.713	(83.558, 91.192)	(80.066, 94.684)

Values of Predictors for New Observations

New Obs	Die Temp	Shot Siz	Inj. Pre	DTemp*NT	DTemp*IP
1	-1.00	-1.00	1.00	1.00	-1.00

The predicted average strength, 87.375, exceeds any in our test matrix.

DOE Strategies

- Randomization
  - Randomize the order of the runs.
  - Reduces the effects of background variables (nuisance variables).
- Blocking
  - Randomize the order of the runs within each block (for example, block of time: AM vs. PM, or Day 1 vs. Day 2).
  - The test combinations to be run within each block must be determined through DOE techniques (see future modules).

Questions

1. What are the pros and cons of randomizing the order of the runs?
2. What is the purpose of blocking?

Blocking

- Blocking is a technique used to determine which test combinations should be run, for example, within a given block of time.

The highest order interaction column is used to create the block.

	A	B	C	AB	AC	BC	ABC	Run order
1	–	–	–	+	+	+	–	1
2	+	–	–	–	–	+	+	5
3	–	+	–	–	+	–	+	6
4	+	+	–	+	–	–	–	2
5	–	–	+	+	–	–	+	7
6	+	–	+	–	+	–	–	3
7	–	+	+	–	–	+	–	4
8	+	+	+	+	+	+	+	8

For example, we may decide to run the ABC_- tests in the morning and the ABC_+ tests in the afternoon.

Question

What is the purpose of running the tests in this manner?

Steps for Designing and Conducting DOE

12. Determine combination of factor levels that best achieves the objective(s).
13. Run a confirmation experiment at this “optimal” combination.
14. Assure these best levels for strong factors are maintained over time by implementing Standard Operating Procedures and visual controls.
15. Re-evaluate process capability (to quantify improvement(s)).

Summary: Keys to Successful Experiments

1. **Good Output Measure**
   Whenever possible, use an output that directly relates to the function of the process. When a measure of the problem or defect must be used, be sure to use variable data.

2. **Sound Experimental Design**
   No amount of data analysis can make up for a poorly designed experiment. Carefully select output response, factors and levels and the DOE layout.

3. **Careful Planning**
   To assure conditions can be controlled as stated in the experimental design, all resources (people, materials, etc.) for conducting the experiment must be prepared ahead of time.

4. **Verified Measurement Systems**
   To assure the data is "good", verify all measurement systems before conducting the DOE.

5. **Track the Experimental Units**
   Label each unit according to the experimental condition by which it was produced. Otherwise, all information is lost.

Training Project Scenario

- Execute the Plan for Analysis
  - Run a Screening (fractional) DOE to evaluate at least 6 factors for their significance on projectile distance.
  - Remember to include factors that may significantly impact average
  - Remember to include factors that may significantly impact variation
  - Remember to randomize noise factors
- Present findings to Instructor.

Training Project Scenario

- Develop & Execute the Plan for Improvement
  - Plan and run an optimizing DOE to fully understand the known significant factors and their interactions.
  - Analyze DOE output for mean and variation
  - Determine optimal settings to meet customer requirements