Forecasting and Comparing the Crude Oil Price Volatility by using the Range and Return Based Volatility Models

Hsiang-Hsi Liu* **Chun-Chou Wu**

Abstract

The nature of petrol prices has been discussed and evaluated in many instances. The evaluation on the volatility of petrol prices gains importance for the periods during which oil prices show unexpected and sharp changes. Future price forecast is especially relevant for the periods of high volatility. This paper aims to examine the forecasting abilities of realized return, realized range and implied volatility in delineating the volatility process for the United States oil fund ETF. Based on the empirical results infer that the realized range and implied volatility both are well-behaved proxies for volatility process. As to the ability in out-of-sample forecasting or in-sample estimating, the range-based CARR model is better than the return-based GARCH model using various performance measures. It indicates that the model with range variable can offer more insights than conventional model with return variable in expressing the structure of crude oil volatility. Finally, incorporating the influence of leverage effect and implied volatility can also indeed improve the model fitting ability for both range and return volatility models.

Keywords: crude oil price, volatility, implied volatility, realized return, realized range implied volatility, petrol prices

I. Introduction

Periods of sharp volatility of petrol prices are usually followed by depressed economic environment, generating a significant impact for the macroeconomics and politics of many nations. Accordingly, it is important to understand the crude oil price trend in advance to make accurate industry management decisions, but, as Regnier (2007) pointed out, after the petrol crisis during 1970's its price is more volatile than other commodities, furthermore there are many kinds of oil with differences in prices, leading to the use of different models for its estimation. To address these differences in petrol prices and reach a more complete analyze of the cruel oil spot price, the United States Oil Fund has created the USO index, composed by many individual crude oil stocks. The aim of this paper is to compare the forecasting efficiency of the most relevant actuarial models in delineating the volatility process for the USO index. The first consideration will be on the realized variance (RV, hereafter) models. RV, defined as the standard deviation of an asset return for some time period, can be thought as a proxy for that asset risk. . The RV is derived by the asset close price. The statistics for RV owns the property of unbiased and well efficiency in theory. However, Martens and van Dijk (2007) proposed that the RV presents bias under the market microstructure noise consideration. Kang, Maysami and Zhao (2011) still showed that the RV estimators exhibit very different magnitudes for stock return volatility pattern.

According to the financial derivatives field, there are many different methods to measure assets volatility. Derivative market participants can compute the implied volatility from a specific option pricing model with several market prices, for example, option prices and its underlying asset prices. Christensen and Prabhala (1998) adopted the same S&P 100 stock index and its option contract to discuss the relationships between implied volatility (IV, hereafter) and realized variance. They pointed out clearly that the implied volatility is a better proxy than realized variance in volatility prediction and evaluation. Meanwhile, Hansen (2002) also verified that the implied volatility is helpful in volatility prediction from the data of S&P 100 index. Christensen and Hansen (2002) proposed that investors will try to source any useful information in evaluating the volatility in order to be under efficient market hypothesis whenever possible. Then, investors will use the expected volatility to make a judgment on the market price of option contracts. In other words, the IV is a well-behaved proxy for the unobservable volatility variable. Christensen and Hansen (2002) also verified that the implied volatility is helpful in volatility prediction from the data of S&P 100 index. Becker, Clements and White (2007) examined whether the S&P 500 implied volatility index (such as VIX) contains any information relevant to future volatility beyond that available from model based volatility forecasts. They argued that the IV might not contain any such additional information relevant for forecasting volatility. Busch, Christensen and Nielsen (2011) studied the forecasting of future realized volatility in the foreign exchange, stock, and bond markets from variables including IV. The results seem to show that under the separate continuous and jump components of realized volatility, the IV then can be important in forecasting future RV components in these three markets. Prokopczuk and Simen (2014) examined the role of the volatility risk premium for the forecasting performance of implied volatility. They find some evidences to suggest that the risk premium implied volatility cannot significantly outperforms other models which implied volatility need to be adjusted.

Another approach applied to obtain the volatility proxy is using the highest and lowest prices for the specific asset during some specific time interval. Such a volatility proxy can be called the realized range (RR, hereafter). The original idea for RR was proposed by Parkinson (1980), who concluded that the RR is unbiased and its degree of efficiency is better than of the RV. His results were confirmed by Alizadeh, Brandt and Diebold (2002) who applied the RR to analyze the exchange rate volatility pattern. Martens and Van Dijk (2007) propose the RR estimator as a more efficient measure of ex post volatility. The results of their forecasting volatility depicted that in a frictionless market the RR estimator is indeed more efficient than the RV estimator when comparing similar sampling frequencies. Papavassiliou (2012) still showed that RR-based measures improve upon the corresponding RV-based ones in most cases, especially for the most actively traded Greek stocks. Additionally, Sheu and Lai (2014) investigated the information content of RR for futures hedging. The out-of-sample forecasting results show that both the statistical and economic hedging effectiveness increase with the inclusion of intraday price ranges. The empirical results indicate that informative RR is valuable for futures hedging.

Surveying from a bundle of past related literature, the ARCH/GARCH family of models have provided effective tools in estimating the volatility of individual assets. Tailored to the needs of different asset classes, these various models have achieved remarkable success (see Bollerslev et al. (1992), Poon and Granger (2003) and Engle (2004), for a more comprehensive review). Some recent studies which related to the analysis of crude oil price such as Kang, Kang and Yoon (2009), Wei, Wang and Huang (2010) and Chen, Choudhry and Wu (2013). However, these studies did not execute the comparisons for different volatility forecasting ability by different validation indicators. The first goal for this study is to perform these related comparisons rigorously.

In addition, the conditional autoregressive range (CARR) model arranges the high/low range data of asset prices during a fixed time interval to set up the model structure. In estimating the volatility of asset prices, there is a growing recognition of the fact that the range data of asset prices can provide sharper estimates and forecasts than the return data based on close-to-close prices. Many insightful studies have provided powerful evidences including Parkinson (1980), Garman and Klass (1980), Kunitomo (1992) and, more recently, Alizadeh et al. (2002), Brandt and Jones (2006), Chou (2005, 2006), Chou and Liu (2010), Lee (2013), Chiang, Chou and Wang (2016), Kurma (2016) and Ng, Seiris, So-kuen-Chan, Allen and Ng (2017). Especially, Chou (2005) and Kumar (2016) who concluded that the CARR model is better than the GARCH model in volatility analysis with the S&P 500 Index or crude oil price movement. Intuitively, the CARR model incorporates more information than the GARCH model for a specific time interval. Thus, the CARR model does well in capturing the volatility pattern. In other words, a range-based volatility model can serve as a workable substitute for the return-based volatility model in delineating the process of volatility. In light of the success of the range-based volatility models, it is natural to inquire whether the efficiency of the range-based CARR structure is better than the return-based GARCH model in estimating and forecasting the volatility of crude oil price, too.

RR, RV and implied volatility have different advantages in depicting the volatility structure; however, it is rare to find out using these different methods to discuss the volatility pattern of crude oil market for the same time period. The second goal of this study is to find out what is the most appropriate approach to analyze the volatility pattern of the USO index among these methods. Hopefully, this research can provide some policy implications for risk management to investors and government agencies. The remaining of this study is organized as follows. Section II introduces the various volatility measurements and volatility models. Section III describes the properties of data used and discusses the empirical results. Section IV discusses the forecasting ability for GARCH and CARR model by using various performance indicators. Finally, the conclusions are arranged in section V.

II. Volatility Measurement and Volatility Models

A popular method to express the volatility is the realized variance (RV). The RV is computed from the asset returns function. The formal definition of RV can be demonstrated as below:

$$RV_t = \left(\ln \left(\frac{C_t}{C_{t-1}} \right) \right)^2 \tag{1}$$

where RV_t is the realized variance of the asset at day t, C_t is the close price of the asset at day t. However, if the close prices are the same for the specific asset at day t and day t-1, then by the previous definition the value of the RV_t is zero, but if the price fluctuated during the day this is a misleading calculation. In other words, we may seriously miss some essential information during the trading period. For example, UMI[1] close price on October 22th, 2009 was NTD16.5. After one day, its close price was still NTD16.5 (Oct 23th). In terms of close prices, they are exactly the same. Namely, the valve of RV is zero for the UMI. It is natural to

[1] UMI is the abbreviation for the United Microelectronics Corporation. The UMI is a famous global semiconductor foundry that provides advanced technology for applications spanning every major sector of the IC industry.

infer that the trading price was ping at NTD16.5 on October 23, 2009, however, when we examine the real trading data during Oct 3th, we discover that the highest trading price was NTD16.7 and the lowest trading price was NTD 16.3. Contrastively, the ranged-based variable better depicts the process of volatility than the price-based one. As for the indicator of realized range is defined by Parkinson (1980). The variability of realized range (RR) can be expressed in equation (2)

$$RR_t = \frac{1}{4\ln 2}\left(\ln\left(\frac{H_t}{L_t}\right)\right)^2 \qquad (2)$$

where RR_t is the realized range at time t. H_t is the highest price for the specific asset during the time t. L_t is the lowest price for the asset during the time t.

CBOE introduced the crude oil volatility index (OVX) in May 10, 2007. The OVX can be thought as an implied volatility which reflects the oscillation of the market price for the United States Oil Fund option. The higher value of the OVX, the bigger the expected volatility. The computation for OVX is similar to the conventional volatility index (VIX). The definition of IVOL can be shown as follows:

$$IVOL_t = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{\pi T} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad (3)$$

where, the $IVOL_t$ is the implied volatility at day t;T denotes the duration of the option contract; π is the riskless discount factor; F is the expected price derived from the option price process; and K_i denotes the exercise price for the i_{th} out of the money option contract. When K_i is greater than F, we take the call option; otherwise, we take the put option into the equation (3). $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$, K_0 represents the first exercise price below the expected price F. The Q (K_i) is the middle point for bid and ask spread for each exercise price K_i. The $IVOL_t$ is an abbreviation for implied volatility at day t. We can therefore obtain the implied volatility estimation from equation (3). For the performance comparisons among implied volatility, realized variance and realized range in stating the process of crude oil price volatility. We design a regression structure that expressed in equation (4) to review the relationship between the realized variance (RV) and the realized range (RR).

$$RV_t = \alpha_0 + \alpha_{RR} RR_t + \alpha_{RV} RV_{t-1} + \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0, \sigma^2) \qquad (4)$$

where α_0, α_{RR} and α_{RV} are parameters to be estimated. The RV_{t-1} can be thought as a control variable. There are few testable hypotheses of interest. Firstly, if realized range contains some information about future realized variance, then the coefficient of the realized range $\hat{\alpha}_{RR}$ in equation (4) should be nonzero. Secondly, if realized range is an unbiased forecast of future realized volatility, then the coefficient of realized range $\hat{\alpha}_{RR}$ should be equal to one and the coefficient of intercept $\hat{\alpha}_0$ should be equal to zero respectively (also see the empirical results of Table 2). To take into account the possible autocorrelation for realized variance, we incorporate the lag term of realized variance skillfully to eliminate the latent disturbance.

The Durbin Watson test is not applicable when the regression model includes lagged dependent variables as explanatory variables. However, Durbin (1970) devised a test statistic (Durbin-h) that can be used for models testing in serial autocorrelation. We will demonstrate more rigorous statements in Section III. Similar to the structure of equation (4), we construct the equation (5) to discuss the relationships between the realized variance (RV) and the implied volatility (IVOL) for the crude oil price.

$$RV_t = \alpha_0 + \alpha_{IVOL} IVOL_t + \alpha_{RV} RV_{t-1} + \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0, \sigma^2) \tag{5}$$

As indicated in many researches, the important feature of many financial asset return series are known as volatility clustering. In other words, the current level of volatility tends to be positively correlated with its level during the immediately preceding periods. How could this phenomenon be parameterized? One approach is to use an ARCH model. The Autoregressive Conditional Heteroscedastic (i.e. ARCH for short) model was introduced by Engle (1982) to accommodate the dynamics of conditional heteroscedasticity. Its advantages are the simplicity of formulation and ease of estimation. In the generalized ARCH (GARCH) extension of Bollerslev (1986), the conditional variance is a linear function of both lagged squares of residuals and lagged volatilities. The GARCH model is more parsimonious and avoids over-fitting compared to the ARCH type. Consequently, the model is less likely to breach non-negativity constraints. Additionally, we can incorporate the exogenous variables to extend the usefulness for the basic GARCH form. In this study, we called it a GARCH-X model and could be constructed as equation (6).

$$\begin{aligned} r_t &= \mu + \varepsilon_t \quad , \quad \varepsilon_t \overset{iid}{\sim} N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma r_{t-1} + \theta IVOL_{t-1} \end{aligned} \tag{6}$$

where r_t denotes return at time t; h_t is the conditional variance at time t; and IVOLt denotes implied volatility (IV) at time t. The full definition for $IVOL_t$ can be referred in equation (3). About equation (6), if the estimate of $\hat{\gamma}$ is significantly smaller than zero, one can infer the fact that the bad news will make the return downward and drive the risk boost up in the next period. Besides, if the implied volatility can enhance the power of explanation for the model, then the coefficient of the implied volatility should be different from zero in statistic. Generally speaking, more regressors will increase the power of explanation for the model, but if the regressor is not worthy it only adds noise. The worthiness of a regressor to the model can be judged by the likelihood ratio test (LR test). The statistic for LR test is listed in equation (7).

$$LR=-2(L_R-L_U)\sim\chi^2(m) \quad (7)$$

where the L_R is the value for the log-likelihood function in the restricted model and the L_U is the value in the unrestricted one. The parameter 'm' denotes the number of constraints. If the value for LR is greater than $\chi^2(m)$ based on the significant level 1% or 5%, the null hypothesis is rejected, thus, we can conclude that the unrestricted model structure is better than the restricted one.

Regarding to the relationships between implied volatility and range-type volatility, we introduce the Conditional Autoregressive Range (CARR) model proposed by Chou (2005) as indicated before to analyze the interactions of the crude oil price volatility and its implied volatility. Meanwhile, to achieve the purpose of comparison with the previously mentioned GARCH-X structure, we establish a range-based CARR-X model in equation (8).

$$R_t = \lambda_t \epsilon_t \quad , \epsilon_t \overset{iid}{\sim} \exp(0,1)$$
$$\lambda_t = \alpha_0 + \alpha_1 R_{t-1} + \beta_1 \lambda_{t-1} + \gamma r_{t-1} + \theta IVOL_{t-1} \quad (8)$$

where R_t is the range variable at time t; the r_{t-1} is the return at time t-1; and the λ_t is the conditional range variable. Based on the conditional range part of equation (8), we can investigate the influence of leverage effect. If γ is negative, then the leverage effect exists in the volatility process.

III. Empirical Results and Analysis

The basic statistics for the USO index return and range variables are shown in Table 1. The full samples start on May 10, 2007 and ends on January 29, 2012. This study period can provide more economic information, and avoid too chaos or fuzzy events which may occur from recent political or noneconomic issues in analytical data. Also, this study period can capture ex-post forecasting ability, and hence give more accurately out-of- sample forecasting the tendency of daily return, realized range and implied volatility for USO index. As indicated in Table 1, it is clear that the standard deviation for the range variable is smaller than the returns. At the first glance, we can see that the range variable is more efficient than the return variable in estimating the confidence interval of parameter because the confidence interval for return is wider than the range variable based on the same significant level. The variables show fat tail distributions with kurtosis values greater than three for both models. The GARCH and CARR models can catch the fat-tail distributions. It seems therefore appropriate to fit these two series with GARCH and CARR model for the time being at this stage.

Refer Table 1

To provide more insightful information from the original trading data, we plot Figure 1 for the pattern of daily return and realized variance of the USO index. Meanwhile, the patterns of the realized range and the implied volatility are shown in the lower panel of Figure 1.

Looking at the Figure 1, we can find out that there are volatility clustering phenomena for daily return, realized variance and realized range. It is reasonable to conjecture that there are auto-correlations for these volatility series. Additionally, when the fluctuation of return series is more volatile, the realized variance, realized range and implied volatility are all getting bigger. These findings imply that these series have some relationships among them. Therefore, it may be appropriate to use the range and return based models to describe the volatility process of the USO index.

Refer Figure 1

Table 2 presents the empirical results using maximum likelihood estimate approach with market trading data, the coefficient of realized range ($\hat{\alpha}_{RR}$) is significantly different from zero. It denotes that the realized range is a meaningful dependent variable to explain the variation of the realized variance. The estimated coefficient of realized range is insignificantly different from unity and the constant term is indifferent from zero, we therefore can write the testing hypothesis below:

$\mathrm{H}_0: \alpha_0 = 0$

$H_0: \alpha_{RR} = 1$

Firstly, for the full model structure, Durbin-Watson (DW) test is valid when the equation does not include a lagged dependent variable as an explanatory variable. Secondly, Durbin (1970) derived a statistic (i.e. Durbin-h statistic) has the following form:

$$\mathrm{Durbin-h} = (1 - \frac{d}{2})\sqrt{\frac{n}{1 - n\sigma_{\hat{\gamma}}^2}}$$

where n is the number of observations, d is the regular DW statistic and $\sigma_{\hat{\gamma}}^2$ is the estimated variance of the coefficient of the lagged dependent variable. For large sample observations, this Durbin-h follows a Gaussian distribution.

Judging from Table 2, it is evident that the models fitting are well-behaved from F statistics. The null hypothesis for all these four models are rejected, i.e., all the coefficients are not equal to zeros. Besides, from the empirical results of DW values and Durbin h, there are not autocorrelation problems. The DW values are 1.87 and 1.96 corresponding to the upper and lower models in Table 2, and the values of Durbin-h are 2.36 and 2.17 respectively. Indicating there is no evidence of serial correlation. As to the testing about individual coefficient of the models, the variate of realized range is a bias estimator to the realized variance. Similarly, the implied volatility still has statistical testing ability to explain the variability of the realized variance from the t value (15.217 and 14.237 respectively) of the implied volatility coefficient. The implied volatility variable is also a bias estimate to the realized variance based on the similar hypothesis tests.

Refer Table 2

From the basic statistics for the variables of range and return in Table 1, it is intuitive to discuss their pattern with advanced GARCH and CARR approaches. We now move to discuss the empirical results by using GARCH models for updating volatility of the USO index. Their fitting results are presented in Table 3.

The maximum likelihood method can be used to estimate the parameters when GARCH scheme is used. We compare the values of log likelihood function (LLF, hereafter) for four latent qualified GARCH models. The GARCH model appears to have done a good job in explaining the data. The LLF value is -1591.003 for GARCH (1, 1) model which is smaller than the LLF value in GARCH (2, 2). When the extra regressors are added, the explanatory ability for conditional variance with such an ad hoc GARCH (2, 2) model has no additional contributions for better fit volatility structure by the judgment of LLF. Thus, by the principle of parsimonious, we choose the GARCH (1, 1) model as the basic structure for other empirical analysis since the Ljung-Box Q-statistics for the estimated series ε_t^2 showing no evidence of autocorrelation[2].

Refer Table 3

Based on the Ljung-Box Q (8) and Q^2(8) of diagnostic testing in Table 3, there is no series correlation for the estimated residual and squared-residual processes until the eighth lag period. The coefficients of ARCH term and GARCH term both are between zero and one. The GARCH emphasizes the point that we must have $0 < \alpha_i < 1$, $0 < \beta_i < 1$ and $\alpha_i + \beta_i < 1$ for a stable structure. The estimated coefficient of ARCH term α_1 is 0.058, the coefficient of GARCH term β_1 is 0.936 and $\alpha_1 + \beta_1$ is 0.994 from the simple GARCH (1,

[2] Ljung-Box Q statistics can be used to detect the autocorrelation for GARCH family models conventionally. As to the linear regression model, the DW testing approach is very popular.

1) model in Table 3. From this viewpoint, the GARCH structure is suitable for modeling the USO index volatility process. To investigate whether the leverage effect exists or not, we test if the estimated coefficient of r_{t-1} is smaller than zero and statistically significant in the variance equation. No matter the variable of implied volatility is incorporated into the conditional variance equation or not, the leverage effect is salient, also $\alpha_1 + \beta_1$ for both GARCHX (1,1)-a and GARCH (1,1)-b are less than that of GARCH(1, 1). Furthermore, we plot the GARCH (1, 1) conditional variance process with/without the lag term of return (r_{t-1}) and implied volatility term $(IVOL_{t-1})$ in Figure 2. It is easy to realize and confirm the fact that the simple GARCH and the GARCH-X models both are appropriate in describing the volatility structure. Also, we cannot neglect the leverage effect in this volatility structure.

Additionally, for the purpose of comparison using return based GARCH model and range based CARR model in fitting ability of the USO index volatility, we shown the empirical results for the CARR model fitting in Table 4. Similar to the analysis of the GARCH models, the value of log likelihood (LLF, hereafter) is -1346.511 with CARR (1, 1). The value of LLF for CARR (2, 2) is -1346.454. It indicates that the ability of explanation for more complicated model structure in data fitting has no overwhelming better than the simple one here. Thus, by the principle of parsimony, we select the basic CARR (1, 1) structure as our baseline model. The estimated parameters of CARR(1,1) model are shown in Table 4, we find the estimate of $\hat{\alpha}_1$ is 0.102 with t-value 5.532 indicating that the estimated coefficient value is different from zero by conventional hypothesis test. It represents that the variable of conditional range will be affected by the lagged range variable, too. Besides, the $\hat{\beta}_1$ amounts to 0.887 with t-value 42.813 which is significantly different from zero within CARR (1, 1) model. Due to the variable of conditional range is one of proxies for volatilities from past literatures, we clearly find that the pattern for conditional range can be modeling by the lag term itself and the lag term of range variable. Namely, the CARR model can be used as a powerful volatility model for forecasting daily return for USO index and the patterns for conditional range, too. Our finding is supported by the research results of Chou (2005, 2006), Chou and Liu (2010), Lee (2013), Chiang, Chou and Wang (2016), Kurma (2016) and Ng, Seiris, So-kuen-Chan, Allen and Ng (2017).

Refer Figure 2

For the same reason, in order to review the leverage effect with CARR model, we will check whether the estimate of $\hat{\gamma}$ is significantly smaller than zero or not. Based on the empirical results shown in Table 4, regardless the variable of implied volatility is incorporated or not, this CARR model with data fitting reject the null hypothesis that there is no leverage effect based on the 5% significant level by t-test. It represents that the leverage effect is essential when exploring the USO index volatility structure for CARR model. As to the estimated coefficient of implied volatility $\hat{\theta}$ is 0.529 with t-value 2.894 which is different from zero under the 5% significant level by t-test. The empirical result supports that the implied volatility variable is helpful in describing the process of conditional range.

Refer Table 4

Figure 3 plots the conditional range patterns for the CARR models with/without the impact of leverage effect. We observe that the volatility oscillation for returns can be handled by the process of conditional range, no matter that the variable for leverage effect or the implied volatility is taken into CARR model or not.

Refer Figure 3

IV. In-sample and out-of sample Forecasting Validation

In this section, we compare the performance for CARR and GARCH models with in-sample and out of sample data. We start to discuss the in-sample situations and the out of sample comparison will be shown later.

Many approaches can be used to compare the performance for in-sample forecasting with different volatility models. Based on the concept and estimation by Chou (2005), Brandt and Jones (2006), Chou and Wang (2016) and Kurma (2016), the daily return squared (DRTS), daily range squared (DRNS) and absolute daily return (ADRT) can be used as a suitable proxy for realized volatility or measured volatility (MV). Their expressions can be rearranged again below:

$$\mathrm{DRTS}_t = \left(\ln\left(\frac{C_t}{C_{t-1}}\right)\right)^2 \tag{9}$$

$$\mathrm{DRNS}_t = \left(\ln\left(\frac{H_t}{L_t}\right)\right)^2 \tag{10}$$

$$\mathrm{ADRT}_t = \left|\ln\left(\frac{C_t}{C_{t-1}}\right)\right| \tag{11}$$

The various realized volatility or measured volatility (MV) indicators can be regarded as function of volatility model estimates which come from the GARCH or CARR structure. Thus, we can devise equations (12) and (13):

$$\mathrm{MV}_t = \gamma_0 + \gamma_1 \mathrm{FV}_t(\mathrm{GARCH}) + u_t \tag{12}$$

$$\mathrm{MV}_t = \gamma_0' + \gamma_2 \mathrm{FV}_t(\mathrm{CARR}) + u_t \tag{13}$$

Where MV_t can symbol $DRTS_t$, $DRNS_t$ and $ADRT_t$, FV_t(GARCH) and FV_t(CARR) are volatilities derived from the GARCH and CARR models respectively. If the volatility estimates were meaningful to explain the pattern of MV_t, then the coefficient for $\hat{\gamma}_1$ or $\hat{\gamma}_2$ will different from zero. Besides, we can extract the indicators for adj-R^2 and Q-statistic, after data applied to estimate the equations (12) and (13), to assist us to compare the performance for GARCH and CARR models.

Table 5 summarizes the results for in-sample regression method forecasting comparison. We can see that regardless which realized volatility is adopted, the volatility proxy from GARCH and CARR are meaningful in explaining the pattern of the realized volatility. From the middle and lower panels in Table 5, the variables for leverage effect and implied volatility are helpful in boosting the explanatory ability for the dynamical process of realized volatility. From the adjusted coefficient of determination (adj-R^2), we find that the adj-R^2 is usually greater in CARR than in GARCH based under the same realized volatility as dependent variable. It conveys a clear fact that the CARR model is more suitable than the GARCH one in volatility forecasting for the in-sample forecasting comparison[3]. From the discussion above, the CARR model is more appropriate and useful than the GARCH model in explaining the process of realized volatilities.

Refer Table 5

With regard to the out-of-the sample forecasting comparison, we can obtain the out-of-sample volatility forecasting values by deriving the general form for the K_{th} prediction through iteration process. The first period (t+1) out-of-sample forecasting expression can be shown as follows:

$$h^f_{t,t+1} = E(h_{t+1}|I_t) = \hat{\omega} + \hat{\alpha}\varepsilon_t^2 + \hat{\beta}h_t + \hat{\gamma}r_t + \hat{\theta}IVOL_t \quad (14)$$

The second period out-of-sample forecasting can be derived as below.

$$\begin{aligned}
h^f_{t,t+2} &= E(h_{t+2}|I_t) = \hat{\omega} + \hat{\alpha}\varepsilon_{t+1}^2 + \hat{\beta}h_{t+1} + \hat{\gamma}r_{t+1} + \hat{\theta}IVOL_{t+1} \\
&= \hat{\omega} + \hat{\alpha}E(\varepsilon_{t+1}^2|I_t) + \hat{\beta}E(h_{t+1}|I_t) + \hat{\gamma}r_{t+1} + \hat{\theta}IVOL_{t+1} \\
&= \hat{\omega} + (\hat{\alpha} + \hat{\beta})E(h_{t+1}|I_t) + \hat{\gamma}r_{t+1} + \hat{\theta}IVOL_{t+1} \\
&= \hat{\omega} + (\hat{\alpha} + \hat{\beta})(\hat{\omega} + (\hat{\alpha} + \hat{\beta})h_t + \hat{\gamma}r_t + \hat{\theta}IVOL_t) + \hat{\gamma}r_{t+1} + \hat{\theta}IVOL_{t+1} \\
&= \hat{\omega} + (\hat{\alpha} + \hat{\beta})\hat{\omega} + (\hat{\alpha} + \hat{\beta})^2 h_t + \hat{\gamma}\left((\hat{\alpha} + \hat{\beta})r_t + r_{t+1}\right) + \hat{\theta}\left((\hat{\alpha} + \hat{\beta})IVOL_t + IVOLr_{t+1}\right) \\
&= \hat{\omega} + (\hat{\alpha} + \hat{\beta})\hat{\omega} + (\hat{\alpha} + \hat{\beta})^2 h_t + \hat{\gamma}\textstyle\sum_{i=1}^{2}(\hat{\alpha} + \hat{\beta})^{2-i} r_{t+i-1} + \hat{\theta}\sum_{i=1}^{2}(\hat{\alpha} + \hat{\beta})^{2-i} IVOL_{t+i-1} \quad (15)
\end{aligned}$$

According to the same reasoning for iterations, we can obtain the K^{th} out-of-sample

[3] Based on the statistics of Durbin-Watson (D-W), they are all approaching two. All of estimated error series have no first-order serial correlation.

forecasting expression:

$$h^{f}_{t,t+k} = E(h_{t+k}|I_t)$$

$$=\frac{\hat{\omega}\left(1-(\hat{\alpha}+\hat{\beta})^{k}\right)}{1-(\hat{\alpha}+\hat{\beta})}+(\hat{\alpha}+\hat{\beta})^{k}h_t+\hat{\gamma}\sum_{i=1}^{k}(\hat{\alpha}+\hat{\beta})^{k-i}r_{t+i-1}+\hat{\theta}\sum_{i=1}^{k}(\hat{\alpha}+\hat{\beta})^{k-i}IVOL_{t+i-1} \quad (16)$$

Where $h^{f}_{t,t+k}$ is the out-of-the sample forecasting of the conditional variance in the t+k period. Additionally, CARR (1, 1) with the variable of leverage effect and implied volatility can be demonstrated as:

$$\lambda^{f}_{t,t+k} = E(\lambda_{t+k}|I_t)$$

$$=\frac{\hat{\omega}\left(1-(\hat{\alpha}+\hat{\beta})^{k}\right)}{1-(\hat{\alpha}+\hat{\beta})}+(\hat{\alpha}+\hat{\beta})^{k}\lambda_t+\hat{\gamma}\sum_{i=1}^{k}(\hat{\alpha}+\hat{\beta})^{k-i}r_{t+i-1}+\hat{\theta}\sum_{i=1}^{k}(\hat{\alpha}+\hat{\beta})^{k-i}IVOL_{t+i-1} \quad (17)$$

Where $\lambda^{f}_{t,t+k}$ is the conditional range forecasting for the period of t+k.

Now, we firstly fit the data from sample one to sample 500 in our study period to get the model estimation of GARCH (1,1), CARR (1,1), GARCH-X (1,1) and CARR-X (1,1). The next step, we estimate period one until period 22 out-of-sample forecasting as the first round. Then, for the second round, by using the method of rolling window, it is possible to further extract the data from period two to 501 to get the model's estimation and estimate period two until 23 period out-of-sample forecasting. Finally, we repeat to roll the sample window and get 188 rounds for out-of-sample prediction values.

As indicated before, the daily return squared (DRTS), daily range squared (DRNS) and absolute daily return (ADRT) are the popular proxies for volatilities. In addition to the point estimate, there are "average" concepts for out-of-sample forecasting comparison, Chou (2005) and Brandt and Jones (2006), Chou and Wang (2016) and Kurma (2016) suggested that the validation statics of MAE and RMSE are useful in the comparison for out of sample forecasting performance. Thus, we compute the root mean square error (RMSE) and mean absolute error (MAE) for out-of-the sample forecasting comparison with GARCH and CARR models respectively. More formal expressions for RMSE and MAE are presented in Table 6.

Due to the scale for volatility proxies obtained from CARR model and GARCH are different[4], it is necessary to adjust the scale up to the same baseline. According to the study of Chou (2005), the adjusted process is illustrated below:

$$FV^{*}_{t} = FV_t \times \frac{\overline{MV}}{\overline{FV}} \quad (18)$$

[4] The volatility proxy from CARR model is estimated from the first order moment. The volatility from GARCH model is estimated from the second order moment.

Where $\overline{MV}$ and $\overline{FV}$ are the average values of MV and FV for the sample period respectively. FV_t is the volatility estimation at time t and MV_t is the realized volatility at time t.

All the results are shown in Table 6. Overall, the RMSE and MAE are mostly smaller with the CARR (1, 1) model than the GARCH (1,1) model. For instance, using the ADRT as a benchmark with the RMSE indicator, the RMSE values for CARRX (1, 1)-a are 0.581, 0.598, 0.601 and 0.602 with out of sample ahead 1, 5, 10 and 22 periods. For the same corresponding out of sample ahead, the RMSE values for GARCH (1, 1) -a model are 0.688, 0.600, 0.665 and 0.661. It appears that the volatility forecasting ability for CARR model is better than the GARCH model in describing the volatility process of USO index. Additionally, no matter what measured volatility is adopted, we find the estimate errors are stable and will not diverge to an unacceptable level. In other words, the range based CARR model can be used to predict the USO volatility process for one day, one week (5 days) and even one month (22 days) ahead. Thus, incorporating the asymmetric effect and implied volatility variable into CARR model is helpful and appropriate in describing the process of United Stated States Oil Fund ETF index (i.e. USO index).

Refer Table 6

In sum, when the implied volatility is incorporated in the GARCH and CARR model, we still find the CARR structure is better than the GARCH model except the DRTS is chosen as realized volatility proxy. This may justify that the CARR model is a better candidate than the GARCH model when capturing the volatility structure of crude oil price for future forecasting.

V. Conclusion and Implications

The volatility of petrol prices has received much attention in the past decades because crude oil is one of the most strategic and most traded commodity around the globe. Rise-and-fall of petrol prices may cause significant effects on everyday economic activities. As a result, crude oil price forecasting is still indeed a very important field of research. Moreover, the modeling/forecasting of petroleum prices is hindered by its intrinsic difficulties of high volatility. Thus it is essential to discuss and find a way to more precisely estimate the volatility structure and forecast of petrol prices. This study uses the USO index to compare the relationships among realized variance, realized range and implied volatility. We found that the realized range and implied volatility are capable of explaining the pattern of realized variance, therefore the realized range and implied volatility can be thought as the proxies of volatilities in describing the crude oil price variation process. Additionally, we compare the GARCH and CARR model in describing the process of USO index volatility. Both of these two models can express the volatility pattern of USO index. we can also observe the leverage effect exist in the process of USO index by GARCH and CARR models. It is noticeable to incorporate the leverage effect into volatility structure to boost the goodness of fit for oil price volatility forecasting model. As to the variable of implied volatility, it is appropriate to add into CARR model for improving the model's forecasting power in explaining the process of realized volatility. Contrastively, the implied volatility with GARCH volatility offer little assistance in describing the structure of realized volatility.

Finally, regarding to the comparison for in sample volatility forecasting, the CARR model is better than the GARCH model. As to the out-of-sample forecasting, we obtain the similar inference. Accordingly, the range based CARR model is better than the return based GARCH one in capturing the volatility process of crude oil price return volatitatity. Hopefully, the research findings about petrol price volatility forecasting models can provide investors more risk management information and practice through our different volatility forecasting model comparisons.

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Tables

Table 1. The statistics for returns and range of the USO index

	Mean	Max	min	St. D.	Skewness	Kurtosis
USO						
Return	-0.044	9.169	-11.300	2.836	-0.204	4.369
Range	3.162	17.917	0.637	1.862	1.994	10.316

Notes：The data for USO index is from May 10, 2007to January 29, 2010. Totally have 687daily observations. $return = 100 \times \ln(C_t/C_{t-1})$，$range = 100 \times \ln(H_t/L_t)$。

Table 2 Information content of realized range, implied volatility: MLE estimate of regression equations (4) and (5)

	$RV_t = \alpha_0 + \alpha_{RR}RR_t + \alpha_{RV}RV_{t-1} + \varepsilon_t$					
oil	$\hat{\alpha}_0$	$\hat{\alpha}_{RR}$	$\hat{\alpha}_{RV}$	$\overline{R^2}$	F value	DW/Durbin-h
	2.289**	1.183**		0.344	359.512**	1.87
	(4.174)	(18.961)				
	2.010**	1.158**	0.050	0.345	180.955**	2.36
	(3.482)	(17.993)	(0.027)			
	$RV_t = \alpha_0' + \alpha_{IVOL}IVOL_t + \alpha_{RV}'RV_{t-1} + \varepsilon_t$					
	$\hat{\alpha}_0'$	$\hat{\alpha}_{IVOL}$	$\hat{\alpha}_{RV}'$	$\overline{R^2}$	F value	DW/Durbin-h
	-12.088**	6.893**		0.252	231.558**	1.96
	(-8.577)	(15.217)				
	-13.029**	7.430**	-0.078*	0.255	118.124**	2.17
	(-8.803)	(14.237)	(-0.037)			

Notes: **denotes 1% significant level, * denotes 5% significant level. The value in parenthesis is t statistics. DW denotes Durbin-Watson statistics for serial correlation test. Durbin-h is a modified statistic when lagged dependent variable as independent variable. $\overline{R^2}$ is the adjusted coefficient of determination. RR_t is realized range at day t. $IVOL_t$ denotes the implied volatility at day t. RV_t is the realized variance at day t.

Table 3 GARCH-X model for the USO index

$r_t = \mu + \varepsilon_t$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \tau r_{t-1} + \theta IVOL_{t-1}$$

	GARCH(1,1)	GARCH(2,2)	GARCHX(1,1)-a	GARCHX(1,1)-b
Oil				
$\hat{\alpha}_0$	0.047(1.460)	0.091(1.538)	0.102*(2.291)	-0.817(-1.508)
$\hat{\alpha}_1$	0.058**(3.512)	0.079**(3.359)	0.061**(3.239)	0.057*(2.023)
$\hat{\alpha}_2$		0.028(0.953)		
$\hat{\beta}_1$	0.936**(54.285)	-0.012(-0181)	0.926**(43.473)	0.787**(8.318)
$\hat{\beta}_2$		0.894**(15.498)		
$\hat{\tau}$			-0.127**(-2.648)	-0.249**(-2.578)
$\hat{\theta}$				0.688(1.768)
Q(8)	12.635(0.396)	12.320(0.420)	13.180(0.356)	13.613(0.326)
$Q^2(8)$	25.771(0.545)	24.331(0.601)	25.332(0.552)	26.112(0.474)
LLF	-1591.003	-1590.011	-1587.418	-1583.611

Notes: $r_t = 100{*}\ln(C_t / C_{t-1})$ **denotes 1% significant level, * denotes 5% significant level. The value in parenthesis is t statistics. GARCHX (1, 1)-a takes the leverage effect into model. GARCHX (1, 1)-b takes the leverage effect and implied volatility into model. The table analyzes data on the USO index during May 10, 2007 and January 29, 2010.

Table 4 CARR-X Model for the USO index

$R_t = \lambda_t \varepsilon_t$

$$\lambda_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i R_{t-i} + \sum_{j=1}^{p} \beta_j \lambda_{t-j} + \gamma r_{t-1} + \theta IVOL_{t-1}$$

	CARR(1,1)	CARR(2,2)	CARRX(1,1)-a	CARRX(1,1)-b
Oil				
$\hat{\alpha}_0$	0.033(1.883)	0.063(1.816)	0.054**(2.833)	-0.008(-0.102)
$\hat{\alpha}_1$	0.102**(5.532)	0.071*(2.212)	0.094**(4.930)	0.075*(2.146)
$\hat{\alpha}_2$		0.126**(4.345)		
$\hat{\beta}_1$	0.887**(42.813)	0.138(0.536)	0.888**(41.342)	0.436**(2.751)
$\hat{\beta}_2$		0.645**(2.754)		
$\hat{\gamma}$			-0.030**(-3.606)	-0.048**(-2.982)
$\hat{\theta}$				0.529**(2.894)
Q(8)	18.217(0.074)	18.142(0.075)	17.663(0.077)	24.688(0.021)
$Q^2(8)$	30.223(0.213)	31.256(0.078)	30.035(0.215)	39.556(0.208)
LLF	-1346.511	-1346.454	-1342.838	-1342.364

Notes: 1. R_t is computed as 100*($\ln H_t$) – ln (L_t). **denotes 1% significant level, * denotes 5%

significant level. The value in parenthesis is t statistics.

2. CARR-X (1, 1)-a takes the leverage effect into the CARR model.
3. CARR-X (1, 1)-b takes the leverage effect and implied volatility into the CARR model.
4. The table analyzes data on the USO index between May 10, 2007 and January 29, 2010.

Table 5. In sample forecasting comparison for the USO index

$MV_t = \gamma_0 + \gamma_1 FV_t(GARCH) + u_t$

$MV_t = \gamma_0' + \gamma_2 FV_t(CARR) + u_t$

MV	$\hat{\gamma}_0(\widehat{\gamma'_0})$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$adj-R^2$	D-W
GARCH(1,1) & CARR(1,1)					
DRTS	0.163(0.207)	0.984**(13.073)		0.199	2.140
	-9.354**(-6.915)		5.523**(13.827)	0.217	2.120
DRNS	-0.646(-0.657)	1.765**(18.746)		0.338	1.999
	-16.821**(-9.939)		9.622**(19.243)	0.350	2.090
ADRT	1.079**(10.783)	0.129**(13.452)		0.208	2.099
	-0.121(-0.697)		0.709**(13.856)	0.218	2.067
GARCHX(1,1)-a & CARRX(1,1)-a					
DRTS	0.198(0.255)	0.975**(13.308)		0.205	2.166
	-9.102**(-6.830)		5.424**(13.874)	0.219	2.125
DRNS	-0.547(-0.566)	1.743**(19.084)		0.347	2.018
	-16.531**(-9.924)		9.494**(19.429)	0.355	2.080
ADRT	1.078**(10.939)	0.128**(13.800)		0.216	2.131
	-0.110(-0.645)		0.703**(14.082)	0.224	2.081
GARCHX(1,1)-b & CARRX(1,1)-b					
DRTS	-0.677(-0.823)	1.120**(13.382)		0.207	2.197
	-9.597**(-6.980)		5.600**(13.771)	0.216	2.144
DRNS	-1.912(-1.853)	1.977**(18.840)		0.341	2.002
	-17.967**(-10.546)		9.983**(19.812)	0.364	2.089
ADRT	0.948**(9.092)	0.150**(14.113)		0.225	2.176
	-0.216(-1.237)		0.740**(14.312)	0.230	2.121

Notes: $DRTS_t = (\log(C_t/C_{t-1}))^2$, $DRNS_t = (\log(H_t/L_t))^2$, $ADRT_t = |\log(C_t/C_{t-1})|$ ◦ The value for Durbin-Watson is more approaching to 2 which denotes farer from one lag autocorrelation. The GARCHX (1, 1)-a and CARRX (1, 1)-a denote the leverage effect is considered. The GARCHX (1, 1)-b and CARRX (1, 1)-b denote the leverage effect and implied volatility both are incorporated.

Table 6 Out-of-sample forecasting comparison for the USO index

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{t=T+1}^{T+n}(FV_t - MV_t)^2}$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{t=T+1}^{T+n}|FV_t - MV_t|$$

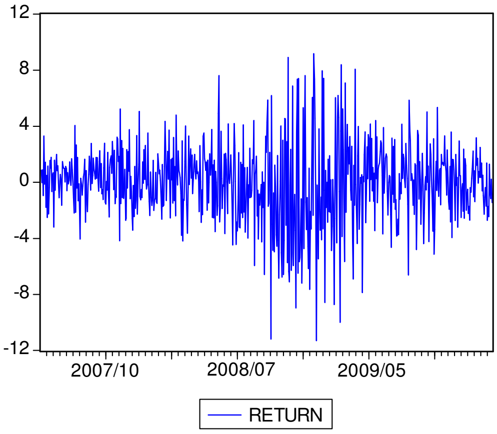
RMSE	DRTS						DRNS						ADRT					
out-of-sample period	GARCH (1,1)	GARCHX (1,1)-a	GARCHX (1,1)-b	CARR (1,1)	CARRX (1,1)-a	CARRX (1,1)-b	GARCH (1,1)	GARCHX (1,1)-a	GARCHX (1,1)-b	CARR (1,1)	CARRX (1,1)-a	CARRX (1,1)-b	GARCH (1,1)	GARCHX (1,1)-a	GARCHX (1,1)-b	CARR (1,1)	CARRX (1,1)-a	CARRX (1,1)-b
22	1.342	1.372	1.321	1.333	1.327	1.323	1.405	1.489	1.393	1.408	1.402	1.394	0.630	0.661	0.609	0.606	0.602	0.600
10	1.331	1.376	1.306	1.317	1.319	1.316	1.388	1.452	1.371	1.376	1.379	1.381	0.620	0.665	0.599	0.597	0.601	0.596
5	1.296	1.309	1.303	1.309	1.313	1.311	1.366	1.422	1.367	1.370	1.376	1.377	0.592	0.600	0.590	0.594	0.598	0.592
1	1.274	1.379	1.271	1.282	1.287	1.288	1.374	1.516	1.335	1.359	1.361	1.368	0.581	0.688	0.573	0.577	0.581	0.580
MAE	DRTS						DRNS						ADRT					
out-of-sample period	GARCH (1,1)	GARCHX (1,1)-a	GARCHX (1,1)-b	CARR (1,1)	CARRX (1,1)-a	CARRX (1,1)-b	GARCH (1,1)	GARCHX (1,1)-a	GARCHX (1,1)-b	CARR (1,1)	CARRX (1,1)-a	CARRX (1,1)-b	GARCH (1,1)	GARCHX (1,1)-a	GARCHX (1,1)-b	CARR (1,1)	CARRX (1,1)-a	CARRX (1,1)-b
22	0.960	0.996	0.944	0.959	0.955	0.950	1.077	1.121	1.052	1.065	1.057	1.053	0.511	0.541	0.491	0.497	0.494	0.492

10	0.947	0.997	0.937	0.938	0.943	0.935	1.037	1.102	1.045	1.028	1.031	1.032	0.504	0.546	0.498	0.488	0.492	0.486
5	0.911	0.922	0.920	0.924	0.929	0.922	1.025	1.048	1.019	1.019	1.021	1.017	0.482	0.492	0.483	0.483	0.488	0.480
1	0.871	0.962	0.876	0.875	0.880	0.878	1.021	1.112	0.987	0.996	1.003	0.998	0.463	0.552	0.461	0.459	0.464	0.460

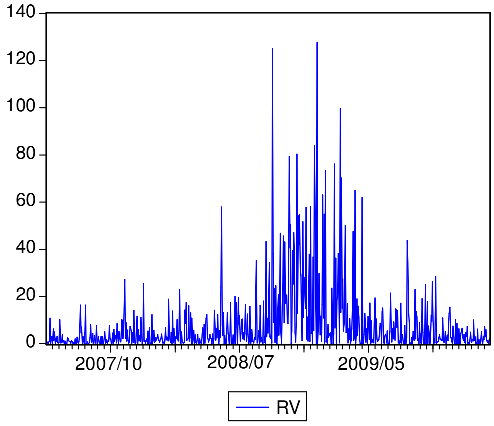
Notes: The data for USO index is from May 10, 2007to January 29, 2010. Totally have188 estimates.

$DRTS_t = \left(\log(C_t/C_{t-1})\right)^2$, $DRNS_t = \left(\log(H_t/L_t)\right)^2$, $ADRT_t = |\log(C_t/C_{t-1})|$. The GARCHX (1, 1)-a and CARRX (1, 1)-a denote the leverage effect is considered. The GARCHX (1, 1)-b and CARRX (1, 1)-b denote the leverage effect and implied volatility both are incorporated.

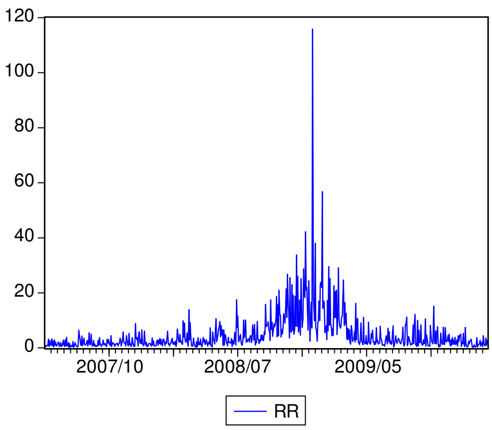
Figures



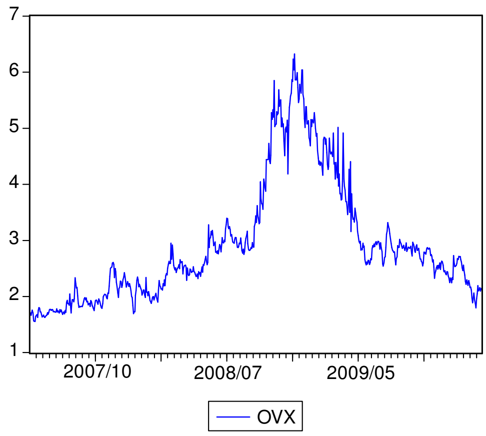
Daily return for USO index



Realized variance for USO index



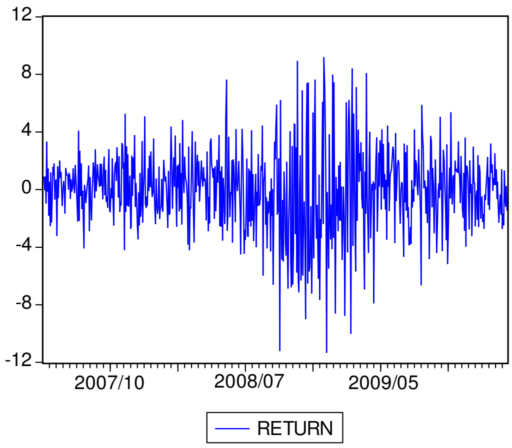
Realized range for USO index



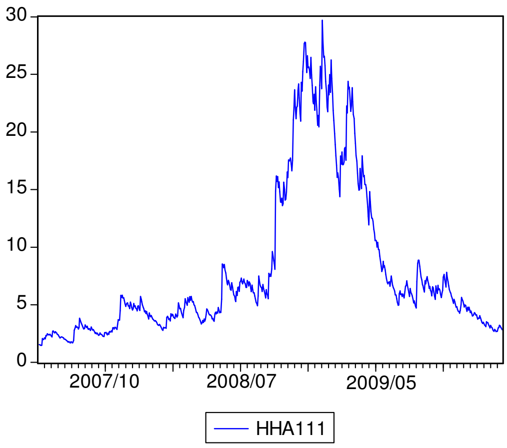
Implied volatility for USO index

Figure 1: Daily return, realized range and implied volatility for USO index

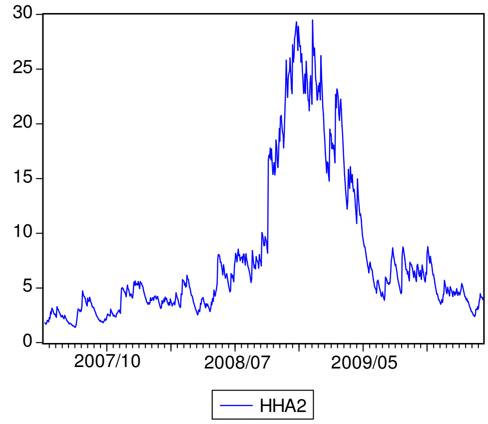
Notes：The data for USO index is from May 10, 2007 to January 29, 2010, it contains 687daily observations.



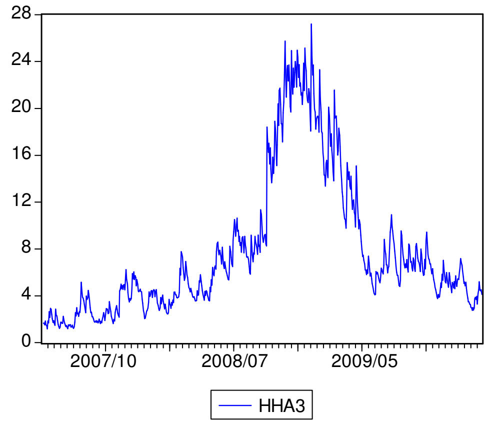
Daily return for USO index



The $\hat{h}_t$ of GARCH(1,1) for USO



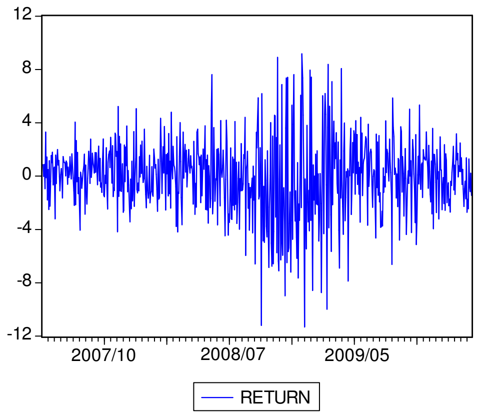
The $\hat{h}_t$ of GARCH (1, 1)-a for USO



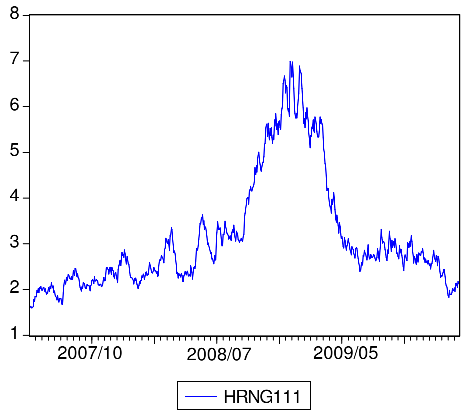
The $\hat{h}_t$ of GARCH (1,1)-b for USO

Figure 2: The daily return of USO index and conditional variance $\hat{h}_t$

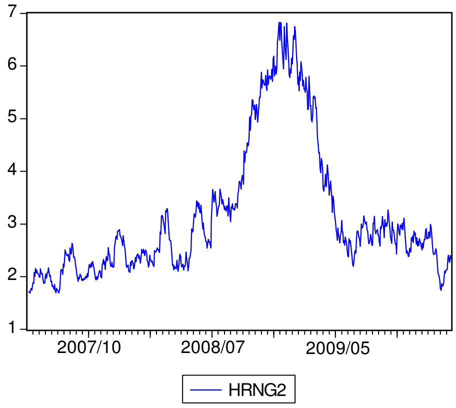
Notes: The data for USO index is from May 10, 2007 to January 29, 2010. Totally have 687daily observations. GARCH (1, 1)-a denotes that the lag term of return (r_{t-1}) is added into the conditional variance equation. GARCH (1, 1)-b denotes that the lag term of return (r_{t-1}) and the implied volatility term ($IVOL_{t-1}$) are added into the conditional variance equation.

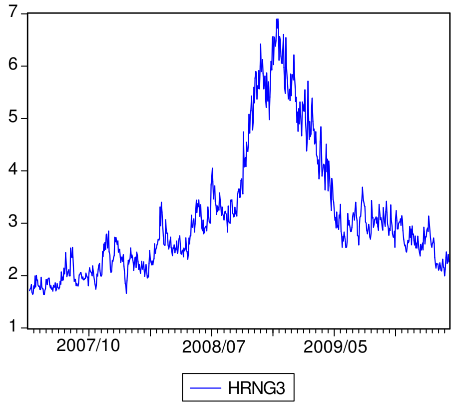


Daily return for USO index



$\hat{\lambda}_t$ for simple GARCH(1,1)





$\hat{\lambda}_t$ for GARCH-X(1,1) with leverage

$\hat{\lambda}_t$ for GARCH-X(1,1) with leverage effect and implied volatility

Figure3. Daily return for USO index and the patterns for conditional range

Notes: The data for USO index is from May 10, 2007 to January 29, 2010. Totally have 687 daily observations.

Authors

Hsiang-Hsi Liu*

Distinguished Professor, Graduate Institute of International Business, National Taipei University, Taiwan, hsiang@mail.ntpu.edu.tw

Chun-Chou Wu

Associate Professor, Department of Finance, National Kaohsiung First University of Science and Technology, Taiwan

* Corresponding author