A Connectivity Preserving Framework For Distributed Motion Coordination in Proximity Networks

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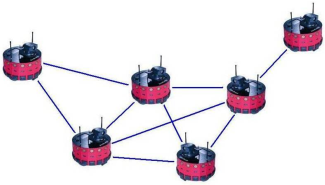
Introduction

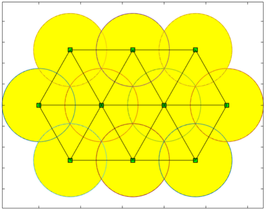
Collection of mobile robots with

- limited sensing and communication,
- limited on-board processing, and
- limited available power

Formation Control



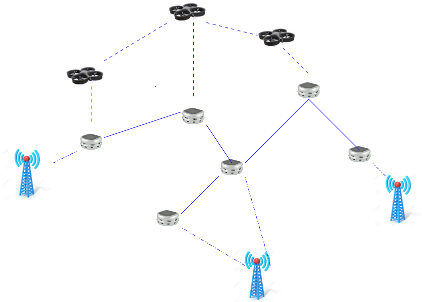
Coverage Control



Motivation

Network Topology

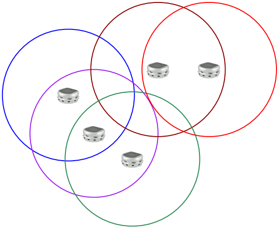
- Graph $G(V, E)$, where
  - $V = \{1, 2, \ldots, N\}$.
  - $E = \{(i, j) \mid i, j \in \text{V}\}$

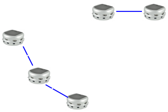


Performance Guarantees

require network topology to be *connected*

Network Connectivity: Proximity Networks





Graph $G(V, E, \Delta)$, where

- $V = \{1, 2, \dots, N\}$.
- $\mathcal{N}_i = \{j \in V \mid dist(x_i, x_j) \leq \Delta\}$

- Neighborhood structure depends on *inter-agent distances*
- *Time varying* network topology
- *Network connectivity needs to be guaranteed*

Motivation

Objective

A framework for *local interaction laws* that can *guarantee network connectivity* in distributed motion planning under *proximity network* model.

Rendezvous Problem

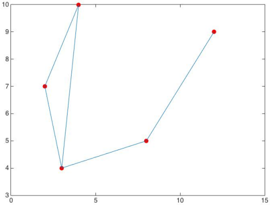
Lemma Saber *et al.* 2007

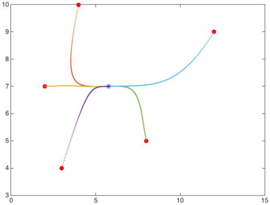
Let G be a *connected undirected graph*. Then,

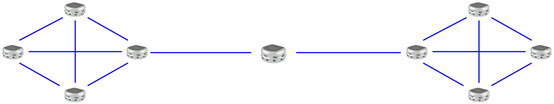
$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j)$$

$$\dot{x} = -\mathcal{L}x$$

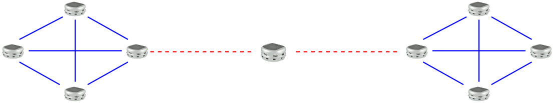
asymptotically solves an average consensus problem for all initial states.





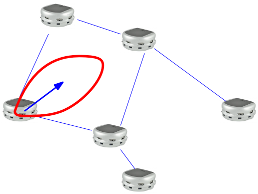


$$\dot{x}_i = - \sum_{(i,j)\in\mathcal{N}_i} (x_i - x_j)$$



Ji & Egerstedt, "Distributed coordination control of multiagent systems while preserving connectedness," *IEEE TAC*, 2007

Optimization Based Approaches



- Each agents computes a *feasible set* by solving a *convex program*.
- Motion is restricted to these feasible sets for each agent

Cortés, Martínez, & Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE TAC*, 2006

Optimization Based Approaches

Fiedler value as a measure of network connectivity

$$\max_{x \in \mathbb{R}^{dn}} \lambda_2(\mathcal{L}(x))$$

- λ_2 is a concave function of $\mathcal{L}$.
- Motion is restricted to the directions that do not decrease λ_2

DeGennaro & Jadbabaie, "Decentralized control of connectivity for multi-agent systems," *Proc. IEEE CDC, 2006*

Potential Function Based Approach

- An edge tension function $\mathcal{E}_{ij}(x)$ is defined between agents i and j if $(i, j) \in \mathcal{N}_i$.
- *Minimize the total edge tension in the system*

$$\mathcal{E}(x) = \frac{1}{2} \sum_{j \in \mathcal{N}_i} \mathcal{E}_{ij}(x_i, x_j)$$

- Local control of each agent has the form

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} w_{ij}(x_i - x_j)$$

- *Edges are never broken*

Ji & Egerstedt, "Distributed coordination control of multiagent systems while preserving connectedness," *IEEE*

Proposed Approach

Optimization Based Approach

Computationally complex

Potential Function Based Approach

Overly restrictive

Proposed Approach

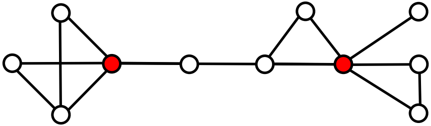
Balance between the two existing approaches

Main Idea

- In a proximity network, determine a small subset of nodes such that maintaining their neighborhoods guarantee that overall network remains connected.
- Design controllers for these as well as other nodes.

Dominating Set

- **Dominating Set:**
  A subset $S \subseteq V$ such that $\bigcup_{s \in S} \mathcal{N}[v_i] = V$



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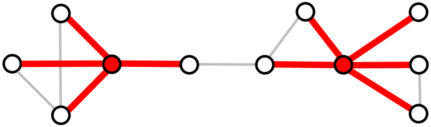
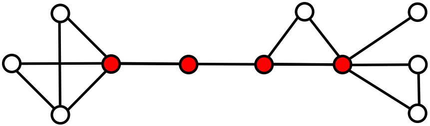


Subgraph induced by dominating nodes is not connected.

Connected Dominating Set

- **Connected Dominating Set:**
  A dominating set whose nodes induce a connected subgraph.



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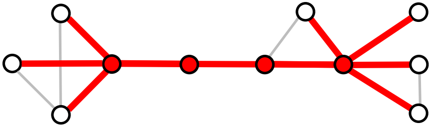
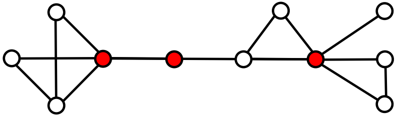


- **Weakly Connected Dominating Set:**



- $\tilde{G}(V, \tilde{E})$ where $\tilde{E}$ is the set of edges incident on the nodes in WCDS, is connected.

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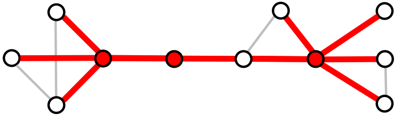


- $\tilde{G}(V, \tilde{E})$ where $\tilde{E}$ is the set of edges incident on the nodes in WCDS, is connected.
- Note that

$$DS \subseteq WCDS \subseteq CDS$$

- Minimum sized WCDS problem is NP-hard[1]

[1] Dunbar et al., "On weakly connected domination in graphs," *Disc. Math.*, 1997.

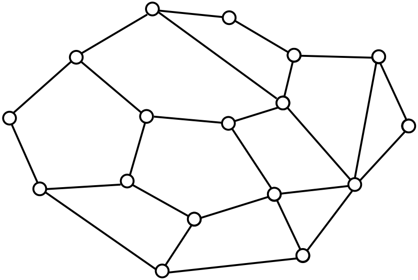
Weakly Connected Dominating Set

(Computation)

- Approximation algorithm: $\mathcal{O}(\log d_{\max})$, where $d_{\max}$ is the maximum degree.
  - Centralized algorithm **(Chen & Liestman 2002)**
  - Distributed algorithm **(Chen & Liestman 2003)**
- Constant factor approximation algorithm: 5 **(Alzoubi et al. 2003)**
  - Time complexity: $\mathcal{O}(n)$
  - Message complexity: $\mathcal{O}(n \log n)$
- Constant factor approx. algorithm: 122.5 **(Alzoubi et al. 2003)**
  - Time complexity: $\mathcal{O}(n)$
  - Message complexity: $\mathcal{O}(n)$
- Constant factor approximation algorithm: 110 **(Han & Jia 2007)**
  - Time complexity: $\mathcal{O}(n)$
  - Message complexity: $\mathcal{O}(n)$

A very simple approach is to

- Construct small sized dominating sets (in a distributed way).
- "Patch up" the components.



Weakly Connected Dominating Set (Construction)

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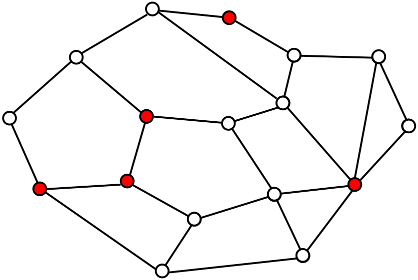

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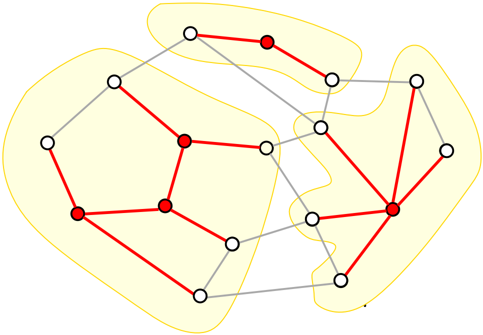

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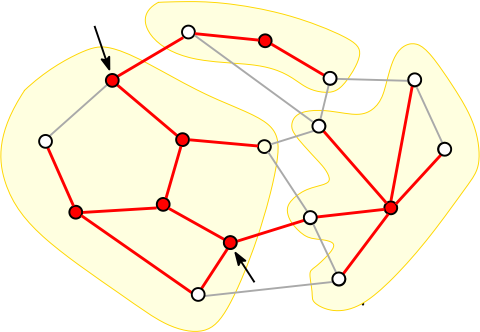

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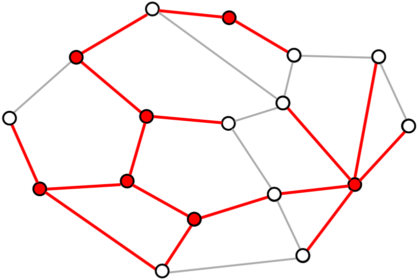


Proposed Approach

- Identify a subset of agents $D \subset V$ such that D is a weakly connected dominated set.
- Define edge tension energies

$$\mathcal{E}_{ij}(x) = \begin{cases} \frac{\|x_i - x_j\|^2}{\Delta - \|x_i - x_j\|} & \text{if } i \in D \text{ and } j \in \mathcal{N}_i, \\ \frac{1}{2}\|x_i - x_j\|^2 & \text{if } i \in V \backslash D \text{ and } j \in \mathcal{N}_i, \\ 0 & \text{otherwise.} \end{cases}$$

Proposed Approach

Lemma

Given a Δ-disk graph $G(V, E, \Delta)$ that is connected at $t = 0$ such that $\|x_i(0) - x_j(0)\| < (\Delta - \epsilon)$ for all $(i, j) \in E(0)$, where ϵ is some small non-negative scalar. Then, under the control law

$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} \frac{\partial \mathcal{E}_{ij}(x)}{\partial x_i},$$

the graph $G(V, E, \Delta)$ remains connected $\forall\ t > 0$.

Proposed Approach

Proof Sketch:

- Total energy of the systems is

$$\mathcal{E}(x) = \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \mathcal{E}_{ij}(x)$$

- We show that

$$\dot{\mathcal{E}}(x) = \frac{\partial \mathcal{E}}{\partial x} \dot{x} < 0,$$

  for the proposed controller.

- *Implication:* If $i \in D$, agent i never loses its edges.
- *All the critical edges are maintained under the proposed controller*

Proposed Approach

Theorem

Given a Δ-disk graph which is connected at $t = 0$ with edge lengths less than $(\Delta - \epsilon)$ for some $0 < \epsilon < \Delta$. Under the controller

$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} \frac{\partial \mathcal{E}_{ij}(x)}{\partial x_i} = -\sum_{j \in \mathcal{N}_i} w_{ij}(x_i - x_j)$$

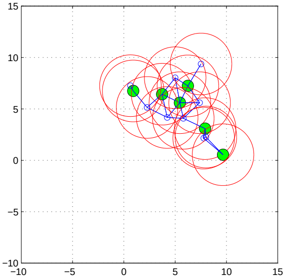
where

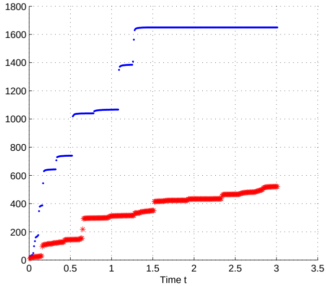
$$w_{ij} = \begin{cases} \frac{2\Delta - \|x_i - x_j\|}{\left(\Delta - \|x_i - x_j\|\right)^2} & \text{if } i \in D \text{ and } j \in \mathcal{N}_i, \\ 1 & \text{if } i \in V \backslash D \text{ and } j \in \mathcal{N}_i, \\ 0 & \text{otherwise.} \end{cases}$$

the system converges asymptotically to the weighted initial centroid of the network.

Illustration

- In a simplistic model, energy consumption due to mobility (E_{acc}) is related directly to the acceleration.
- $N = 15$ and $\Delta = 3$





– P. Tokekar et al. "Energy-optimal trajectory planning for car-like robots," *Autonomous Robots*, 2013.

– M. Ji and M. Egerstedt, "Distributed coordination control of multiagent systems while preserving connectedness," *IEEE Tran. on Robotics*, 2007.

Conclusions

- Connectivity is ensured if a small subset of nodes maintain their edges instead of all the nodes maintaining all edges.
- Such a small subset of nodes constitute a (minimum) weakly connected dominating set.
- Using the edge tension function, appropriate weights can be designed for the connectivity preserving weighted consensus equation.

Thank you