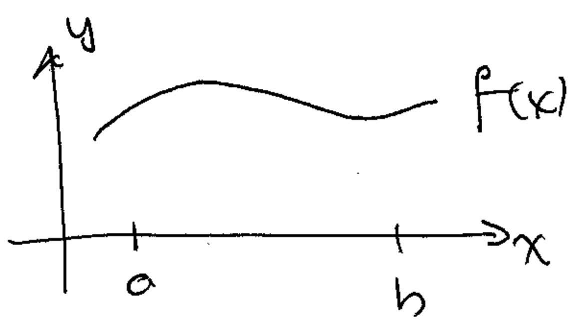
Math 1496 - Calc I

Section 2.4 Continuity

When we hear of something running continuously we think of it not starting and stopping. In mathematics, it's similar

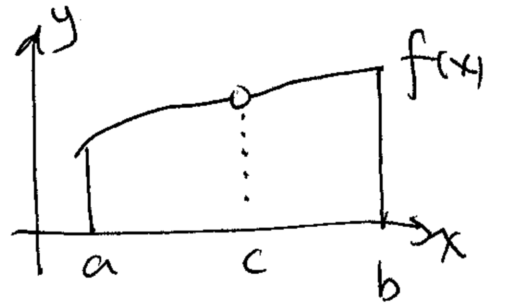
consider the graph below

Fig 1



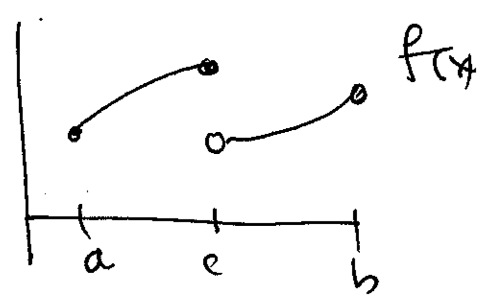
As we go from $x=a$ $\to$ $x=b$, can we travel along the curve w/o stopping

Fig 2



How about now. Well we have to stop at $x=c$ b/c there's a hole.

Fig 3



Here there is a jump at $x=c$

In figure 2 $\lim_{x \to c} f(x)$ exist

but $f(x)$ is not defined

In figure 3 $\lim_{x \to c} f(x)$ DNE.

Def^n Continuity

A function $f(x)$ is said to be cont^s at $x = c$ if

(1) $f(c)$ is defined

(2) $\lim_{x \to c} f(x)$ exists

(3) $\lim_{x \to c} f(x) = f(c)$ ((1) = (2))

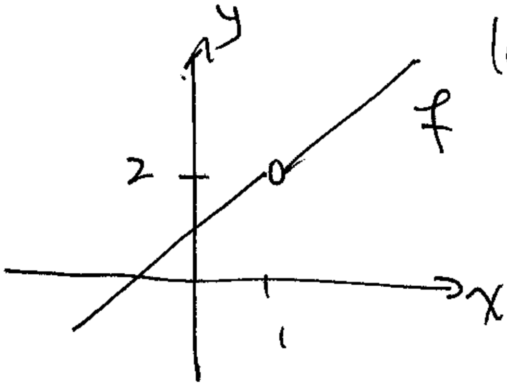
If a function is cont^s at each pt in (a,b) we say f is cont^s on (a,b) and if it's $(-\infty, \infty)$ we say f is continuous everywhere.

If it is not cont^s we say its discontinuous.

Removable Discontinuity

A discontinuity is said to be removable if we can redefine $f(x)$ to make it continuous.

Ex $\quad f(x) = \dfrac{x^2-1}{x-1}, \; x \neq 1$



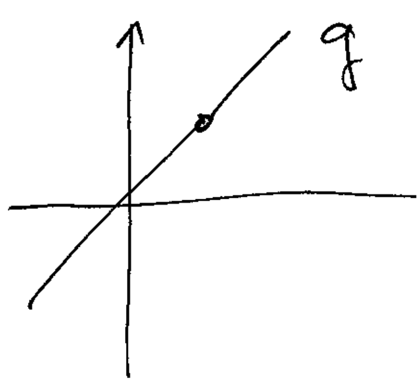
last week we saw

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

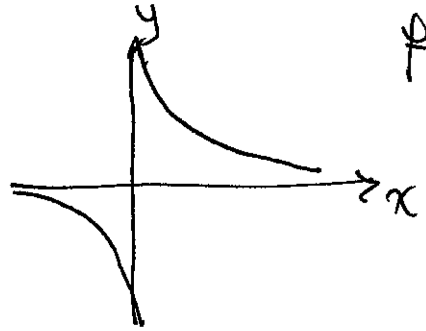
so define

$$g(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 2 & x = 1 \end{cases}$$

← this now fills in the hole.



Ex $f(x) = \dfrac{1}{x}$



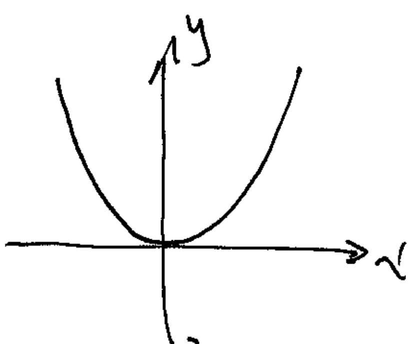
f is cont^s everywhere except $x = 0$

$$\lim_{x \to 0} \frac{1}{x} = DNE$$

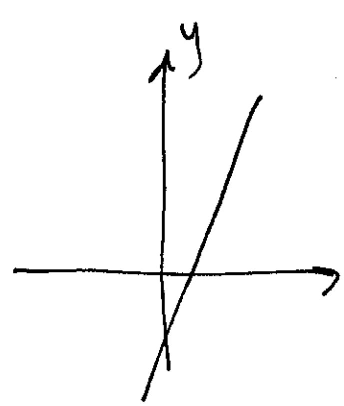
No matter how we define at $x = 0$ we can't eliminate the jump.

so the discontinuity at $x = 0$ is non removable.

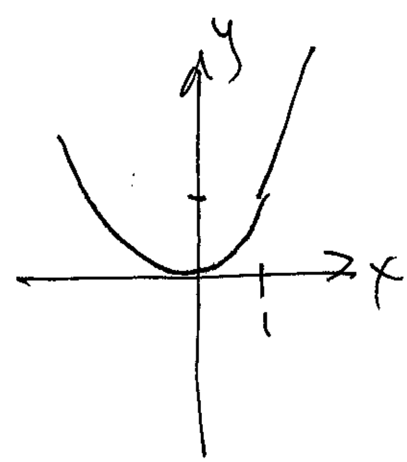
Ex consider $f(x) = \begin{cases} x^2 & x < 1 \\ 2x-1 & x \geq 1 \end{cases}$



$y = x^2$



$y = 2x - 1$



is $f(x)$ cont^s at $x = 1$?

1st limits

(1) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2x - 1 = 1$

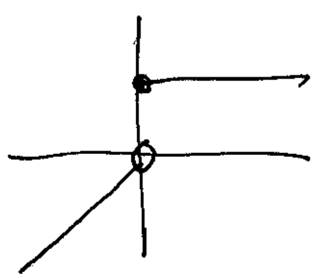
same so $\lim_{x \to 1} f = 1$

the limit exists

(2) $f(1) = 2(1) - 1 = 1$

(3) $\therefore \lim_{x \to 1} f(x) = f(1)$ $f(x)$ is cont^s at $x = 1$

Ex 2 $f(x) = \begin{cases} x & x < 0 \\ 1 & x \geq 0 \end{cases}$



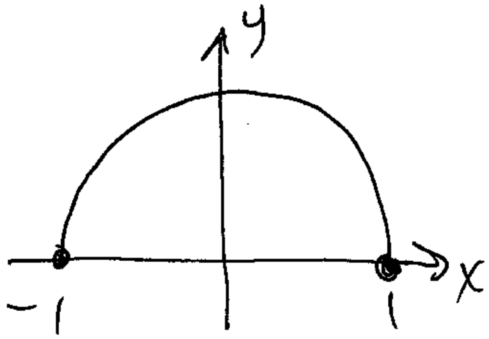
$\lim_{x \to 0^-} x = 0$

$\lim_{x \to 0^+} 1 = 1 \neq 0$

so $\lim_{x \to 0} f(x)$ DNE so no, not cont^s at $x = 0$

One sided Limits

Consider $f(x) = \sqrt{1-x^2}$



it only makes sense to have

$\lim\limits_{x \to -1^+} f(x) = 0 = f(-1)$, $\quad \lim\limits_{x \to 1^-} f(x) = 0$

So we say f is cont^s from the left (right)

Some Properties of Continuity

Suppose f & g are cont^s at $x = c$ & k const.

(1) kf is cont^s at $x = c$

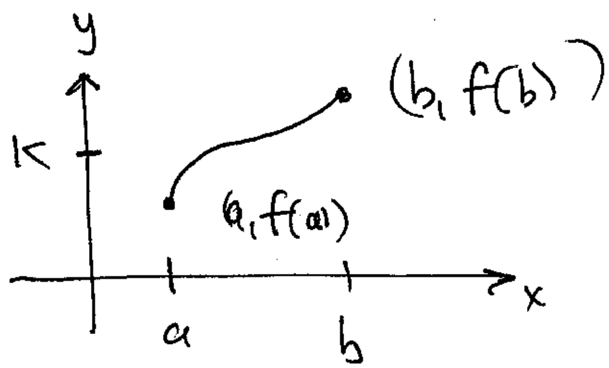
(2) $f \pm g$ " "

(3) fg " "

(4) $\frac{f}{g}$ " " provide $g(c) = 0$

same with $f(g(x))$

Intermediate Value Th^m



If $f(x)$ is cont^s on $[a,b]$ and $f(a) \neq f(b)$

if K is any value between $f(a)$ & $f(b)$

then exists a c such that $f(c) = K$

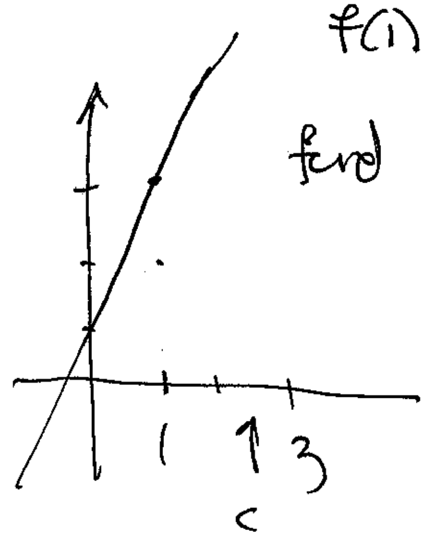
There could be more than 1 c but there is at least 1

Ex $f(x) = 2x+1$ on $[1,3]$

$f(1) = 3 \quad f(3) = 7$

find c such that

$f(c) = 6$



$2c+1 = 6$

$2c = 5$

$c = 5/2$

Aside

yes f is cont^s on $[1,3]$

so c exists