The Asymptotic Existence of Orthogonal Designs

Ebrahim Ghaderpour

University of Lethbridge

March 18, 2013

Example

- Let $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, so $AA^t = (a^2 + b^2)I_2$ which is an $OD(2;1,1)$.

Example

- Let $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, so $AA^t = (a^2 + b^2)I_2$ which is an $OD(2; 1, 1)$.
- Let $B = \begin{bmatrix} a & bi \\ bi & a \end{bmatrix}$, so $BB^* = (a^2 + b^2)I_2$ which is a $COD(2; 1, 1)$.

Example

- Let $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, so $AA^t = (a^2 + b^2)I_2$ which is an $OD(2; 1, 1)$.
- Let $B = \begin{bmatrix} a & bi \\ bi & a \end{bmatrix}$, so $BB^* = (a^2 + b^2)I_2$ which is a $COD(2; 1, 1)$.
- Let $C = \begin{bmatrix} a & b & c & c \\ -b & a & c & -c \\ c & c & -a & -b \\ c & -c & b & -a \end{bmatrix}$, so $CC^t = (a^2 + b^2 + 2c^2)I_4$ which is an $OD(4; 1, 1, 2)$.

Example

Let $D = \begin{bmatrix} ai & b & b & b & b & b \\ b & ai & b & -b & -b & b \\ b & b & ai & b & -b & -b \\ b & -b & b & ai & b & -b \\ b & -b & -b & b & ai & b \\ b & b & -b & -b & b & ai \end{bmatrix}$, so $DD^* = (a^2 + 5b^2)I_6$

which is a $COD(6; 1, 5)$.

Orthogonal Designs

Definition

A *complex orthogonal design* of order n and type $(s_1, \ldots, s_k)$, denoted $COD(n;\ s_1, \ldots, s_k)$, is an n by n matrix X entries in $\{0, \epsilon_1 x_1, \ldots, \epsilon_k x_k\}$, where the x_j's are commuting variables and $\epsilon_j \in \{\pm 1, \pm i\}$ for each j, that satisfies

$$XX^* = \left(\sum_{j=1}^{k} s_j x_j^2 \right) I_n,$$

where X^* denotes the conjugate transpose of X and I_n is the idetity matrix of order n.

- A complex orthogonal design in which $\epsilon_j \in \{\pm 1\}$ for all j is called an *orthogonal design*, denoted $OD(n;\ s_1, \ldots, s_k)$.
- A complex orthogonal design (=COD) in which there is no zero entry is called a *full* COD.
- An orthogonal design (=OD) in which there is no zero entry is called a *full* OD.

Definition

- Equating all variables to 1 in any COD of order n gives us a *complex weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the COD. We denote this complex weighing matrix by $CW(n,k)$.

Orthogonal Designs

Definition

- Equating all variables to 1 in any COD of order n gives us a *complex weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the COD. We denote this complex weighing matrix by $CW(n,k)$.
- Equating all variables to 1 in any OD of order n gives us a *weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the OD. We denote this weighing matrix by $W(n,k)$.

Orthogonal Designs

Definition

- Equating all variables to 1 in any COD of order n gives us a *complex weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the COD. We denote this complex weighing matrix by $CW(n, k)$.
- Equating all variables to 1 in any OD of order n gives us a *weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the OD. We denote this weighing matrix by $W(n, k)$.
- Equating all variables to 1 in any full COD results in a *complex Hadamard* matrix. In other word, a $CW(n, n)$.

Definition

- Equating all variables to 1 in any COD of order n gives us a *complex weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the COD. We denote this complex weighing matrix by $CW(n, k)$.
- Equating all variables to 1 in any OD of order n gives us a *weighing* matrix of weight k, where k is the number of nonzero entries in each row or each column of the OD. We denote this weighing matrix by $W(n, k)$.
- Equating all variables to 1 in any full COD results in a *complex Hadamard* matrix. In other word, a $CW(n, n)$.
- Equating all variables to 1 in any full OD results in a *Hadamard* matrix. In other word, a $W(n, n)$.

Definition

A *circulant matrix* $B = [b_{ij}]$ of order n with the first row $(a_1, a_2, \ldots, a_n)$ is one for which $b_{ij} = a_{j-i+1}$, where $j - i + 1$ is reduced modulus n. We denote this matrix by $circ(a_1, a_2, \ldots, a_n)$.

Definition

A *circulant matrix* $B = [b_{ij}]$ of order n with the first row $(a_1, a_2, \ldots, a_n)$ is one for which $b_{ij} = a_{j-i+1}$, where $j - i + 1$ is reduced modulus n. We denote this matrix by $circ(a_1, a_2, \ldots, a_n)$.

Example

$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_3 & a_1 & a_2 \\ a_2 & a_3 & a_1 \end{bmatrix}$ is a circulant matrix of order 3.

Definition

A *circulant matrix* $B = [b_{ij}]$ of order n with the first row $(a_1, a_2, \ldots, a_n)$ is one for which $b_{ij} = a_{j-i+1}$, where $j - i + 1$ is reduced modulus n. We denote this matrix by $circ(a_1, a_2, \ldots, a_n)$.

Example

$$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_3 & a_1 & a_2 \\ a_2 & a_3 & a_1 \end{bmatrix}$$ is a circulant matrix of order 3.

Fact

- *If* $B = circ(a_1, a_2, \ldots, a_n)$, *then* $B^* = circ(a_1^*, a_n^*, \ldots, a_2^*)$.

Definition

A *circulant matrix* $B = [b_{ij}]$ of order n with the first row $(a_1, a_2, \ldots, a_n)$ is one for which $b_{ij} = a_{j-i+1}$, where $j - i + 1$ is reduced modulus n. We denote this matrix by $circ(a_1, a_2, \ldots, a_n)$.

Example

$$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_3 & a_1 & a_2 \\ a_2 & a_3 & a_1 \end{bmatrix}$$ is a circulant matrix of order 3.

Fact

- *If* $B = circ(a_1, a_2, \ldots, a_n)$, *then* $B^* = circ(a_1^*, a_n^*, \ldots, a_2^*)$.
- *Any two complex circulant matrices commute.*

Amicable and Anti-Amicable Matrices

Definition

- Two matrices A and B are called amicable if $AB^* = BA^*$.
- They are called anti-amicable if $AB^* = -BA^*$.

Definition

- Two matrices A and B are called amicable if $AB^* = BA^*$.
- They are called anti-amicable if $AB^* = -BA^*$.

Example

Let $P := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $Q := \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix}$, and $R := \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix}$. Then

- $PQ^t = -QP^t$,
- $PR^t = RP^t$,
- $RQ^t = QR^t$.

Amicable and Anti-Amicable Matrices

Definition

- Two matrices A and B are called amicable if $AB^* = BA^*$.
- They are called anti-amicable if $AB^* = -BA^*$.

Example

Let $P := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $Q := \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix}$, and $R := \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix}$. Then

- $PQ^t = -QP^t$,
- $PR^t = RP^t$,
- $RQ^t = QR^t$.

Example

Any two Hermitian circulant matrices are amicable. This follows from the fact that any two complex circulant matrices commute.

Kronecker Product

Definition

The *Kronecker product* of two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ of orders respectively $m \times n$ and $r \times s$, denoted by $A \otimes B$ is a matrix of order $mr \times ns$ and is given by

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & & & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}.$$

Kronecker Product

Definition

The *Kronecker product* of two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ of orders respectively $m \times n$ and $r \times s$, denoted by $A \otimes B$ is a matrix of order $mr \times ns$ and is given by

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & & & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}.$$

Example

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ then

Kronecker Product

Definition

The *Kronecker product* of two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ of orders respectively $m \times n$ and $r \times s$, denoted by $A \otimes B$ is a matrix of order $mr \times ns$ and is given by

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & & & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}.$$

Example

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ then

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Property

Suppose that A and B are $n \times n$, and C and D are $m \times m$ matrices. Then, we have the following properties

Property

Suppose that A and B are $n \times n$, and C and D are $m \times m$ matrices. Then, we have the following properties

- $A \otimes (C + D) = A \otimes C + A \otimes D$,

Kronecker Product

Property

Suppose that A and B are $n \times n$, and C and D are $m \times m$ matrices. Then, we have the following properties

- $A \otimes (C + D) = A \otimes C + A \otimes D$,
- $(A \otimes C)^* = A^* \otimes C^*$,

Property

Suppose that A and B are $n \times n$, and C and D are $m \times m$ matrices. Then, we have the following properties

- $A \otimes (C + D) = A \otimes C + A \otimes D$,
- $(A \otimes C)^* = A^* \otimes C^*$,
- $(A \otimes C)(B \otimes D) = (AB \otimes CD)$.

Definition

Suppose that A and B are two square matrices of order n. We denote the Hadamard product of A and B by $A * B$ which is a square matrix of order n such that it's (i, j) entry is the product of the (i, j) entry of A with the (i, j) entry of B.

Disjoint Matrices

Definition

Suppose that A and B are two square matrices of order n. We denote the Hadamard product of A and B by $A * B$ which is a square matrix of order n such that it's (i, j) entry is the product of the (i, j) entry of A with the (i, j) entry of B.
A and B are called disjoint if $A * B = 0$.

Disjoint Matrices

Definition

Suppose that A and B are two square matrices of order n. We denote the Hadamard product of A and B by $A * B$ which is a square matrix of order n such that it's (i, j) entry is the product of the (i, j) entry of A with the (i, j) entry of B.
A and B are called disjoint if $A * B = 0$.

Example

$P := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $Q := \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix}$ are disjoint.

Disjoint Matrices

Definition

Suppose that A and B are two square matrices of order n. We denote the Hadamard product of A and B by $A * B$ which is a square matrix of order n such that it's (i, j) entry is the product of the (i, j) entry of A with the (i, j) entry of B.
A and B are called disjoint if $A * B = 0$.

Example

$P := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $Q := \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix}$ are disjoint.
$I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R := \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix}$ are disjoint.

Theorem

A necessary and sufficient condition that there exists an $OD(n; s_1, \ldots, s_k)$ is that there exist $\{0, \pm 1\}$ matrices $A_1, \ldots, A_k$ of order n such that

Theorem

A necessary and sufficient condition that there exists an $OD(n; s_1, \ldots, s_k)$ is that there exist $\{0, \pm 1\}$ matrices $A_1, \ldots, A_k$ of order n such that

- $A_i * A_j = 0$ *for* $1 \leq i \neq j \leq k$,

Theorem

A necessary and sufficient condition that there exists an $OD(n; s_1, \ldots, s_k)$ is that there exist $\{0, \pm 1\}$ matrices $A_1, \ldots, A_k$ of order n such that

- $A_i * A_j = 0$ *for* $1 \leq i \neq j \leq k$,
- $A_i A_i^t = s_i I_n$, *for* $1 \leq i \leq k$,

Theorem

A necessary and sufficient condition that there exists an $OD(n; s_1, \ldots, s_k)$ is that there exist $\{0, \pm 1\}$ matrices $A_1, \ldots, A_k$ of order n such that

- $A_i * A_j = 0$ *for* $1 \leq i \neq j \leq k$,
- $A_i A_i^t = s_i I_n$, *for* $1 \leq i \leq k$,
- $A_i A_j^t = -A_j A_i^t$, *for* $1 \leq i \neq j \leq k$.

'

Radon number

Theorem (Radon)

The maximum number of variables in an orthogonal design of order $n = 2^a b,\ b$ *odd, where* $a = 4c + d,\ 0 \leq d < 4$, *is* $\leq \rho(n) = 8c + 2^d$. *This bound is achieved!*

Radon number

Theorem (Radon)

The maximum number of variables in an orthogonal design of order $n = 2^a b$, *b odd, where* $a = 4c + d$, $0 \leq d < 4$, *is* $\leq \rho(n) = 8c + 2^d$. *This bound is achieved!*

Example

The maximum number of variables in orthogonal designs of order 2,4,8,16,32,64,128 *are* 2,4,8,9,10,12,16, *respectively.*

Example

Let

$$A_1 = I \otimes I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_2 = I \otimes R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ - & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & - & 0 \end{bmatrix},$$

Example

$$A_3 = R \otimes Q = \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & - \\ - & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$A_4 = R \otimes P = \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & - & 0 & 0 \\ - & 0 & 0 & 0 \end{bmatrix}.$$

Orthogonal Designs

Example

$$A_3 = R \otimes Q = \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & - \\ - & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$A_4 = R \otimes P = \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & - & 0 & 0 \\ - & 0 & 0 & 0 \end{bmatrix}.$$

It is easy to check $A = aA_1 + bA_2 + cA_3 + dA_4$ is an $OD(4; 1, 1, 1, 1)$:

$$A = \begin{bmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{bmatrix}.$$

Lemma (Wolfe)

Given an integer $n = 2^s d$, where d is odd and $s \geq 1$, there exist sets $A = \{A_1, \ldots, A_{s+1}\}$ and $B = \{B_1, \ldots, B_{s+1}\}$ of signed permutation matrices of order n such that

- *A consists of pairwise disjoint anti-amicable matrices,*

Lemma (Wolfe)

Given an integer $n = 2^s d$*, where* d *is odd and* $s \geq 1$*, there exist sets* $A = \{A_1, \ldots, A_{s+1}\}$ *and* $B = \{B_1, \ldots, B_{s+1}\}$ *of signed permutation matrices of order* n *such that*

- A *consists of pairwise disjoint anti-amicable matrices,*
- B *consists of pairwise disjoint anti-amicable matrices,*

Lemma (Wolfe)

Given an integer $n = 2^s d$, where d is odd and $s \geq 1$, there exist sets $A = \{A_1, \ldots, A_{s+1}\}$ and $B = \{B_1, \ldots, B_{s+1}\}$ of signed permutation matrices of order n such that

- *A consists of pairwise disjoint anti-amicable matrices,*
- *B consists of pairwise disjoint anti-amicable matrices,*
- *For each i and j, $A_i {B_j}^t = B_j {A_i}^t$.*

Non-Periodic Autocorrelation Function

Definition

The *non-periodic autocorrelation function* of a sequence $A = (x_1, \ldots, x_n)$ of commuting square complex matrices of order t, is defined by

$$N_A(j) := \begin{cases} \sum_{m=1}^{n-j} x_{m+j} x_m^* & \text{if } \; j = 1, 2, \ldots, n-1 \\ 0 & \text{otherwise} \end{cases}$$

where x_m^* denotes the Hermitian conjugate of x_m, and N_A maps the set of natural numbers into the set of complex matrices of order m.

Example

Let $A = (1, i, 1)$. *Consider* $circ(A) = \begin{bmatrix} 1 & i & 1 \\ 1 & 1 & i \\ i & 1 & 1 \end{bmatrix}$. *Now consider it's upper triangular matrix; i.e,* $U = \begin{bmatrix} 1 & i & 1 \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$.

Example

Let $A = (1, i, 1)$. *Consider* $circ(A) = \begin{bmatrix} 1 & i & 1 \\ 1 & 1 & i \\ i & 1 & 1 \end{bmatrix}$. *Now consider it's upper triangular matrix; i.e,* $U = \begin{bmatrix} 1 & i & 1 \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$.

So, $N_A(1) = 0$ *(the inner product of first row and conjugate of second row of* U*),*

Example

Let $A = (1, i, 1)$. *Consider* $circ(A) = \begin{bmatrix} 1 & i & 1 \\ 1 & 1 & i \\ i & 1 & 1 \end{bmatrix}$. *Now consider it's upper triangular matrix; i.e,* $U = \begin{bmatrix} 1 & i & 1 \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$.

So, $N_A(1) = 0$ *(the inner product of first row and conjugate of second row of* U*),*

and $N_A(2) = 1$ *(the inner product of first row and conjugate of third row of* U*).*

Near Type 1 Matrices

Definition

A set of near type 1 matrices is a set X of commuting square complex matrices of order m with $cd^* = d^*c$ for all $c, d \in X$.

Near Type 1 Matrices

Definition

A set of near type 1 matrices is a set X of commuting square complex matrices of order m with $cd^* = d^*c$ for all $c, d \in X$.

Example

- *The set of all circulant matrices is a set of near type 1 matrices.*

Near Type 1 Matrices

Definition

A set of near type 1 matrices is a set X of commuting square complex matrices of order m with $cd^* = d^*c$ for all $c, d \in X$.

Example

- *The set of all circulant matrices is a set of near type 1 matrices.*
- *The set of negacirculant matrices in two variables is a set of near type 1 matrices, i.e, the set of all matrices of the form*

$$\begin{bmatrix} x_i & y_j \\ -y_j & x_i \end{bmatrix}.$$

Complex Golay Sequences

Definition

Let $A = (x_1, \ldots, x_n)$ and $B = (y_1, \ldots, y_n)$ be two $\{\pm 1, \pm i\}$ sequences such that $N_A(j) + N_B(j) = 0$ for all j. Then A and B are called *complex Golay complementary sequences of length n.*

Complex Golay Sequences

Definition

Let $A = (x_1, \ldots, x_n)$ and $B = (y_1, \ldots, y_n)$ be two $\{\pm 1, \pm i\}$ sequences such that $N_A(j) + N_B(j) = 0$ for all j. Then A and B are called *complex Golay complementary sequences of length n.*

Example

Let $A = (1, i, 1)$ *and* $B = (1, 1, -)$. *Then*
$N_A(1) + N_B(1) = 0 + 0 = 0$ *and* $N_A(2) + N_B(2) = 1 + (-1) = 0$.
Thus, A *and* B *are complex Golay complementary sequences of length 3.*

Complex Golay Sequences

Definition

Let $A = (x_1, \ldots, x_n)$ and $B = (y_1, \ldots, y_n)$ be two $\{\pm 1, \pm i\}$ sequences such that $N_A(j) + N_B(j) = 0$ for all j. Then A and B are called *complex Golay complementary sequences of length n.*

Example

Let $A = (1, i, 1)$ *and* $B = (1, 1, -)$. *Then*
$N_A(1) + N_B(1) = 0 + 0 = 0$ *and* $N_A(2) + N_B(2) = 1 + (-1) = 0$.
Thus, A *and* B *are complex Golay complementary sequences of length 3.*
In other word, if we let $A_1 = circ(A)$ *and* $B_1 = circ(B)$, *then we have* $A_1 {A_1}^* + B_1 {B_1}^* = 3I_3$. *We say that* A_1 *and* B_1 *are complementary circulant matrices of order 3.*

Asymptotic Existence for OD's

Conjecture (P. Eades and J. Seberry)

For any k-tuple $(v_1, v_2, \ldots, v_k)$ *of positive integers, if all of* $v_1, v_2, \ldots, v_k$ *are sufficiently divisible by 2, then there is an*

$$OD\left(\sum_{j=1}^{k} v_j;\ v_1, v_2, \ldots, v_k\right).$$

Asymptotic Existence for OD's

Conjecture (P. Eades and J. Seberry)

For any k-tuple $(v_1, v_2, \ldots, v_k)$ *of positive integers, if all of* $v_1, v_2, \ldots, v_k$ *are sufficiently divisible by 2, then there is an*

$$OD\left(\sum_{j=1}^{k} v_j;\ v_1, v_2, \ldots, v_k\right).$$

Theorem (P. Eades and J. Seberry)

Suppose that $(w_1, \ldots, w_k)$ *is a binary expansion of* t *and there is an* $OD(t; w_1, \ldots, w_k)$*. Then, for every m-tuple* $(u_1, \ldots, u_m)$ *such that* $u_1 + \cdots + u_m = 2^a t$*, there is an integer N such that for each* $n \geq N$*, there is an*

$$OD(2^{n+a}t; 2^n u_1, \ldots, 2^n u_m).$$

Asymptotic Existence for OD's

Theorem

For any k-tuple $(u_1, u_2, \ldots, u_k)$ of positive integers, there exists an integer N such that for each $n \geq N$ there is an

$$OD\left(2^n \sum_{j=1}^{k} u_j;\ 2^n u_1, 2^n u_2, \ldots, 2^n u_k\right).$$

Thank you for your attention!