Mathematical Study of Blood Flow through Atherosclerotic Artery in the Presence of Porous Effect

Sapna Ratan Shah
Department of Mathematics,
Harcourt Butler Technological Institute, Kanpur, (India),
Fax No.0512-2533812.
E-mail: sapna1980jan@rediffmail.com

Abstract
In the present study a mathematical model for the blood flow through atherosclerotic artery in the presence of porous effects is proposed. The laminar, incompressible, fully developed, non-Newtonian in an artery having axially non-symmetric but radially symmetric stenosis is numerically studied. Here the blood is represented as Casson’s fluid model and flow model is shown by the Navier-Stokes and the continuity equations. Using appropriate boundary conditions, numerical expression for volumetric flow rate, pressure drop and wall shear stress have been derived. The expressions are computed numerically and the computational results are presented graphically. The effects of porous parameter on wall shear stress, stenosis length, stenosis size and stenosis shape parameter are discussed. The study reveals that the wall shear stress increases as the porous parameter, stenosis size and stenosis length increases, but as the stenosis shape parameter increases, the wall shear stress decreases. Few comparisons with the existing results have been made in order to validate the applicability of the present model. This model helps for the people working in the field of biomedical science as well as to the medical practitioners.

Keywords: Blood flow, porous effects, resistance to flow, wall shear stress, Atherosclerotic artery.

1.INTRODUCTION:
The study of blood flow through atherosclerotic artery is very important because of the fact that the cause and development of many cardiovascular diseases are related to the nature of blood movement and the mechanical behavior of the blood vessel walls. A stenosis is defined as a partial occlusion of the blood vessels due to the accumulation of cholesterols and fats and the abnormal growth of tissue. ”Atherosclerosis” stenosis (shown in fig.1.(a)) is one of the most frequently anomaly in blood circulation. Once the constriction is formed, the blood flow is significantly altered and fluid dynamical factors play important roles as the stenosis continues to enlarge leading to the development of cardiovascular diseases such as heart attack and stroke etc. In 1968 Young analyzed the effect of stenosis in circular tube. Localized narrowing of the blood vessel is commonly referred to as a stenosis. It is well known that, at various locations in the arterial system, stenosis may due to abnormal intvascular growths. This vascular disease is of frequent occurrence, particularly in mammalian arteries. If this disease takes a severe form, it may lead morbidity and fatality. The exact mechanism for the development of this vascular disease is unclear. Various investigators emphasized that the formation of the intravascular plaques and the impingement of ligaments and spure on the blood vessel wall are some of the major factors for the initiation and development of this vascular disease. Shukla et al. (1980) used several different non-Newtonian models for simulations of blood flow in large arteries and they observed that there is no effect of the yield stress of blood on either the velocity profiles or
the wall shear stress. Cavalcanti (1995) has discussed the inadequacies of such studies for the determination of

![Figure 1(a). Atherosclerosis](image)

the model for plaque growth. These observations provide future direction for research. Quarteroni et al. (2000), Neofytou (2003) founded the important hemo-dynamical characteristics like the wall shear stress, pressure drop and frictional resistance in catheterized coronary arteries under normal as well as the pathological conditions due to stenosis being present. Leuprecht (2001) measured wall shear stress downstream of axi-symmetric stenoses in the presence of hemo-dynamics forces acting on the plaque, which may be responsible for plaque rupture. Dwyer et al. (2001) and Abraham et al. (2004) founded the influence of stenosis morphology on flow fields and on quantities such as wall shear stress among stenotic vessels with very mild stenosis. Sharma et al. (2001), Daripa et al. (2002) considered a mathematical analysis of blood flow through arteries using finite element Galerkin approaches. Pontrelli (2000, 2001) utilized the Brinkman model to investigate the two-dimensional interstitial flow through the tunica media of an artery wall in the presence of an internal elastic lamina. Sharan and Popel (2001) studied flow and heat transport in porous media using mass diffusion and different convective flow models such as Darcy and the Brinkman models. Energy transport in tissue is also analyzed. Yakhot (2004) studied the pulsatile flow of blood under the influence of body acceleration treating blood as a third grade fluid. Sarojani and Nagarani (2008) studied the flow of a casson fluid in a tube filled with a porous medium under periodic body acceleration with application on artificial organs. Johnston (2004), Grigioni et al. (2002) studied the effect of body acceleration on pulsatile flow of non-Newtonian fluid through a stenosed artery and observed that all the instantaneous flow characteristics are affected by the application of body acceleration. Sanjeev et al. (2009) worked on the Pulsatile flow of blood in a constricted artery with body acceleration and observed that the velocity and flow rate increases but effective viscosity decreases, due to a slip wall. The aim of the present investigation has been to study the effect of porous parameter on flow of blood considered as blood as a Casson’s fluid model. In this model problem has been
sorted out numerically. Numerical expressions are shows the variation of the velocity profile, the volumetric flow rate, the wall shear stress.

2. FORMULATION OF THE PROBLEM:
Consider the axisymmetric flow of blood in a uniform circular artery with an axially non-symmetric but radially symmetric mild stenosis. The geometry of the stenosis as shown in [Fig. 1.(b)] is assumed to be manifested as:

\[
\frac{R(z)}{R_0} = 1 - A[L_0^{(m-1)}(z - d) - (z - d)^m], \quad d \leq z \leq d + L_0
\]

\[
= 1, \quad \text{otherwise,}
\]

\[
A = \frac{\delta}{R_0^m L_0^{m/(m-1)}},
\]

where

- \( R_0 \) : Radius of normal tube
- \( R(z) \) : Radius of stenotic region
- \( L \) : The length of the artery
- \( L_0 \) : The length of the stenosis
- \( d \) : Distance between equispaced points
- \( \delta \) : Maximum height of stenosis (\( \delta \ll R_0 \))
- \( m \) : Parameter determining the shape of stenosis (\( m \geq 2 \))

**Casson’s fluid model:** For Casson’s fluid, the relationship between stress and strain is given by:

\[
\tau^{1/2} = \tau_0^{1/2} + (\mu e)^{1/2}, \quad \tau \geq \tau_0
\]

\[
e = \left( -\frac{du}{dr} \right)
\]

Where,

- \( \tau_0 \) = Measure of yield stress
- \( \tau \) = Stress tensor
- \( e \) = Strain rate (-du/dr)
- \( u \) = Velocity of fluid
- \( \mu \) = Viscosity of blood (Casson’s viscosity coefficient)
- \( r \) = radius of the artery

3. DARCY FLOW MODEL:
The Darcy model of flow through a porous media is,

\[
U = \left( -\frac{kp}{\mu} \right)
\]

Where

- \( k \) = Porous parameter
- \( \mu \) = Viscosity of blood
- \( p \) = Pressure gradient
3.1 Conservation equations:

In the present investigation the flow formulation for Navier-Stokes equations for incompressible non-Newtonian fluids in the porous effect has been considered. The basic equations of motion in cylindrical polar coordinates are:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right) \tag{4}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right) \tag{5}
\]

\[
\frac{\partial u}{\partial z} + \frac{1}{r} \frac{\partial (vr)}{\partial r} = 0 \tag{6}
\]

3.2 Boundary conditions:

The no slip condition on the stenosis surface gives the following boundary conditions

\[ u = 0 \quad \text{at} \quad r = R_0 \]
\[ u = 0 \quad \text{at} \quad r = R(z) \tag{7} \]

3.3 Solution of the problem:

Solving these equation, the velocity of blood, rate of flow and pressure in (7, 8, 9),

\[ v = -\frac{P}{4\mu} \left( R^2 - r^2 - \frac{8}{3} R^{1/2} \left( R^{3/2} - r^{3/2} \right) + 2R (R-r) \right) \tag{8} \]

Now total flow flux

\[ Q = \int_0^R 2\pi u r \, dr = \pi \int_0^R r \left( -\frac{du}{dr} \right) \, dr, \tag{9} \]

By using equation (8) and equation (9), we have,
\[ Q = -\frac{\pi p R^4}{8 \mu} \left( \frac{16}{7} \left( \frac{r}{R} \right)^{1/2} + \frac{4}{3} \left( \frac{r}{R} \right) - \frac{1}{21} \left( \frac{r}{R} \right)^4 \right) \]  

(10)

\[ \frac{dp}{dz} = \left( -\frac{8 \mu Q}{\pi R^4} \phi(z) \right) \]  

(11)

With

\[ \phi(z) = \frac{1}{\left( 1 - \frac{16}{7} \left( \frac{r}{R} \right)^{1/2} + \frac{4}{3} \left( \frac{r}{R} \right) - \frac{1}{21} \left( \frac{r}{R} \right)^4 \right) } \]  

The pressure drop, \( p = p \text{ at } z = 0 \) and \( -p \text{ at } z = L \) across the stenosis in the tube of length, \( L \) is calculated from equation (9) as

\[ \Delta p = \int_{0}^{L} \left( \frac{dp}{dz} \right) dz = \left( \frac{8 \mu Q}{\pi R^4} \right) \psi \]  

(12)

Where

\[ \psi = \int_{0}^{L} \phi(z) dz = \int_{R/R_0 = 1}^{d} \phi(z) dz + \int_{d+L_0}^{d+L_0} \phi(z) dz + \int_{R/R_0 = 1}^{L} \phi(z) dz \]  

The first and third integrals in the expression for obtained above are straight forward whereas evaluation of the second integral is a formidable task and therefore will be evaluated numerically.

\[ p = \frac{8 \mu Q}{\pi R^4} \int_{0}^{R} \left( \frac{R}{R_0} \right)^4 \left( \frac{16}{7} \left( \frac{r}{R} \right)^{1/2} + \frac{4}{3} \left( \frac{r}{R} \right) - \frac{1}{21} \left( \frac{r}{R} \right)^4 \right) dz \]  

(13)

Using equation (1) and equation (3) we have,

\[ \tau = \left( \frac{U \mu R_0}{2k} \right)^{1/2} \int_{d}^{(d+L_0)} \left( \frac{16}{7} \frac{\delta}{R_0 L_0^{(m-1)}} \left[ L_0^{(m-1)} (z - d) - (z - d)^m \right] \right)^{1/2} \]  

\[ \int_{d}^{(d+L_0)} \left( \frac{4}{3} \frac{\delta}{R_0 L_0^{(m-1)}} \left[ L_0^{(m-1)} (z - d) - (z - d)^m \right] \right)^{4} \]  

(14)
4. RESULTS AND DISCUSSIONS:

The motivation behind developing this mathematical model has been to study some aspects of blood flow through a stenosed artery in the presence of porous parameter having an axially non-symmetric but radially symmetric mild stenosis. Most of theoretical result such as velocity, volumetric flow rate, wall shear stress and pressure gradient are obtained in this analysis. Out of these results only the numerical solution of wall shear stress are shown. Wall shear stress is an important factor in the study of blood flow. Accurate predictions of the distribution of wall shear stress are particularly useful for the understanding of the effect of blood flow through stenosed artery in the presence of porous effects. In order to estimate the quantitative effects of porous parameter, stenosis size, stenosis length and stenosis shape parameter on wall shear stress, computer codes were developed and to evaluate the numerical results for wall shear stress for diseased system associated with stenosis due to the local deposition of lipids have been determine. In order to understand the complete nature of blood flow in the presence of porous parameter, the computed numerical results by using the values of parameter based on experimental data in stenosed artery (U = 50 cm/sec, R₀ = 1.2 cm and μ = 0.04 dyne/cm²) are plotted in Fig 2-5.

In order to analyze the wall shear stress along the stenosed arterial segment under study, Fig.2 exhibits the variation of wall shear stress with porous parameter. It is observed that in the stenosed portion of the artery, the wall shear stress increases with the increase in porous parameter.

Fig 2. Variation of wall shear stress with porous parameter
Fig 3. Variation of wall shear stress with size of the stenosis

Fig 4. Variation of wall shear stress with stenosis length
The result is consisting with the result of Shukla et al. (1980). The variations of wall shear stress with stenosis size are shown in figure 3. It is evident that wall shear stress increases as stenosis size increases. As the stenosis grows, the wall shearing stress increases in the stenotic region. Our results are similar to those obtained by Sanjeev (2009). Fig.4 gives a comparison of our results with those reported by Johnston et al. (2004). It is shown that the wall shear stress increases with increasing value of stenosis length. In Fig.5 the variation of wall shear stress with stenosis shape parameter has been shown. The wall shear stress gives the reverse trend of stenosis size and stenosis length. This figure illustrates that wall shear stress decreases as stenosis shape parameter increases, maximum wall shear stress occurs at \((m = 2)\), i.e. in case of symmetric stenosis. These results are similar with the results of Pontrelli (2000).

5. CONCLUSIONS:

This analysis presented numerical results for blood flow through a stenosed artery in the presence of porous effects. In this paper we used the laminar, incompressible, fully developed, non-Newtonian flow having axially non-symmetric but radially symmetric stenosis. Here the blood is represented as Casson’s fluid model and flow model is shown by the Navier-Stokes and the continuity equations. The advantage of this study is that here we calculated the effect of porous parameter on wall shear stress, stenosis shape parameter, stenosis size, stenosis length in an stenosed artery. It has been concluded that the wall shear stress increases as porous parameter, stenosis size and stenosis length increases but decreases as stenosis shape parameter increases. it has shown that the results were greatly influenced by the change of porous parameter and stenosis shape parameter. This model is able to predict the main characteristics of the physiological flows and would be helpful for the people working in the field of biomedical science as well as to the medical practitioners.
6. REFERENCES: