

# Math 3331 – ODEs

## Sample Test 2

1. A large tank is filled with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at a faster rate of 10 gal/min. Find the number of pounds of salt in the tank at any time. Find the amount of salt in the tank at the moment when the tank is empty.

2. A population of rabbits satisfies the logistic eq. i.e. the population grows proportional to the amount of rabbits present and to the space available (which we will assume is 1000 rabbits). If the initial population of rabbits is 100 and it is observed that after 1 week there are 120 rabbits, how many rabbits will there be after 2 weeks? How long will it take for the populations to reach 90% of the limiting population? (This model also applies to spreading of rumors and epidemics)

3. A roast is removed from an oven and placed in a room where the temperature is  $70^\circ\text{F}$ . A meat thermometer indicated the internal temperature of the roast to be  $160^\circ\text{F}$ . Twenty minutes later the meat thermometer indicates  $150^\circ\text{F}$ . Assuming Newton's law of cooling, find the temperature of the roast at any given time

4. Solve the following

$$(i) \quad y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$(ii) \quad y'' + 2y' + 10y = 0, \quad y(0) = -1, \quad y'(0) = 4$$

$$(iii) \quad 4y'' - 4y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

5. (i) Solve

$$x^2y'' - xy' + y = 0,$$

given that  $y_1 = x$  is one solution.

5. (ii) Solve

$$xy'' - (x + 1)y' + y = 0,$$

given that  $y_1 = e^x$  is one solution.