

COMMON CORE BASICS

Mathematics Lesson Plans

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BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Use place value to read and write whole numbers
- Compare and order whole numbers

✓ Determine Student Readiness

In this lesson, students apply place-value concepts to read, write, compare, and order numbers through the billions. To determine their readiness, call out numbers through the millions and have volunteers write the numbers in order on the board. Or, invite students to work in pairs, taking turns calling out and writing numbers through millions. Observe students as they work.

Key Concept

Represent, compare, and order whole numbers to better understand the meaning and value of whole numbers.

Concept Background: Understanding place value is key to working successfully with numbers. Direct students to the place-value chart on page 12. Discuss the place values indicated in the chart, and work with students to explain the total value of each digit. If students appear to struggle, you may want to have them use math manipulatives, desk supplies such as paper clips, or pictures to represent the value of each digit within a whole number.

Develop Core Skills

Core Skill: Apply Number Sense Concepts

Write the term *number sense* on the board. Point to each word in the term separately, and ask students to explain what it means. Guide them toward understanding that sense is an understanding or awareness of meaning. Number sense refers to an understanding of the meanings of numbers and their relationships. For example, explain that the symbols 5 and 2 each represent a quantity, and that the symbol + represents an action, or operation. Ask: *What does your number sense tell you about the expression $5 + 2$?*

Core Practice: Model with Mathematics

Explain to students that there are a variety of tools to help support or explain mathematical thinking. A chart or diagram is a model. So is an expression like $5 + 2$. Point to the place-value chart that appears in the lesson. Invite students to discuss how a chart can be a helpful tool in understanding math concepts.

Pre-Teach Vocabulary

Apply Prior Knowledge

Read the vocabulary words for students, and ask them to raise their hands when they hear a familiar word. Pause to respond to each show of hands, inviting a volunteer to explain the word. Record the answer on the board. Then, allow other students who raised their hands to support, clarify, or revise the definition. If necessary, write the correct definition for each word and help students resolve discrepancies.

Tier 2 Words:

approximate
(p. 13)
period (p. 12)
value (p. 12)

Tier 3 Words:

digit (p. 12)
number line (p. 15)
whole number
(p. 12)

Test Words:

chart (p. 14)

DURING THE LESSON

PAGE 12

Place Value

Read the first two paragraphs of "Place Value" with students. Direct their attention to the model of a place-value chart. Ask questions to help students interpret what they see, such as: *How many hundreds are in this number?*

Guide students through Examples 1 and 2. You may want to draw or project the place-value chart on the board to allow volunteers to walk through the steps with help from the class. After completing both examples, write a new number in the millions on the board. Ask students to identify the value of each digit in the number, referring to the place-value chart in their books, if necessary.

PAGE 13

Identify the Main Idea

Create a chart on the board with five columns: **Reading, Writing, Mathematics, Science, Social Studies**. Tell students that in all subject areas, being able to find the main idea of nonfiction texts, such as what they will find in mathematics, science, and social studies, as well as reading and writing, will help them understand hard concepts.

Read the boxed paragraph on page 13 together as a class. Review which sentence in this paragraph from a mathematics book is the main idea. Ask for student volunteers to give tips for finding the main idea in mathematics texts. Write the tips in a numbered list on the board.

PAGE 14

THINK ABOUT MATH

ANSWER KEY

1. 70
2. 80,000
3. 6
4. 300,000,000
5. 8,000,000, 000

Read and Write Whole Numbers

Read the first sentence aloud. Guide students through the steps in Examples 3 and 4. To reinforce understanding, tell students that you are going to read some numbers aloud, and you want them to write what they hear. Afterward, write the numbers you read aloud on the board and give students time to check their work. Encourage students to ask questions about any problems they may have. Next, write numbers in word form on the board, and challenge students to write them in number form. Invite volunteers to write the answers on the board, and discuss any discrepancies that may occur.

THINK ABOUT MATH

ANSWER KEY

1. E
2. D
3. A
4. C
5. B

Core Skill: Model with Mathematics

Remind students that a chart is a visual model of a concept, such as place value. Have students refer to the place-value chart on page 12 to explain how the value of each digit in the number 5,555 changes as the digits move from the right to the left. Challenge students to suggest new numbers and then read and explain the value of each digit in those numbers.

MATH LINK

Remind students that when a zero appears in a number, it tells the reader that there is no value associated with that place in the number. So, that place is not expressed when a number is written in words. Write the number 345,017 on a board or chart. Ask a volunteer to write the number in words and then read it. Discuss the written and spoken forms as a class.

Compare and Order Whole Numbers

PAGE 15

Ask students to identify an example of a mathematical model on the page. Direct attention to the number lines, and explain that these lines serve as mathematical models to assist students in understanding the relationships among numbers. Read the number line for students and then guide them through the steps in the example.

Engage and Extend

ELL Instruction: State Preferences Ask: *Which do you think is easier: using a number line to compare numbers or using place value chart to compare numbers? Why?* Have volunteers state their preference and offer reasons for it. Encourage students to use the terms *number line*, *compare*, and *place value* in their responses.

21st Century Skill: Access Information

Give students time to read the text independently. Then invite them to talk about a mathematical idea or process that they would like to know how to do very well. Ask them where they might go for more information on the topic. Engage students in a discussion about the many kinds of information that are available to them, both in print and online. Encourage students to think of people, too, understanding that it is often possible to get information directly from experts.

MATH LINK

Write the word *equality* on the board, and invite students to suggest a meaning. They may say, for example, that it contains the base word *equal*, so it means “being equal.” Next, write the prefix *in* on the board, and ask students what it means. Give students some examples of words that begin with the prefix, such as *incorrect*. Now, add the prefix *in* to the word *equality*, and ask students to define it. Then, read the Math Link and explain that the symbols $<$ and $>$ represent inequalities. They show that one value is either less than or greater than another.

PAGE 16

THINK ABOUT MATH

ANSWER KEY

1. Compare the digits of the numbers from left to right until the digits in the same column are different. The digits in the thousands place, 3 and 4, are not the same. Compare those digits. $203,478 < 204,210$.
2. 698,321; 698,432; 701,286

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 390 and 391.

Extension Activity: Relate a Process Have students write an explanation of a process other students can follow to compare numbers through the billions. Encourage students to share their explanations and test them by having listeners follow the steps to compare two numbers.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Add whole numbers
- Subtract whole numbers

✓ Determine Student Readiness

In this lesson, students learn to add and subtract whole numbers through the millions. To determine student readiness, give students an opportunity to demonstrate their mastery of basic addition and subtraction facts. You may want to have students construct addition and subtraction tables. Or you may want to play a class game, such as baseball facts, in which you assign “bases” in the room, and organize students into two teams of players. You, or someone you appoint, calls out an addition or a subtraction problem. If the player answers correctly, he or she moves to first base. Play continues until a team earns three outs, or incorrect answers. Then play moves to the next team. If students appear to struggle, allow teams to work together to give sums and differences, rather than asking individuals to demonstrate proficiency.

Key Concept

Addition and subtraction are basic operations in mathematics.

Concept Background: Addition and subtraction are the two most frequently used operations in mathematics. Ask students to use their own words to define each operation and then describe the strategies they use to complete each operation. For example, some students may draw pictures, use objects they can manipulate, or rely on mental math. Ask students to give examples of addition and subtraction they use every day, such as calculating how much change they will receive after paying for lunch.

Develop Core Skills

Core Skill: Perform Operations

Write the symbols $+$, $-$, \times , and \div on the board. Ask students if they recognize these symbols. Have volunteers identify the meaning of each symbol and offer an example of a simple problem that represents each operation. Tell students that as they solve problems, it is important to remain aware of symbols and their meanings if they are going to solve problems successfully.

Core Practice: Attend to Precision

Write the word *precision* on the board. Tell students that *precision* means a “high level of accuracy.” Then write the following examples on the board: An engineer is building a highway; A carpenter is building kitchen cabinets; A clothing designer is cutting a length of fabric. Ask students what the word *precision* has to do with these examples. Explain to students that precision is related to measured values and how close they are to perfection. An engineer, for example, must measure precisely to make sure drivers have enough time to slow down before leaving a highway. A carpenter must measure precisely to avoid wasting wood. A clothing designer makes precise measurements so clothes fit well. Invite students to describe personal examples of when precise measurements are critical.

Pre-Teach Vocabulary

Relate to Other Words

Ask students to look at the list of vocabulary words to identify words they already know. Ask students to share examples of when those words have applied to something they did. To promote student thinking, suggest an example, such as: I will calculate the sum of coins I have to determine if I can buy an ice-cream cone. The difference between the cost and the change I received was 25 cents.” Encourage students to give personal examples, pointing to vocabulary words as they use them.

Tier 2 Words:

addition (p. 18)
calculate (p. 18)
difference (p. 19)
subtraction (p. 19)

Tier 3 Words:

operations (p. 18)
sum (p. 18)

Test Words:

context clue (p. 19)

DURING THE LESSON

PAGE 18

Add Whole Numbers

Read the first two paragraphs for students and then, work through Example 1 on page 18 and Examples 2 and 3 on page 19 as a class. If students appear to have difficulty, provide hands-on materials, such as desk supplies, that students can use to represent numbers.

If you find that students understand the concept of addition but cannot align digits by place value, suggest they use a ruler to separate place value columns before they attempt to add. Provide practice problems, allowing students to add two 2-digit numbers without regrouping. Once students demonstrate mastery, provide practice problems that require regrouping. Ask: *What are you doing when you carry numbers over? How are regrouping, or carrying over, and place value related?*

THINK ABOUT MATH

ANSWER KEY

1. 79 2. 85 3. 553

Subtract Whole Numbers

Read the first paragraph with the class. Afterward, ask: *How does the illustration relate to what we have read? What operation does the illustration show us? Which number in the subtraction sentence represents the difference?*

Guide students through the steps in Examples 4 and 5 on page 20. Emphasize the need for special attention to regrouping. For students who appear to struggle, provide objects they can use to represent and manipulate numbers. You may even want to pair struggling students with students who demonstrate proficiency in subtraction with regrouping. Offer practice problems, such as $2,304 - 1,756$, take time to discuss the subtraction process, and give students time to find the difference (548). Continue providing practice problems, or invite students to write and solve their own problems while you observe their work.

Finally, guide students in using a calculator to work through Example 6.

Core Practice: Attend to Precision

Have students read the text. Next, show them an image of a skyscraper or a set of highway overpasses. Then have them explain the relationship between what they see and mathematical precision. Invite students to discuss the possible consequences of using the wrong operation when calculating measurements, whether in building enormous structures or something simpler, such as a birdhouse.

Engage and Extend

ELL Instruction: Use Models to Talk about Math

Distribute desk supplies or math manipulatives to students. Call out numbers and have students use the materials to represent the numbers. Then assign the operation of either addition or subtraction, and invite students to use their models to complete it. Ask pointed questions to help students explain their thinking as they work. For example, ask: *Why did you move those objects to the tens place after adding the ones?*

THINK ABOUT MATH

ANSWER KEY

1. 25 2. 1,932 3. 629

MATH LINK

Help students recognize that they cannot change the position of numbers within a subtraction problem without changing the results. Write a simple problem such as $5 - 2 = 3$ on the board. Then have students explain the consequences of reversing the order of 5 and 2.

MATH LINK

Explain that because addition and subtraction are opposite operations, they can be used to check answers. Invite students to think of subtraction as “undoing” addition. Point to the example in the Math Link, and then have students work in pairs to practice. Ask one student in each pair to write and solve an addition problem, and have the other student use subtraction to check the sum. Then have them repeat the activity using a subtraction problem, and using addition to check the difference.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 391.

Extension Activity: Estimate Numbers Organize students into pairs. Give each pair a transparent plastic cup that you have marked with a different letter of the alphabet. Have students fill their cups with common objects, such as paper clips. Tell students to count the objects they put in the cups, record the number, and keep it a secret until later. Have pairs exchange cups, record the letter of the cups they receive, estimate the total number of objects in the cups, and write their estimates. Continue the activity until all pairs have estimated the number of objects in each cup. Then, write the letters on the board and beneath them, have pairs write their estimates. Finally, have them compare estimates to exact quantities and determine their differences. Discuss different strategies students used to estimate and calculate the differences between estimates and actual counts.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Multiply whole numbers
- Divide whole numbers

✓ Determine Student Readiness

In this lesson, students multiply and divide multi-digit numbers. To determine student readiness, use prepared flash cards or have students make flash cards for multiplication and division facts through 12. Show students the cards, and ask students to find the product or quotient. There are also numerous online games that students can use to refresh their recall of basic facts.

Key Concept

Multiplication is the operation of adding a certain quantity a set number of times. Division is the operation that is used to separate a quantity into parts.

Concept Background: Model examples of multiplication as repeated addition and division as the separation of a whole into parts. Students may draw pictures or use hands-on objects, such as desk supplies, to follow along with the process. Have students share and explain their models. Ask questions to help students. For example, ask: *How does your model demonstrate the relationship between multiplication and repeated addition?*

Develop Core Skills

Core Skill: Find Reverse Operations

Explain to students that like addition and subtraction, multiplication and division are also opposite operations. This means they can use multiplication to check division, and division to check multiplication. Each operation “undoes” the other. Have a volunteer demonstrate checking an addition sum using subtraction.

Reading Skill: Draw Evidence from Texts

Write the word *context* on the board. Explain to students that a context is a setting or location. Give an example, such as: *I read a book about the construction of the Eiffel Tower, which served as the entrance to a world’s fair. The context of the book is Paris, France, in the late 1800s.* Then explain that math problems are often set in specific contexts, too, and that students can sometimes get clues from the context to figure out how to solve a problem. Ask students: *How might the context of a math problem help you solve it? What should you look for?*

Pre-Teach Vocabulary

Look for Connections

Have students examine the vocabulary words and discuss possible relationships among them. Encourage students to rely on their prior mathematics knowledge and experience to find connections. Write the words *Multiplication* and *Division* in separate circles on the board and explain the terms, if necessary. Then invite students to add circles to connect examples and ideas to the central circles. Challenge students to make connections between the two operations, too, if they are able.

Tier 2 Words:

dividend (p. 23)
factor (p. 22)
product (p. 22)

Tier 3 Words:

division (p. 22)
divisor (p. 23)
multiplication (p. 22)
quotient (p. 23)

Test Words:

context (p. 24)

DURING THE LESSON

PAGE 22

Multiply Whole Numbers

Read the introductory text with students, emphasizing the boldfaced words. Work through Example 1 as a class. Ask: *When we multiply large numbers, why do we write sets of numbers in the problem vertically instead of horizontally? Why do we write the larger number on top? As you complete the problem together, point out that the order of the factors doesn’t matter, but putting the larger factor first and stacking the factors on top of each other make multiplication easier. For students who appear to have difficulty aligning factors and regrouping, encourage them to draw vertical lines to distinguish place values before they attempt to solve problems.*

Next, organize students into pairs. Have students complete Example 2 on page 23. Have pairs compare answers, and if they find discrepancies, have them determine where they might have made a mistake and try again.

MATH LINK

Explain to students that when they hear or read the term regrouping in mathematical operations, they are actually carrying. Also help students see how important it is to write the values they are regrouping in the correct place-value column. Use the problem in Example 1 to demonstrate regrouping and placing all of the regrouped values correctly.

THINK ABOUT MATH

ANSWER KEY

- | | |
|-----------|------------|
| 1. 68 | 6. 684 |
| 2. 414 | 7. 3,552 |
| 3. 1,560 | 8. 9,936 |
| 4. 19,593 | 9. 48,200 |
| 5. 11,426 | 10. 51,345 |

Divide Whole Numbers

Point to the division problem represented four different ways. Ask students which formats they have used in the past. Ask: *What are the common elements in each of the representations?*

Guide students through Example 3. Offer additional practice problems that give you time to observe students' skill at finding quotients without remainders. Identify and help students resolve common problems, such as aligning numbers correctly, and applying basic rules.

Guide students through Example 4 on page 24. Point out the remainder and how the remainder is represented in the quotient. Then have students complete Example 5. Afterward, have students use calculators and reverse operations to check the answers in both examples on this page.

Core Skill: Find Reverse Operations

Remind students that in Lesson 1.2, addition and subtraction were called opposite operations. In this lesson, multiplication and division are called reverse operations. Explain that as they read math texts, they may come across another term that also means opposite or reverse operations. It is *inverse operations*. Whether using the term opposite, reverse, or inverse, the action is the same, and it can be applied to check addition and subtraction, as well as multiplication and division. Ask: *What inverse operation can we use to check our work in Example 3?* Allow a volunteer to demonstrate the process.

Engage and Extend

ELL Instruction: Model Operations Ask students to use pictures or objects to demonstrate and explain the processes of multiplication and division. Students may want to model original problems or use examples from the lesson. Ask questions to encourage them to talk about their thinking and understanding as they progress.

THINK ABOUT MATH

ANSWER KEY

- | | |
|----------------------|-----------|
| 1. 13 R (remainder)2 | 6. 34 R1 |
| 2. 31 | 7. 20 |
| 3. 32 | 8. 15 R8 |
| 4. 202 R14 | 9. 109 R3 |
| 5. 200 | 10. 16 R1 |

Reading Skill: Draw Evidence from Text

Read the sidebar as a class. Examine the problem involving Jorgé and ask students to identify words from the problem that tells them how to solve the problem. For example, students might note the expressions “every week,” “total,” and “at the end of 17 weeks.” Explain that students should look for evidence such as this to determine their strategies for solving problems. Repeat the process with the problem involving Keisha. Then have students record words they can look for as evidence of the need to divide to solve a problem.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 391 and 392.

Extension Activity: Draw Conclusions Based on Factors

Organize students into small groups. Randomly assign five different digits to each group. Have groups work together to use their digits to create two factors, one with two digits and a second with three digits. Tell students that their goal is to find two factors with the greatest product. Encourage students to share and justify their work.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Determine the set of factors of a number
- Determine the greatest common factor of two numbers
- Identify and apply patterns

✓ Determine Student Readiness

This lesson requires students to determine factors of whole numbers and identify common factors. To determine student readiness, ask students to use mental math to test their proficiency with basic multiplication. Call out a multiplication problem and choose a student to answer. Then have that student call out a multiplication problem and choose another student to answer. Continue playing until all students have had an opportunity to come up with and answer a problem.

Key Concept

A whole number is the product of two or more factors, and the greatest common factor is the greatest factor shared by those whole numbers.

Concept Background: Explain to students that every whole number has a set of factors, or numbers that can be multiplied to equal that whole number. This set of factors can be determined by examining all of the numbers that can be multiplied to yield the whole number in question.

Develop Core Skills

Core Skill: Apply Number Sense Concepts

Explain to students that when they talk about numbers, relationships between numbers, and how operations affect numbers, they are applying number sense concepts. Draw or project a multiplication table on the board. Shade a diagonal from the upper left to lower right on the table, covering the numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, and 144. Ask: *What do you see when you compare numbers above and below the diagonal line I have shaded?* Help students see that the numbers are the same.

Core Skill: Build Solution Pathways

Explain that in math, as in daily life, we often follow a set of steps in order to find a solution or accomplish a goal. To aid students in visualizing a set of sequential steps in a process, ask them to describe something they do regularly that requires five or more steps. For example, students might describe the steps involved in doing a specific chore or filling a car's tank with gasoline at a gas station.

Pre-Teach Vocabulary

Make Meaning

Draw a two-column chart on the board. List the meanings below of the vocabulary words in the first column and the actual words in the second column. Have students guess or use prior knowledge to link each meaning to its word.

an act upon numbers, such as addition (operation); to find the value of (evaluate); a group of numbers and symbols (expression); to make the same, or equal (equation); a number that is multiplied (factor)

Tier 2 Words:

equation (p. 26)
evaluate (p. 29)
factor (p. 26)

Tier 3 Words:

Commutative Property of Multiplication (p. 27)
Distributive Property of Multiplication (p. 29)
greatest common factor (p. 28)

Test Words:

expression (p. 29)
operation (p. 29)

DURING THE LESSON

PAGE 26

What Is a Factor?

Read the section aloud, and ask a volunteer to define the term *factor* as it's used in the text. Ask: *What are the factors in the equation, or mathematical statement, $6 \times 4 = 24$?*

Apply Number Sense Concepts: Finding the Factor of a Number

Write the equations at the bottom of page 26 on the board. Point out that the product of each pair of factors is the same. Have volunteers circle each factor the first and only the first time it is used.

Guide students through the text on page 27, pausing after introduction of the Commutative Property of Multiplication to discuss how it applies to the list of equations. Ask: *When we consider the factors of a number, why do we cross out equations that use identical factors?* Encourage students to refer to the Commutative Property of Multiplication in their answers, helping them see that because the order of the factors doesn't matter, eventually, the same factors are repeated.

PAGE 27

Core Skill: Apply Number Sense Concepts

Ask students to explain how the associative and commutative properties apply to addition and multiplication. Point out that the name of each property describes how it works. The associative property, for example, describes how numbers associate, or group with each other, and associations don't affect the product. The commutative property describes how the order in which numbers commute, or move back and forth, doesn't affect the product.

Comparing Sets of Factors

Ask students: *What do apples and oranges have in common?* Students may suggest that they are fruits, they grow on trees, they have skins, and they have seeds. Help students see that the term *common* means that they share the same or similar characteristics. Then explain that when they look for common factors, they are looking for factors that are the same in two or more products.

Guide students through the section, pausing often to ask students to summarize their learning. Afterward, invite volunteers to suggest other numbers to factor. Have students work in pairs or small groups to factor the suggested number. Discuss the results and then allow another volunteer to suggest a new number to factor. Continue until you are confident that students understand the concept of factoring.

Core Skill: Build Solution Pathways

Read the text as a class, pausing at the end of each paragraph to determine meaning. Explain that this is an effective comprehension strategy students can use to understand any text, including math text.

After reading the text, ask students to explain how applying a property, such as the Commutative Property of Multiplication, to the solving of a problem can simplify a solution pathway and save time.

Factoring Mathematical Expressions

Write the problem $9 + 2 = 11$ on the board to review the meanings of the vocabulary words. Then ask students, before they read this section, to recall how applying a mathematical property can actually make finding a solution easier.

As they read the text, pause whenever necessary to respond to students' questions. Then ask students how they could use factoring and the Distributive Property of Multiplication to solve expressions more easily. Invite students to demonstrate their explanations.

Engage and Extend

ELL Instruction: Choose a Number Write a set of numbers on separate pieces of paper and put the papers in a box. Organize students into pairs and have each pair choose a number from the box. Engage students in a conversation of how to find the factors of the number. Invite them to go to the board and demonstrate the process for finding factors, explaining each step in their own words.

The factors of 63 are: 3, **7**, 9, and 21.

The factors of 28 are: 2, 4, **7**, and 14.

The only common factor is 7.

So, the greatest common factor is 7.

$$63 - 28 = (7 \times 9) - (7 \times 4)$$

$$(7 \times 9) - (7 \times 4) = 7 \times (9 - 4)$$

Finally, you can rewrite the original expression as:

$$63 - 28 = 7 \times (9 - 4)$$

21st Century Skills: Critical Thinking and Problem Solving

Explain to students that some learning does not have a direct application in daily work. However, it may have an indirect application. Read the text aloud and ask students to summarize the purpose of learning strategies such as factoring and mathematical properties such as the Distributive Property of Multiplication. Ask students to explain the relationship between algorithms and complex mathematical tasks.

MATH LINK

Ask students how they can use the meaning of the word *commute* to apply the Commutative Property of Multiplication. Then have students apply the property to determine the factors of 20: 2, 4, 5, 10.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 392.

Extension Activity: Construct Factoring Problems Ask students to follow the parameters below to create original problems and identify the greatest common factor of each set of three numbers.

- What is the greatest common factor of nn , nn , and nn ? (*where nn is a number equal to 100 or less*)
- Factor the following expression $nn \times (nn + nn)$ (*where nn is a number equal to 100 or less*)

Have students explain their work in writing. Encourage them to share their explanations with others, and invite students to ask questions.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Identify situations in which rounding or estimating is appropriate
- Round numbers to the nearest specified place value
- Use estimation appropriately

✓ Determine Student Readiness

In this lesson, students round numbers to a given place value and estimate sums and quotients. To demonstrate readiness for estimating, have students solve addition and division problems such as: $500 + 200$; $200 + 300 + 600$; $200 + 600 + 100$; $1,200 \div 40$; and $8,000 \div 20$; $5,400 \div 90$.

Key Concept

Rounding and estimation are useful when an answer does not need to be exact or when checking an exact answer.

Concept Background: Ask students to explain what someone means if they offer a ballpark estimate. After students have had time to respond, explain that the expression means anyplace inside a huge space in which baseball is played. A ball can go a variety of distances and still be in the ballpark, so a ballpark estimate isn't exact. Have students give examples of when a ballpark estimate works as well as an exact answer. For example, students might see an item that costs \$1.75. Knowing that they need to buy 5 of the items, they can guess that the total cost will be less than \$10.00, a ballpark estimate.

Develop Core Skills

Core Skill: Paraphrase Data

Explain to students that a teacher will sometimes paraphrase information in a textbook, or restate it in different words, to make it easier for students to understand. Ask students if they can recall a time during their childhood when an adult paraphrased a story for them. Tell students that if they are able to paraphrase something they have learned, it will help them understand and remember the information themselves.

Core Practice: Use Appropriate Tools Strategically

Ask students to describe a time when they estimated how much time it would take to do something, like complete a chore or finish a homework assignment, and then compared their estimate to the actual amount of time. Explain that sometimes, an estimate is useful in itself, and no exact measurement is needed. But at other times, a tool is required to be more exact. A clock, for example, is a tool for determining exactly how much time it takes to complete something.

Pre-Teach Vocabulary

Relate Words to Other Words

Ask students to use their prior knowledge and experiences to build group word maps. Organize students into groups and have each group design and build a word map to explain vocabulary words they already know. Encourage students to share and explain their maps.

Tier 2 Words:

rounding (p. 32)

Tier 3 Words:

compatible numbers (p. 34)
front-end digits (p. 34)

Test Words:

estimate (p. 32)

DURING THE LESSON

PAGE 32

Rounding

Read the explanation of rounding aloud. Ask students to compare the text to the illustration of the hilly number line. Ask students to explain how the text and illustration work together to explain the concept of rounding.

Then work through Example 1 together. If you find that some students are struggling, have them work with proficient students to make and use number lines to demonstrate each example. If necessary, give them time to practice rounding additional numbers to the nearest ten and hundred.

Tells students that being able to round numbers prepares them for estimating the answers to operations. It also makes mental math easier.

THINK ABOUT MATH

ANSWER KEY

1. 60
2. 90
3. 130
4. 1,350

PAGE 33

Draw Conclusions

Make up an example of a situation that can lead to a conclusion. For example, say, *When I woke up this morning, my slippers were gone. I found them at the bottom of the stairs, along with the morning paper, torn to shreds. I concluded that my new puppy woke up before I did.* Ask students to repeat the conclusion. Then have them identify facts in your story that made the conclusion logical.

Begin reading the text in the lesson and then give students time to read the passage independently. Ask them to choose an appropriate conclusion and underline evidence in the text that they can use to support their choice. Discuss students' responses as a class and resolve cases where students formed an incorrect conclusion.

Core Skill: Paraphrase Data

Read a brief news article to students. Then paraphrase the article, or invite a volunteer to do it. Explain that paraphrasing is an effective strategy for understanding a text. Have students paraphrase what they have learned about rounding.

PAGE 34

Estimation

Guide students through the steps in both examples. For Example 2, explain that the numbers are rounded so that there is only one non-zero digit in each number. This makes the numbers easier to add mentally. However, the answer is only an estimate. The actual sum is 142 more than the estimated sum.

Tell students that if they estimate to check their work, they are attempting to determine if their answers are reasonable. Point out that when they use front-end estimation, the estimated sum will always be less than the actual sum.

Explain to students that Example 3 shows how to use compatible numbers. Knowing how to multiply and divide will make this easier. Work through the steps together, taking note of students who may be struggling. You may need to provide additional practice problems before students complete the Think about Math activity.

THINK ABOUT MATH

ANSWER KEY

Sample answers:

1. 700 3. 1,100
2. 5 4. 14

21st Century Skill: Understand Media Messages

Help students find media stories that contain numbers. Have students determine whether the numbers are exact values or rounded or estimated amounts. Help them recognize that they are applying skills they learned in class to real-life examples, and that these skills are useful in understanding a variety of media.

Engage and Extend

ELL Instruction: Build an Estimation Number Line Invite students to help you draw a number line to 50. Point to a number on the line and ask a volunteer to round it to the nearest 10. Then write two 2-digit numbers on the board. Ask students to round each number to the nearest ten before finding the sum. Ask students to explain how they used front-end estimation to add. Invite volunteers to choose numbers, round the numbers to the nearest ten, and add numbers using front-end estimation. Encourage them to use the vocabulary words in explanations of their work.

Core Practice: Use Appropriate Tools Strategically

Explain to students that although there are a variety of math tools they can use to solve a given problem, some tools work better than others. When selecting a tool, it is important to consider the problem first, and the kind of solution the tool is likely to help you find. Read the text together and invite students to summarize why front-end estimation would not be the best tool to use if they are working with large numbers.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 393.

Extension Activity: Solve Multiplication Problems Have students look in local newspapers to find advertisements for computers or electronic devices that cost less than \$1,000. Then have students imagine buying enough quantities to furnish a school or business. Have them write and solve multiplication problems to determine total costs and then use estimates to check each other's work. Afterward, have students write an explanation of their work process.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand that operations must be performed in a specific order
- Solve problems involving the order of operations
- Use mental math to solve problems without paper and pencil

✓ Determine Student Readiness

In this lesson, students will learn to find the value of arithmetic expressions including multiple operations. Play a game with students to determine their readiness. Present problems that involve a series of operations. For example, say: *Divide 6 by 3. (2) Add 7 to the quotient. (9) Multiply the sum by 2. (18) Then subtract 3 from the product. (15)* Continue providing practice problems or offer intervention until you are confident that students are ready to move on.

Key Concept

Basic mathematical operations must be performed in a specified order to obtain the correct answer.

Concept Background: The order of operations is essential in evaluating mathematical expressions. Suggest that students discuss other situations in life where order is important. For example, students might say putting on socks before shoes and slicing bread before buttering it. Explain to students the relationship between the order of operations in real-world examples and the order of operations in evaluating mathematics expressions. Help students recognize that unless they follow the order of operations in math expressions, the results will be incorrect.

Develop Core Skills

Core Practice: Make Use of Structure

Students may not be aware of when they encounter relationships among quantities using numbers and symbols to explain or represent the relationships. For example, students may have observed that among 10 objects in a set, those objects can be represented by 5 groups of 2, or 5×2 . They can also be represented by $5 + 5$. Mathematical structures include relationships among values. Write the following set of numbers on the board and ask students to identify relationships among the numbers: 2, 6, 12. Students may say $2 \times 6 = 12$; or $12 \div 2 = 6$; or $12 \div 6 = 2$, for example.

Core Skill: Solve Real-World Arithmetic Problems

Students may be surprised by how often they use math in daily life without even realizing it. Have them brainstorm all of the ways they use math in a single day, and then think about how they figure out the answers to math problems.

For example, students may say that they calculate how much money they need to pay for lunch each day, or how much they need to save each week to purchase something they want. Ask: *Do you visualize problems in your head? Do you draw quick pictures to find answers? Do you solve problems mentally and then use objects to check your answers?*

Pre-Teach Vocabulary

Word Bench

Explain that you can take a phrase apart and put it back together. Have students tell you what each word in each vocabulary term in Tier 3 means. Then have them tell you the meaning of the individual parts combined. For example, students may say that the word *mental* refers to the mind and the word *math* refers to calculations. So, mental math means “calculating in your mind.”

Tier 2 Words:

compensation
(p. 38)

Tier 3 Words:

mental math (p. 37)
order of operations (p. 36)

Test Words:

strategy (p. 37)

DURING THE LESSON

PAGE 36

Order of Operations

Ask students to watch as you give your face a particular “expression.” Ask students to interpret the expression. Explain that other kinds of expressions have meaning, too. For example, write the expression *cool as a cucumber* on the board, and invite students to interpret its meaning. Finally, point out that mathematics is filled with expressions, or “phrases” of numbers and operations. Read the definition of the arithmetic expressions on page 36, and emphasize the term as you complete Example 1 together.

Give students an opportunity to complete Example 2 on page 37 independently. Observe students as they work to determine if intervention is necessary. You may want to provide additional practice opportunities for struggling students. Start with simple problems, such as $(4 + 3) - 2$, and then increase the complexity of the expressions as students gain understanding.

MATH LINK

Tell students to use caution when they apply the acronym PEMDAS to problem-solving situations. The acronym implies that multiplication is always performed before division, and addition is always performed before subtraction. Have students write PEMDAS in their notebooks and circle the letters MD and AS to remind them that these operations are completed in the order they appear in an expression.

Core Practice: Make use of Structure

After students have read and discussed the Math Link, ask them to explain the relationship between PEMDAS and patterns, or structures. Help students recognize that when they apply PEMDAS to solving expressions that require multiple operations, they are applying a problem-solving structure. Write the expression $12 \div 2 + (1 + 5) - (6 \times 2)$ on the board, and ask students to apply PEDMAS to evaluate it.

PAGE 37

THINK ABOUT MATH

ANSWER KEY

- 2
- 16
- 7
- 44

Mental Math Strategies

Tell students that they already apply mental math strategies in their everyday lives. Invite students to talk about how they solve mathematical problems without the use of objects or paper and pencil. Highlight the usefulness of mental math strategies, not only in test taking, but also in other areas of their lives. For example, ask students how they can use mental math when deciding how many supplies are needed for a party.

Read about the zero strategy with students. Point out how useful it is when working with numbers that end in zeros. Complete Example 3 as a class.

Core Skill: Solve Real-World Arithmetic Problems

Before reading the sidebar, ask students if they have their own mental math strategies. An example might be calculating the tax on an item they want to purchase. Perhaps they round the local tax rate to the nearest dollar to estimate the total cost of the item. Encourage students to share examples of strategies they use to solve real-world arithmetic problems.

PAGE 38

Break Apart Numbers

Guide students through the steps in Examples 4 and 5. Some students, especially those without strong number sense, may struggle with breaking apart numbers and compensation. For breaking apart numbers, it may help to reinforce place value.

Engage and Extend

ELL Instruction: Complete Sentences Write the following cloze paragraph on the board or on a chart, and have students fill in the blanks with the vocabulary words from this lesson: It is helpful to have a _____ when solving problems. _____, or substitution, is one of them. Being able to do _____ is helpful when you don't have a pencil, paper, or calculator. No matter what strategy you use to evaluate arithmetic expressions, you need to know the proper _____. Pair English language learners with students who are fluent in English.

Help students see that the numbers are being broken apart into tens and ones. This is done so that they only need to add one non-zero digit in each number at a time. Have students begin with smaller numbers, such as $12 + 13$, until they are more proficient.

For compensation, tell students that the numbers are changed to avoid regrouping, making it easier to perform problems mentally. Again, start with smaller numbers if students appear to have difficulty.

MATH LINK

If students have trouble working with parentheses within parentheses, have them use brackets to rewrite expressions. The regular parentheses show what to put in a calculator so that the calculator will apply the order of operations correctly.

MATH LINK

Read and discuss the text as a class. Ask students to paraphrase the text, allowing you to determine their understanding. If it is clear that students do not understand the strategy of compensation, review Example 5 before working through another practice problem.

PAGE 39

THINK ABOUT MATH

ANSWER KEY

- 336
- 4,500

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 393 and 394.

Extension Activity: Modify Equations

Write the following equations on the board:

$$5 + 9 \times 6 - 4 + 14 = 42$$

$$7 \times 10 - 7 \times 2 + 5 = 47$$

Have students copy the equations and insert parentheses to make them true.

$$(5 + 9) \times (6 - 4) + 14 = 42$$

$$7 \times (10 - 7) \times 2 + 5 = 47$$

Have students make, exchange, and solve similar exercises.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Use the five-step approach to solve word problems
- Use various strategies to solve word problems
- Judge the reasonableness of answers

✓ Determine Student Readiness

In this lesson, students will apply the skills they have learned in previous lessons to solve word problems. To determine their readiness, provide practice problems that ask students to apply the operations of addition, subtraction, multiplication, and division. You may want to organize students into four groups. Assign one operation to each group, and have students write simple problems that require the assigned operation. Invite students to write their problems on the board and challenge the class to solve them. Students who create the problems must be able to provide the correct answers.

Key Concept

Problem solving is an important part of the study of mathematics and an important part of everyday life.

Concept Background: Explain to students that effective problem solving is a fundamental life skill. Invite students to share experiences of solving real-world problems, like repairing a bike or finding lost keys. Then, as a class, discuss some other ways the same problems might have been solved, emphasizing that there can be multiple ways to solve a problem.

Develop Core Skills

Core Practice: Attend to Precision

Ask students to imagine being mountain climbers. They are preparing for a new adventure, which will require special supplies. Those supplies include ropes of specific lengths and devices capable of holding specific weights. Ask: *Why would mountain climbers want to be sure their measurements were precise?* Discuss with students what types of problems could arise if the climbers used imprecise measurements.

Core Skill: Build Lines of Reasoning

Explain to students that when they build a line of reasoning, they apply logic to the solution of a problem. So, for example, they might decide that two gallons of paint will be necessary to paint a room in their home. How did they come to that conclusion? What reasoning did they use? What logical procedures did they follow to reach that conclusion? Invite students to explain the relationship between building lines of reasoning in solving real-life problems and mathematical problems.

Pre-Teach Vocabulary

Connect to Life Experiences

Write the vocabulary words and definitions below on the board. Then ask students to think about personal experiences in which they found a solution to a problem. Have them describe their experiences by writing and completing the following sentences: I had a problem, but found a *solution* by...; My solution was *reasonable* because...; One factor related to the problem that turned out to be *irrelevant* was...

irrelevant: unrelated; beside the point; inapplicable

reasonable: logical; showing good judgment

solution: answer; a process for solving a problem

Tier 2 Words:

irrelevant (p. 41)

reasonable (p. 40)

Tier 3 Words:

solution (p. 40)

DURING THE LESSON

The Five-Step Approach

PAGE 40

Introduce the five-step approach as an effective strategy for finding solutions. Discuss the steps as a class. Then have students work in pairs to complete Example 1.

Identify Irrelevant Information

PAGE 41

Explain that in real-life problem-solving situations, students must often sift through a lot of information before planning how they will solve a problem. Help them understand that the information they don't use is irrelevant because it isn't useful in solving their problem.

Read the boxed text on page 41 aloud and then give students time to identify the irrelevant information in the problem. Discuss the answers as a class to determine if students recognize the irrelevance of some information.

Core Practice: Attend to Precision

Point out the word *precision*. Explain that the word comes from a Latin word *praecisionem*, which means "a cutting off." Ask a volunteer to define the term *precision* in his or her own words. Then explain that in mathematics, the term *precision* refers to the number of significant figures in a number. So the number 4172.98 has a precision of 6, while 2.19 has a precision of 3.

Also point out that in mathematics, some people refer to precision as careful attention to details, such as aligning decimals vertically. Or they may use the word in place of *correctness*, such as completing operations correctly. However, a better term for correctness is *accuracy*.

Invite students to read the text and discuss why aligning digits and using correct labels are valuable skills for successful problem solving.

Choose an Operation

While key words can be useful, encourage students to read word problems for understanding. It is more important that students actually understand the meaning of the words in the list than memorize them and try to rely on their mental list for solving problems. Go over each word or phrase in the list, having students define each one. Then guide students through the steps in Example 2.

THINK ABOUT MATH

addition; \$7

ANSWER KEY

Problem-Solving Strategies

Explain to students that drawing a picture can be an excellent way to understand a problem. Guide students through the steps in Example 3, pointing out how the diagram is labeled with information in the problem.

Guide students in the application of the Guess and Check strategy on page 44 for solving problems. Explain that this strategy is often used when a problem asks for a certain amount and the student needs to adjust conditions to get this amount. In that case, students can use a table to record guesses.

You may want to give students a problem to practice, such as: *Rosa spends \$18 on glue and paper. Bottles of glue cost \$3, and packs of paper cost \$2. Rosa buys 8 items in all. How many bottles of glue did she buy?*

Guess	Bottles of Glue	Packs of Paper	Total Cost	Number of Items
1	5	3	\$21	8
2	4	4	\$20	8
3	3	5	\$19	8
4	3	6	\$18	8

Applying place-value concepts is a strategy for solving the problem in Example 4. The greatest possible sum is not given, so there is nothing to compare the guesses to.

Engage and Extend

ELL Instruction: Work with a Partner Pair English language learners with students who are fluent in English. Have the pair work through the Skill Review together. Have them use the Five-Step Approach and identify each step as they complete it.

To get the greatest sum, students need the two greatest numbers. Since the tens place has a greater value than the ones place, students can use the two greatest digits, 4 and 5, in the tens places of the numbers. Then, they can use the next two greatest digits in the ones places: 53 and 42 (or 52 and 43)

Core Skill: Build Lines of Reasoning

Ask students to relate the strategy of drawing pictures to building lines of reasoning. Help them see that effective pictures include only the most critical information required to solve a problem. The decisions to insert specific details in their drawings are logical, and all intended for the same purpose as building lines of reasoning—finding a solution. Invite students to share experiences in which drawing a picture or any visual led to a logical solution.

THINK ABOUT MATH

1. guess and check; 4
2. draw a picture; 20 times

ANSWER KEY

21st Century Skill: Technology Connection

After reading the text as a class, share some of the websites you visit to learn more about mathematics education. Invite students to share the sites they visit to receive tutorial help or to learn more about something they've studied in class. Encourage discussion of how expertise is related to reliable websites, and signs students can look for to assure them of the quality of the material they're reading, listening to, or viewing.

MATH LINK

Explain to students that their pictures do not have to look perfect as long as they understand what the pictures represent. Encourage students to share pictures and learn what others find most valuable when using this problem-solving strategy.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 394.

Extension Activity: Use Concepts to Solve Non-Routine Problems Have students write addition, subtraction, multiplication, and division expressions with variables. Have them exchange the expressions with a partner. The partner then writes a scenario that the arithmetic expression could represent. For example, one partner writes $x + 7$ and the second partner could write *seven years greater than my age*.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand decimals as part of the place-value system
- Use strategies for rounding decimals
- Compare and order decimal numbers

✓ Determine Student Readiness

For this lesson, students need to know how to round and compare whole numbers. To determine student readiness, draw a number line on the board or on a chart. Mark the number line from 1 to 100 in multiples of ten. Choose several numbers and ask students to round the number to the nearest ten. Continue having students round whole numbers until you are confident they are ready to move on.

Key Concept

Decimals represent a part of a number. They are an extension of the place-value system.

Concept Background: Tell students that decimals, which represent part of a whole, are used in metric measurement and in our monetary system. Share grocery receipts or look at food advertisements as a class. Point out the decimals and have students read them. Ask students to share other ways decimals are used, such as in track and field events, swimming, and batting averages.

Develop Core Skills

Core Practice: Model with Mathematics

Draw a place-value chart on the board. Explain to students that charts, such as place-value charts, are useful tools for organizing and explaining information. A place-value chart, for example, organizes a number's digits by place value, making it easy to identify the digit's value. Give volunteers several whole numbers and have them use the place-value chart to show the numbers.

Core Skill: Apply Number Sense Concepts

Explain to students that the term *number sense* applies to a general understanding of numbers, the quantities they represent, and their relationships with each other. Use the number 435 as an example. Tell students that when they apply their number sense, they know that the written representation of the number tells them that there are 4 groups of 100, 3 groups of 10, and 5 ones. Together, the groups represent a quantity that is greater than 400 but less than 500. Ask: *What does your number sense tell you about 25×4 ?*

Pre-Teach Vocabulary

Define and Model

Write the definition of each vocabulary word on the board. Then to support the Core Practice of Modeling with Mathematics, have students work in pairs or small groups to draw a model to support or explain each definition. Encourage students to compare definitions and drawings and discuss any discrepancies that occur.

Tier 2 Words:

cent (p. 50)
decimal (p. 50)

Tier 3 Words:

decimal point
(p. 51)
hundredth (p. 50)
tenth (p. 50)

Test Words:

compare (p. 51)

DURING THE LESSON

Understand Decimals

PAGE 50

Read the section with students and discuss the models that help explain the text. Emphasize that if students find it helpful, they can think of decimals through hundredths in terms of money.

Invite students to discuss the models and comment on their usefulness in understanding decimals as parts of a whole.

MATH LINK

Discuss the text as a class. Then have students shade the hundredths decimal model to represent 0.25, or 25 cents.

PAGE 51

Place Value in Decimals

Ask a volunteer to read and interpret the place-value chart. Help students recognize that like a whole number place-value chart, the value of a digit in a decimal is 10 times the value of the place directly to the right, and one tenth of the value of the place directly to the left.

Tell students that when they read a decimal, they read the whole number part, add *and* for the decimal point, read the decimal part as if it was a whole number, and then say the rightmost place of the number. For example, the decimal 12.345 is read as *twelve and three hundred forty-five thousandths*. Invite students to write and read more examples of decimal numbers.

Compare Decimals

After reading the text together and walking through Example 1, ask students to explain why it is important to insert a zero at the end of a decimal to give it the same number of decimal digits as the number it is being compared to. Help students see that without adding the zero as a placeholder, it is easy to think mistakenly that because $145 > 17$, then $1.145 > 1.17$.

Core Practice: Model with Mathematics

Read the text as a class. Then help students make their own place-value charts through millionths. They can use their charts to help compare and round decimals later in the lesson.

THINK ABOUT MATH

ANSWER KEY

1. > 2. > 3. = 4. < 5. =

PAGE 52

Round Decimals

Guide students through the steps in Examples 2 and 3. Help students see that rounding decimals is like rounding whole numbers, except that any zeros should be dropped to the right of the place being rounded to. As shown in Example 2, 1.537 rounded to the nearest tenth is 1.5, not 1.500. This is more important for problems involving rounding measurements, because 1.500 indicates a much greater level of precision than 1.5.

THINK ABOUT MATH

ANSWER KEY

1. 6.1 4. 5.01
2. 3.0 5. 4.24
3. 16 6. 12.37

MATH LINK

Write the word *annex* on the board. Explain that the word comes from the Latin prefix *ad*, meaning “to,” and the Latin base word *nectere*, meaning “to tie, or to bind.” In later Latin, the word became *annexare*. The word *annex* means “to attach.”

Read the Math Link as a class. Students are accustomed to working with whole numbers that can be lined up by place value at the rightmost place. Help students see that annexing zeros at the ends of decimals to give them the same number of decimal places allows for this with decimals, too.

Core Skill: Apply Number Sense Concepts

Give students time to read the text and in pairs discuss who is right, Amelia or Rory. Have pairs apply reasoning to their decisions and then share and justify their conclusions with the class.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 396.

Engage and Extend

ELL Instruction: Practice Pronunciation Have students practice saying place-value names, such as *hundred*, *hundredth*, *thousand*, *thousandth*, and so on. Make sure they properly aspirate the *th* sound in the decimal place values. Have them practice listening for the *th* sound as well.

Extension Activity: Compare Data Remind students that many sports use statistics that involve decimals. For example, batting averages, earned run averages, and win-loss statistics use decimals. Share a print or online stats sheet for a particular sport and have students write statements comparing win-loss records for two particular seasons or batting averages for the best and worst hitters on a team.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Add decimals
- Subtract decimals

✓ Determine Student Readiness

Write addition and subtraction problems on the board and have students copy and solve them. To start, write problems that require no regrouping. When students can demonstrate mastery, write problems that require regrouping. Observe students as they work so that you can intervene if necessary.

Key Concept

Decimals are added and subtracted by using place value much as whole numbers are added and subtracted.

Concept Background: Write the following whole numbers on the board horizontally: 15,342; 367; 12; 45,098. In their notebooks, have students align the numbers vertically as they would for addition. Ask students to explain why they aligned the numbers in the way they have. Then place decimal points in the numbers to make them 153.42; 0.367; 1.2; 450.98. Ask students to re-align the numbers and explain their thinking.

Develop Core Skills

Core Skill: Perform Operations

Ask a volunteer to identify the four basic math operations (addition, subtraction, multiplication, and division). Explain that just as they have applied these operations to whole numbers, they can also apply them to decimals.

Core Practice: Attend to Precision

Remind students of the meaning of *precision*, a word they learned in an earlier lesson. Write the number 12.7 on the board. Ask: *What is the precision, or number of significant digits, in this value?* (3) *What word do people sometimes use as a synonym for precision?* (accuracy) *What is the difference between the two terms in mathematics?* (Accuracy describes how closely a number comes to a true value. Precision refers to the number of significant digits in a number.) Say: *You buy 16 ounces of sugar. You use two scales to measure the sugar's weight. One scale reads 15.5 ounces. The answer is accurate because it is close to the true value, or weight. An electronic scale reads 17.428 ounces. The second measure is more precise because it gives more significant digits. However, it isn't as accurate as the first measurement because it's farther from the true value. The scale may need to be adjusted, or recalibrated.*

Also remind students that the words *accuracy* and *precision* are often used as synonyms in other areas of study, so they need to consider the context in which the words are used.

Pre-Teach Vocabulary

Apply Prior Knowledge

Read the vocabulary words aloud. Ask students to identify the words that they have heard or read before, or used in conversation. Ask students to identify familiar words and to recall the context in which they were used.

Tier 2 Words:

align (p. 55)
annexed (p. 55)

Tier 3 Words:

place value (p. 54)
vertically (p. 55)

Test Words:

organize (p. 55)
topic (p. 55)

DURING THE LESSON

PAGE 54

Add Decimals

Guide students through the steps in Example 1. Encourage students to use the inverse operation of subtraction to check their answers.

As students read Example 2, point out how the numbers are aligned by place value. If some students appear to have difficulty aligning numbers, encourage them to draw pictures or use math manipulatives to solve the problem.

Give students an opportunity to use a calculator to add the decimals in Example 3 on page 56.

Core Skill: Perform Operations

Have students draw pictures or use some other visual to determine whether Evariste is correct. Ask students to comment on Evariste's thinking, and suggest reasons why Evariste might have come to his incorrect conclusion.

PAGE 55

Identify Topic Sentences

Explain to students that the word *topic* means "subject." When they look for a topic sentence in a paragraph, they are looking for a sentence that identifies the paragraph's subject. Have students cover the information below the sample paragraph with one hand as they read the paragraph and identify the topic sentence. Then invite them to remove their hands and check their thinking. Discuss the results as a class.

Core Practice: Attend to Precision

Write the following on the board: 3.5 dollars and 2.05 dollars. Tell students to imagine that they earned this money doing chores. How much did they earn in all? Have students align the numbers to the right and add to find a sum. Then have them align the numbers by the decimal point and find the sum. Ask which sum they would rather receive. (Aligned to the right, the sum is 2.4 dollars. Aligned by place value, the sum is 5.90 dollars.) Explain to students that there is only one correct sum, and that must be found by adding the digits in each place.

THINK ABOUT MATH

ANSWER KEY

1. \$0.60 2. 7.9 3. 2.005

Subtract Decimals

Guide students through the steps in Example 4. Be sure students understand the importance of alignment.

Work through Example 5 with students. Then complete Example 6 together, reminding students of the importance of alignment. For students who have difficulty aligning numbers correctly, encourage them to write the numbers inside place-value charts.

21st Century Skill: Productivity and Accountability

Have students read the text and then ask them why schedules are helpful in accomplishing specific personal goals. If students have difficulty giving personal examples, ask them to think of people they depend on who use schedules to organize their workday. A dentist, for example, schedules appointments, assigning more time for patients who require more complicated procedures. A sports coach schedules games for a season. A teacher schedules steps in a big project so students can be successful. Then return to the subject of personal schedules, and discuss their value.

MATH LINK

Use this tip if your students' calculators show the solution as a fraction. Demonstrate for students that they can also divide the numerator by the denominator to change a fraction to an equivalent decimal.

Engage and Extend

ELL Instruction: Use Real-World Examples Bring in a grocery receipt listing multiple items. Cut or tear off the total value. Discuss the items on the list and their costs with students. Then show them that part of the receipt is missing and ask them to help you find the total cost. Ask: *What process could I use to find the total cost?* Give students an opportunity to respond, and then give volunteers an opportunity to walk you and other students through the process.

Add and Subtract Decimals Summary

Have students read the summary. Then ask them to write a topic sentence to precede the sentences in the paragraph. Help students recognize that the subject of this paragraph is the steps they should remember when adding and subtracting decimals.

THINK ABOUT MATH

ANSWER KEY

1. 3.3 2. 8.53 3. 1.985

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 397.

Extension Activity: Modify Decimals On the board, write several three-digit numbers. Have students choose three numbers and place a decimal point in each. Tell them that no number may be a whole number, so each number must show a decimal amount. Have students place each decimal point to create a combination whose sum is closest to, but not greater than 100. Ask students to explain their work.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Multiply decimals

✓ Determine Student Readiness

Have students multiply whole numbers in the hundreds. Observe students as they align numbers, find and write partial products, and regroup. Intervene, if necessary. Then have students add and subtract decimals through the hundredths to determine whether students understand how to align decimals before completing an operation.

Key Concept

Multiplying decimals is a process that is similar to multiplying whole numbers.

Concept Background: Remind students that when they multiply whole numbers, they are finding a total amount. Then explain that they are also finding total amounts when they multiply decimals. Project a 100 grid on the board (see the example in lesson 2.1). Write the problem 0.2×0.3 and use the grid to model the product. Project two new grids, and work with students to find the product of 0.2×6 . Then show students how to find the product of 0.4×0.4 . Use as many grids as you think your students need to understand what they will be doing when they multiply decimals.

Develop Core Skills

Core Skill: Apply Number Sense Concepts

Ask a volunteer to explain what the term *number sense* means (an understanding of numbers, their values, and their relationships). Then ask students to explain the relationship between multiplication and repeated addition, and why using multiplication is often easier and more efficient than using repeated addition. If necessary, share an example with students such as $4 + 4 + 4 + 4 + 4 = 5 \times 4 = 20$.

Core Skill: Represent Real-World Problems

Explain to students that decimals serve a purpose in many real-world problems. Write the following locations on the board: Grocery Store; Gas Station; Sports Track; and Swim Meet. Ask students to describe the link between these locations and decimals.

Pre-Teach Vocabulary

Use Prior Knowledge

Review the vocabulary words with students. Point out that they read and used these words in earlier lessons. Ask students to rely on prior knowledge to help them define each term. Encourage students to come to some agreement about the meaning of each word, and write the definitions on the board.

Tier 2 Words:
product (p. 60)

Tier 3 Words:
factor (p. 61)
multiplication (p. 61)

DURING THE LESSON

Multiply Decimals

PAGE 60

Guide the students through the steps of Example 1. Some students find it helpful to write a little number to the right side of each factor, indicating how many decimal places each has. They can then find the sum and write that number to the right of the product, so they know how many decimal places there should be.

When students learned to multiply whole numbers, they placed a zero to the right of the second partial product as a placeholder. Students may leave that space blank, as shown, or write a zero there to make sure the numbers stay in the correct places. In Step 4, the numbers are rounded to the nearest whole number to make the mental math easy.

Some students may benefit from using math manipulatives or drawing pictures to model the problem. This will help them make a connection between the algorithm and what it means to multiply two decimals.

For Example 2, explain that sometimes there are fewer digits in the product than the necessary number of decimal places. Make sure students place any needed zeros to the left of the product, not the right.

Give students an opportunity to use calculators to find the product of decimal amounts in Example 3.

Guide students through Example 4. You may want to remind students that they can use zero as a placeholder when they find partial products during multiplication.

MATH LINK

Helps students apply what they learned in the example to calculating other monetary units. Again, you may want to encourage students to use a zero as a placeholder when they find partial products.

THINK ABOUT MATH

ANSWER KEY

- | | |
|-----------|-----------|
| 1. \$2.30 | 5. 0.15 |
| 2. \$10 | 6. 20.4 |
| 3. \$3.75 | 7. 0.0325 |
| 4. \$3.50 | 8. 2.294 |

Core Skill: Apply Number Sense Concepts

Have students read the section about Nina calculating the math score by using both repeated addition and multiplication. Ask students to verify Nina's calculations. Then challenge students to work in pairs to find a way to use repeated addition to find the product of another two decimals. Observe students as they work, prompting their thinking, if necessary.

Multiply Decimals Summary

Have students review the steps in multiplying decimals. As a class, discuss a suitable topic sentence for this paragraph. Write the topic sentence on the board and ask students to determine how well it summarizes the paragraph's main idea or subject.

MATH LINK

Explain to students that although 0.025 and .025 represent the same thing, it is generally considered good practice to keep the leading zero. This decreases the likelihood of a decimal point getting missed or lost.

Core Skill: Represent Real-World Problems

Give students time to read the text. Have them explain why decimals are important in real-world situations. Have students draw a picture to model the problem before solving it. Explain to students that area describes the space that something covers, and to find the area of a rectangle, they can multiply the length by its width.

Engage and Extend

ELL Instruction: Explain Relationships Write the vocabulary words *factor*, *multiplication*, and *product* on the board. Have students say the words aloud and then explain the relationships among them. Encourage students to use numbers to support their explanations.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 398.

Extension Activity: Show the Relationship Between Problems Organize students into pairs. Have each pair write eight different problems in which a decimal is multiplied by a decimal. Then have them select pairs of problems to write product comparisons, leaving a blank for the $<$, $>$, or $=$ sign. For example: 0.2×0.8 _____ 0.3×0.8 . Have pairs exchange problems with another pair and use the correct sign to make each statement true. Discuss the results as a class.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Divide decimals

✓ Determine Student Readiness

Give students time to find quotients for a few long-division problems, including those with remainders. Start with simple problems, such as two digits divided by one digit, and then make the problems increasingly complex to include two-digit divisors. Observe students as they work to determine if intervention is required.

Key Concept

Dividing decimals is similar to dividing whole numbers. The key difference is the placement of the decimal point in the quotient, or answer.

Concept Background: Remind students that when they divide whole numbers, they are making equal groups. Explain that when they divide decimals, they are also making equal groups. Write the problem $16 \div 2$ on the board. Ask students to explain how to get the answer. Tell them that this same process will be used to divide decimals.

Develop Core Skills

Core Skill: Apply Number Sense Concepts

Remind students that when they apply understandings of operations such as division, they are applying number sense. Ask: *What does the operation of division do?* (separates a quantity into equal groups) *How might the operation change if you work on decimals instead of whole numbers?* (It doesn't.) *What do you expect to do when you divide decimals?* (separate decimals into equal parts) Explain to students that their number sense tells them that division is division, whether they are working with whole numbers or parts of whole numbers.

Core Skill: Evaluate Reasoning

Explain to students that when they evaluate a procedure or an answer, they judge the methods that are used to implement the procedure or come up with an answer. This helps to ensure that all of the necessary details in the steps toward a solution are included. Remind students to evaluate, or check the details of their work, to determine if the solutions they get are reasonable, meaning they can be verified, or proven to be true.

Pre-Teach Vocabulary

Display Prior Knowledge

Read the vocabulary terms aloud. Ask students to share their prior knowledge of these words by building a word map together. Write the lesson title *Divide Decimals* in a large circle on the board. Invite students to help you write the vocabulary words around the central circle and build connections among them. Ask students to explain the connections they make.

Tier 2 Words:

dividend (p. 64)

Tier 3 Words:

divisor (p. 64)

quotient (p. 64)

Test Words:

evaluate (p. 65)

reasoning (p. 65)

summarize (p. 65)

DURING THE LESSON

PAGE 64

Divide Decimals

Guide students through the steps of Examples 1 and 2. Explain to students that the divisor must be modified to be a whole number. If it is a decimal, they need to move the decimal point all the way to the right and count the number of places it moved. Then the decimal point in the dividend must move the same number of spaces. This keeps the divisor and dividend equivalent to the divisor and dividend in the original problem. Finally, tell students that wherever the decimal point is in the dividend, it should be directly above that place in the quotient.

Guide students through the evaluation of the problem in Example 3.

Guide students through Example 4 on page 66. Allow students who struggle with the algorithm to use play money to model problem solving.

Have students use calculators to work through Example 5. Then organize students into pairs. Have each student in each pair write and solve three division problems without the help of a calculator. Then have students within each pair exchange their problems and use calculators to check the work. Have students work together to resolve any errors they find.

Core Skill: Apply Number Sense Concepts

Explain to students that when they move a decimal point the same number of spaces in both the divisor and dividend, the resulting division problem is equivalent to the original problem, and this is why the answer is correct. Not moving the decimal point the same number of places in both the divisor and dividend results in a problem that is not equivalent to the original, and the quotient will be incorrect.

Remind students of their knowledge of place value. Tell them that moving a decimal point in a number one place to the right is the same as multiplying the number by 10. Moving it two places to the right is the same as multiplying by 100, and so on.

Tell students to solve the problem $\$27.36 \div 3$ and check their answers for reasonableness.

Summarize Ideas

PAGE 65

Ask students to name a movie or TV show they've seen recently. Write the title that the most students give on the board. Ask students to state the show's main idea in one sentence. Then when they agree upon that, ask them to choose a few of the most important details to support the main idea. Encourage discussion about what details are truly the most important. Write students' responses on the board, and read the finished work aloud. Explain that by identifying a main idea and supplying a few key details in their own words, they successfully summarized the movie or show. Then have students work in pairs or small groups to read the sample paragraph on page 65 and summarize it. Encourage students to compare summaries.

Core Skill: Evaluate Reasoning

Remind students that including details in their work makes it easier to evaluate, or judge, how reasonable the solution process and answer are. Have students examine the solution process in the sidebar and record an error in reasoning. Then discuss their thoughts as a class, reaching a consensus about the error and the value of checking the answer to determine its reasonableness.

Engage and Extend

ELL Instruction: Retell Problems and Solutions Invite students to select a particular example from the lesson and explain the problem and steps of the solution in their own words. Give all students an opportunity to explain an example. Encourage the use of the words *dividend*, *divisor*, and *quotient* in the retellings. Provide help if students are having difficulty with words representing math concepts.

PAGE 66

21st Century Skill: Communication and Collaboration

Read the text and ask volunteers to describe an experience in which they worked with others to brainstorm the solution to a problem. Help students see that in this case, you are using the term *solution* to mean "the process for finding an answer." Give students an opportunity to think quietly about how they might solve this problem in a way unlike the one used in Example 4. Then bring students together to brainstorm. Record their ideas on the board. Let students choose one or two solutions.

PAGE 67

Divide Decimals Summary

Have students read the summary and write a topic sentence for the paragraph. Encourage students to share their topic sentences.

THINK ABOUT MATH

ANSWER KEY

- | | |
|---------|-----------|
| 1. 0.09 | 5. 4.8 |
| 2. 30 | 6. \$6 |
| 3. 60 | 7. 23 |
| 4. 0.9 | 8. \$0.80 |

MATH LINK

Ask students to determine if the keys in the Math Link are also on their calculators. If they are, read the text together. If they are not, point to the x^2 button shown in the text and explain that it refers to exponents, a topic they will study later. Meanwhile, they can use any calculator to divide decimals. Ask students to use their calculators to check the answers they found in the Think about Math activity.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 398 and 399.

Extension Activity: Identify Patterns Write the following problems on the board. Challenge students to identify a pattern as they solve the problems:

$$56 \div 7 \quad 56 \div 0.7 \quad 56 \div 0.07$$

Ask students to reveal the pattern they observed. Help them see that as the divisor decreased by a power of 10, the quotient increased by a power of 10. Then have students use the pattern to write and solve the next three problems in the sequence: $56 \div 0.007$; $56 \div 0.0007$; and $56 \div 0.00007$.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand fractions
- Name equivalent fractions
- Compare and order fractions

✓ Determine Student Readiness

Draw or project three blank hundred grids on the board. Call out a decimal, such as 1.5, and ask a volunteer to shade the grids to represent the decimal. Ask another volunteer to explain the relationship between the decimal and the shaded area. Call out additional decimals, giving more students an opportunity to respond. Use students' responses to determine if intervention is required.

Key Concept

Represent, compare, and order fractions to understand and develop the meaning and value of fractions.

Concept Background: Hold up a piece of paper for students to see. Fold the paper in half and open it. Ask students to identify how much of the whole each part represents. Then refold the paper and fold it again to make fourths and then eighths. Open the paper and ask students to count how many parts make up the whole. Then shade or use a marker to mark a given number of fractional parts and ask students how many parts of the whole are shaded. Tell students that when they identify parts of a whole, they are naming fractions.

Develop Core Skills

Core Skill: Interpret Data Displays

Explain to students that data are specific pieces of information that can be represented in words, numbers, and even pictures. Ask students turn to page 12 in their books. Ask them to identify the data display on this page (place value chart). Then ask students to interpret the place-value chart. For example, ask: *How many hundred millions are in the number 9,876,543,210?* Next, have students turn to page 15 in their books. Point out the number lines on the page. Explain that these numbers lines are also examples of data displays. Ask questions related to the numbers lines, giving students opportunities to interpret what they see. For example, ask: *Is 45 less than or greater than 40?*

Core Skill: Perform Operations

Ask students to describe examples of operations. Students may mention surgical, military, or local disaster preparedness operations, for example. Help students understand that an operation is a process, and that its meaning applies to mathematics as well. Write the symbols $+$, $-$, \times , and \div on the board. Ask students to interpret the operations these symbols represent.

Pre-Teach Vocabulary

Word Bench

Explain that you can take a word apart and put it back together. Have students identify the vocabulary words they know or words that are similar to the words in the vocabulary list. They may know, for example, that a fraction is part of a whole. Or they may say that the word *equivalent* looks similar to the word *equal*, and so may have similar meanings.

Tier 2 Words:

represent (p. 75)

Tier 3 Words:

common multiple

(p. 79)

denominator (p. 74)

equivalent fractions

(p. 76)

fraction (p. 74)

lowest terms (p. 76)

numerator (p. 74)

Test Words:

diagram (p. 75)

DURING THE LESSON

Understand Fractions

PAGE 74

Guide students through the steps of Example 1, which continue to page 75. Explain to students that a fraction represents a part of a whole, and that it has two parts. Point to the diagram of $\frac{3}{4}$. Invite a volunteer to explain the diagram, its labels, and what it tells him or her about what a fraction is. Then ask another volunteer to explain the relationship between the circle shaded to show $\frac{3}{4}$ and the rectangle shaded to show $\frac{7}{10}$.

Recognize Details

PAGE 75

Show students a picture from a magazine or website that they will find interesting, such as a photograph of a sporting event or a close-up of an exotic plant or animal. Write the word *detail* on the board and ask students to define it. Then have students share details, or specific pieces of information, about the picture, such as "the ball is in the air" or "the wing is red." Explain that texts also include details and that the purpose of details is to describe, quantify, or in some other way, support a main idea. Read the text preceding the passage as a class. Then have students read the passage and record details from the passage before checking their work.

Core Skill: Interpret Data Displays

Guide students through the explanation of diagrams, asking them to explain the varied purposes a diagram can have. Then have them draw a diagram to represent the fraction $\frac{2}{5}$ and explain how the diagram fulfills its purpose.

MATH LINK

Read the Math Link as a class. Invite volunteers to draw a number of shapes and explain whether it would be possible to divide the shapes into fractional parts.

PAGE 76

THINK ABOUT MATH**ANSWER KEY**

- $\frac{5}{8}$
- $\frac{1}{8}$
- $\frac{3}{10}$
- $\frac{11}{12}$

Name Equivalent Fractions

Help students understand that equivalent fractions represent equivalent amounts of a whole. Use the diagram to explain equivalence. Then relate the concept of equivalence to the concept of lowest terms. Use the diagram to list the equivalent fractions and then identify the fraction written in lowest terms. Guide students through Example 2. If students appear to struggle, encourage them to use diagrams or fraction bars to solve each problem.

Continue guiding students through Examples 3 and 4. If you observe students having difficulty, distribute grid paper. Have students draw models of the problems to confirm answers. When you are confident that students understand the concept of equivalence and lowest terms, have students use a calculator to work through Example 5.

21st Century Skill: Collaborate with Others

Read the text and ask volunteers to describe successful experiences in which they collaborated with other students to complete a process or solve a problem. Ask students to consider the advantages and disadvantages of both collaboration and working independently. Record students' responses to promote discussion.

PAGE 77

MATH LINK

Read the Math Link as a class. Write the fraction $\frac{50}{100}$ on the board and ask students to find the greatest number by which both the numerator and denominator are evenly divisible. Ask students to explain the advantage of dividing by the largest possible number to rewrite a fraction in its lowest terms.

Engage and Extend

ELL Instruction: Retell Have students explain the diagrams on page 76 again, using the term *equivalent fraction* in their explanations. Then have them use the diagrams to identify $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{12}{16}$ in their *lowest terms*. Ask them to explain the relationship between the two terms in their own words. If students struggle, invite them to draw pictures to show the relationship and explain their drawings.

PAGE 78

THINK ABOUT MATH**ANSWER KEY**

- C
- A
- E
- D
- B

Compare and Order Fractions

Read the explanation as a class. Then guide students through Example 6. Use the number line to have students locate other pairs of fractions on the line and write them as inequalities.

Work through Examples 7 and 8 as a class. Point out the term *common multiple*, and ask students to use the term in their explanations of the steps.

MATH LINK

Read the Math Link as a class. Then invite volunteers to come to the board to write inequalities without their symbols. Have other volunteers come to the board to insert the appropriate symbols and explain their choices.

PAGE 79

THINK ABOUT MATH**ANSWER KEY**

- Draw a number line from 0 to 1 and divide it into fourths and eighths. Locate each fraction on the line. The fraction farther to the right is the greater fraction.
- After finding a common multiple to rewrite the fractions with a common denominator, compare the numerators. The fraction with the smaller numerator is less than the fraction with the greater numerator.
- $\frac{5}{6}$, $\frac{7}{9}$, $\frac{2}{3}$

Core Skill: Perform Operations

Discuss the explanation of equivalent fractions as a class. Then give students an opportunity to solve the problem. Afterward, ask volunteers to show and explain their work.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 400 and 401.

Extension Activity: Make Observations about Fractions

Write the fraction $\frac{8}{24}$ on the board. Ask students to use the fraction to explain the process of using common multiples to rewrite the fraction in its lowest terms. Challenge students to support their explanation with one or more diagrams.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Add and subtract fractions with like denominators
- Add and subtract fractions with unlike denominators

✓ Determine Student Readiness

Write the fraction $\frac{4}{5}$ on the board and have students write a set of equivalent fractions (e.g., $\frac{8}{10}$, $\frac{12}{15}$, $\frac{16}{20}$). Then write the fraction $\frac{9}{12}$ on the board and ask students to rewrite the fraction in its lowest terms ($\frac{3}{4}$). Ask students to explain their thinking as they work. Use students' responses to determine if intervention is required.

Key Concept

Understand and apply strategies for finding the sums and differences of fractions that have like or unlike denominators.

Concept Background: Draw or project an image of a hundred grid. Shade and label $\frac{3}{10}$ of the grid. Then tell students you want to add $\frac{5}{10}$. Shade $\frac{5}{10}$. Ask students to identify how much of the grid is shaded ($\frac{8}{10}$). Explain that when they identified the total shaded area, they added like fractions. Now, tell them you want to remove $\frac{2}{10}$ of the grid. Ask students how much of the grid will be left shaded ($\frac{6}{10}$). Ask students to explain what operation they completed in order to answer your question.

Develop Core Skills

Core Skill: Perform Operations

Direct students' attention to the hundred grid you used when you introduced the lesson's key concept. Ask students to identify the operations they used in the activity. Help students understand that addition and subtraction are processes, or operations. Invite students to identify other operations they can use to work with numbers (multiplication, division).

Core Skill: Apply Number Sense

Explain to students that number sense is an understanding of numbers, what they represent, and how they relate to each other. Someone with good number sense can predict reasonable answers to problems, based on their understandings of number and operational relationships. Ask students what they know about fractions, and how this understanding contributes to their general number sense.

Pre-Teach Vocabulary

Semantic Map

Give students the definition of *denominator*. Have students also write what they think *like*, *unlike*, and *common* mean. Use these definitions to help students understand the terms *common denominator*, *like denominators*, and *unlike denominators*.

Tier 2 Words:

simplify (p. 82)

Tier 3 Words:

common denominator (p. 83)

like denominator (p. 82)

unlike denominator (p. 82)

DURING THE LESSON

PAGE 82

Add and Subtract Fractions with Like Denominators

Guide students through the steps of Example 1. Afterward, have students review the steps and draw pictures in their notebooks to help explain each step. Observe students as they work, offering additional guidance when necessary.

Give students an opportunity to work through Example 2 independently, and observe students as they work. If students need additional help, ask pairs of students to draw pictures in their notebooks to explain each step. Finally, read the paragraph below Example 2 aloud. Ask a volunteer to identify the topic sentence, or main idea, of the paragraph. Then have students number the steps in their notebooks.

MATH LINK

After completing Example 2, have students look at the diagram in the Math Link. Have them read through the example again and explain how the diagram can help clarify the steps in the example.

PAGE 83

THINK ABOUT MATH

ANSWER KEY

1. $\frac{8}{12}$, $\frac{2}{3}$
2. $\frac{26}{36}$, or $\frac{13}{18}$
3. $\frac{6}{18}$, or $\frac{1}{3}$
4. $\frac{18}{20}$, or $\frac{9}{10}$

Add and Subtract Fractions with Unlike Denominators

Walk through Example 3 on page 84 with students. Some students may need to use fraction blocks or draw pictures to make meaning of the text. If students appear to struggle, pair students with different competency levels and have them work through the problems together.

When you are confident that students can add fractions with unlike denominators, guide them through the steps in Example 4. Once again, students may need to use fraction blocks or draw pictures to support their learning. Students may need to be paired with more proficient learners to practice subtracting fractions with unlike denominators.

Have students use a calculator to work through Example 5. You may want to ask students to add or subtract additional problems to be confident that they understand the procedure.

Then allow students to work through Example 6 independently. Observe their work, and intervene when students appear to need help. If students appear to struggle, ask volunteers to write problems on the board. Then solve the problems as a class, or allow students to work in pairs to complete the task. Continue to observe students as they work to respond to their needs.

MATH LINK

Read the Math Link at the top of page 83 as a class. Ask students to explain how they can determine whether an answer should be simplified. Write a fraction such as $\frac{4}{6}$ on the board. Ask students to apply what they described in their explanations to simplify the fraction.

MATH LINK

Read the Math Link at the bottom of page 83 as a class. Write the problem $\frac{1}{2} + \frac{2}{4}$ on the board. Ask students to apply what they read in the Math Link to solve the problem and write it in lowest terms. ($2 \times 4 = 8$;
 $\frac{1}{2} = \frac{4}{8}$, $\frac{2}{4} = \frac{4}{8}$, $\frac{4}{8} + \frac{4}{8} = \frac{8}{8} = \frac{1}{1}$ or 1)

Engage and Extend

ELL Instruction: Pair and Explain Tell students to turn to page 84. Have students work in pairs. Have one student in each pair explain the procedure for adding fractions with unlike denominators. Then have the other student explain the procedure for subtracting fractions with unlike denominators. Have them support each other if a partner is having difficulty with explanations.

Core Skill: Perform Operations

PAGE 84

Have students read the text. Then invite them to share experiences in which they were first learning to add and subtract whole numbers. Ask: *Did you draw pictures to help you understand the concept, or idea, of addition/subtraction? How did the pictures help? When did you first realize that you could perform addition/subtraction without the help of pictures? What do you think will happen if you use pictures to help you learn to add and subtract fractions, especially fractions with unlike denominators?* Guide the discussion to help students recognize that once they understand a concept, they are able to perform operations easily.

PAGE 85

THINK ABOUT MATH

ANSWER KEY

1. 8, 3, 24, $\frac{17}{24}$

2. 6, 3, 6, $\frac{1}{2}$

Core Skill: Apply Number Sense

Ask students to read the first paragraph. Then ask them how recognizing the falsity of Michael's conclusion is related to an understanding of what 100% really means. Invite students to talk about how they apply their understanding of numbers and what they represent to checking their work for reasonableness.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 401.

Extension Activity: Apply a Plan to Solve a Multi-step Problem

Write the following problems on the board: $\frac{3}{4} + \frac{2}{7}$ and $\frac{6}{8} - \frac{2}{3}$. Have students write a plan that can be applied to solve both problems. Then have pairs of students exchange their plans to determine how it can be improved to make it easier for someone else to follow.

Multiply and Divide Fractions

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Multiply fractions
- Divide fractions

✓ Determine Student Readiness

Write the fractions $\frac{8}{10}$, $\frac{12}{20}$, and $\frac{6}{24}$ on the board. Ask students to simplify each fraction, or write it in its lowest terms. Walk among students to observe their work, intervening whenever necessary. Students will need to be able to simplify the products and quotients they find in this lesson.

Key Concept

Extend and develop ideas about multiplication and division to include multiplying and dividing fractions.

Concept Background: Write the problem $\frac{1}{2} \times \frac{1}{2}$ on the board. Circle the operation symbol and read the problem as $\frac{1}{2}$ of $\frac{1}{2}$. Then distribute grid paper. Have students trace two 4 by 4 squares. Then tell them to shade $\frac{1}{2}$ of the first grid horizontally, and shade $\frac{1}{2}$ of the second grid vertically. Have students cut out the second rectangle, place it upon the first, and determine what fraction of the rectangle $\frac{1}{2}$ of $\frac{1}{2}$ represents ($\frac{1}{4}$). Then write the product next to the problem on the board: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Develop Core Skills

Core Skill: Apply Number Sense

Remind students that number sense is an understanding of numbers, what they represent, and the kinds of relationships they have with each other. Ask students to explain the value of using objects and diagrams to explain fractions. Invite students to offer examples.

Core Skill: Perform Operations

Explain to students that the operations they apply to whole numbers can also be applied to fractions. Invite volunteers to explain the processes of multiplication and division of whole numbers, and have them predict how these processes will apply to fractions.

Pre-Teach Vocabulary

Predict Meanings

Write the Latin word part *in-* on the board and write “in, on” above it. Then write the Latin word *vertere* on the board, and write “to turn” above it. Ask students to put the meanings together to predict the definition of the Latin word *invertere* (to turn about, or to turn upside down). Next, write the Latin word *reciprocus* on the board, and write “alternating” above it. Ask students to apply this information to predict and write definitions for *invert* and *reciprocal*. Encourage students to share their thinking. At the end of the lesson, have students revise their definitions, if necessary.

Tier 2 Words:

invert (p. 89)
reciprocal (p. 89)

Tier 3 Words:

multiplicative
inverse (p. 90)

DURING THE LESSON

PAGE 88

Multiply Fractions

Guide students through the steps of Example 1. Then write additional multiplication problems on the board, or ask students to write them. Have students work in pairs to find the products and write the products in lowest terms. Observe students as they work to determine if intervention is required. For students who struggle, encourage them to use models such as decimal grids to multiply simple fractions, such as $\frac{1}{2} \times \frac{1}{4}$ and $\frac{1}{3} \times \frac{1}{3}$.

When students can demonstrate proficiency in multiplying fractions, have them complete Example 2 using a calculator.

MATH LINK

Read the Math Link as a class. Invite volunteers to write whole numbers in fractional form and convert fractions with 1 as the denominator to whole number form.

THINK ABOUT MATH

ANSWER KEY

1. C
2. D
3. A
4. B

Divide Fractions

Draw a circle on the board. Invite a volunteer to divide the circle into halves and describe the number of halves that are in the circle. Then draw three new circles on the board. Invite another volunteer to divide the three circles into halves and describe the total number of halves that are in three wholes. Next, write the problem $3 \div \frac{1}{2}$ on the board. Give students a moment to find the quotient, and then ask students to share their answers. Discuss and resolve any discrepancies.

Guide students through the steps of Example 3, and when you are confident that they understand the process, complete Example 4 as a class.

Core Skill: Apply Number Sense

Read the text with students and then give them time to find the answer before discussing the problem as a class. Encourage students to copy and use the diagram to help them find the answer. You may want to give students a sum that Juan might earn in one day of snow shoveling and have students calculate how much Juan and each of his employees earned that day.

MATH LINK

Read the Math Link as a class. Invite students to explain the relationship between the diagrams and the multiplication process.

THINK ABOUT MATH

ANSWER KEY

1. $9 \div \frac{1}{3}; 9 \times \frac{3}{1} = 27$; There are 27 thirds in 9
2. 18

MATH LINK

Read the Math Link as a class. Invite students to return to the Pre-Teach Vocabulary activity to explain the relationship between the words *reciprocal* and *multiplicative inverse*.

Core Skill: Perform Operations

Read the text as a class. Help students understand that the steps they follow to divide and multiply fractions are algorithms, or processes that they can follow to find the answer to a multiplication or division problem. Give students time to write the algorithm for multiplying two fractions. Discuss their algorithms as a class.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 402.

Engage and Extend

ELL Instruction: Elaborate Using the Vocabulary

Words Write the words *invert*, *reciprocal*, and *multiplicative inverse* on the board. Review the definitions listed in the lesson. Explain to students that inverting a fraction creates its multiplicative inverse, or reciprocal. Have students practice inverting fractions. Ask them to explain the steps of the process using the vocabulary words.

Extension Activity: Sketch Math Models Challenge students to create visual models to demonstrate the process of multiplying and dividing fractions. Some students may want to use paper or transparent plastic sheets to make models. Others might prefer to use digital drawing tools, or to use a free online animation tool to demonstrate the processes.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Add and subtract mixed numbers
- Multiply and divide mixed numbers

✓ Determine Student Readiness

Invite volunteers to write, solve, and explain the processes involved in adding, subtracting, multiplying, and dividing fractions. Continue the activity until students demonstrate proficiency.

Key Concept

Understand mixed numbers and perform the basic operations of addition, subtraction, multiplication, and division with mixed numbers.

Concept Background: Use students' familiarity with adding and subtracting whole numbers and adding and subtracting fractions to introduce adding and subtracting mixed numbers. Have students predict how adding and subtracting mixed numbers will be similar to what they already know. Have students practice changing mixed numbers to improper fractions in preparation for multiplying and dividing mixed numbers.

Develop Core Skills

Core Skill: Represent Real-World Problems

Ask students to visualize themselves in a grocery or hardware store. Ask: *What can you buy in the store that has a per-pound cost? How often does your purchase weigh a whole number amount?* Explain that when students buy something by the pound, they may purchase whole amounts and parts of a whole, such as $2\frac{1}{2}$ pounds of bananas. Write the term *mixed number* on the board and ask students to relate the name to the value $2\frac{1}{2}$.

Reading Skill: Evaluate Arguments

Ask students to share an advertisement they have seen or heard in which a celebrity endorses a product. Ask: *Why do you think manufacturers use celebrities to sell products? What questions should you ask before accepting a claim or argument put forth by a celebrity?*

Pre-Teach Vocabulary

Use Word Parts to Define

List the prefixes *re-* and *im-* and the base words *ducere* and *proprius* on the board. Next to *re-*, write: Latin meaning "again, back, against." Next to *im-*, write: a variant of the Latin *in-*, meaning "not," used before a word that begins with *b*, *m*, or *p*. Next to *ducere*, write: Latin meaning "to bring, or lead." Next to *proprius*, write: Latin meaning "particular to itself."

Discuss the word parts and their meanings. Then as a class, define the words *reduce*, *rename*, *proper fraction*, and *improper fraction*. At the end of the lesson, have students revise their definitions, if necessary.

Tier 2 Words:

reduce (p. 92)
rename (p. 92)

Tier 3 Words:

improper fraction (p. 92)
mixed number (p. 92)
proper fraction (p. 92)

Test Words:

detail (p. 93)

DURING THE LESSON

Add and Subtract Mixed Numbers

PAGE 92

Guide students through the steps of Example 1. Engage students in a discussion of the steps in the solution process. Some students may need help writing equivalent fractions before adding mixed numbers. They may also need help simplifying answers. You may want to have these students draw diagrams as they explain the steps of writing equivalent fractions and simplifying.

Guide students through Example 2 on page 94. Pause occasionally to ask students to retell the steps in their own words or to explain the purpose of each step. Afterward, invite a volunteer to use the inverse operation of addition to check the answer.

MATH LINK

Read the Math Link as a class. Then write the improper fractions $\frac{6}{3}$ and $\frac{7}{3}$ on the board. Invite volunteers to follow the steps in the Math Link to rewrite each fraction as a whole number or mixed number.

PAGE 93

Summarize Supporting Details

Ask: *When you read a paragraph, where is the paragraph's main idea frequently stated? (in the first or last sentence) What do we call this sentence? (topic sentence) When you identify the topic sentence, or main idea, what is the purpose of the remaining sentences in the paragraph? (to support or explain the main idea) What do we call these sentences? (details) Have students read the text and the passage. Afterward, work with students to select the most important details to summarize the passage.*

Core Skill: Represent Real-World Problems

Read the text with students and then give them time to answer the question before discussing the problem as a class. Encourage students to use diagrams as a problem-solving strategy.

Multiply and Divide Mixed Numbers

Explain to students that multiplying and dividing mixed numbers is similar to multiplying and dividing fractions. Before they perform the operations, they must rewrite each mixed number as an improper fraction. You may want to have students practice rewriting mixed numbers as improper fractions before completing Example 3 together.

Work with students to complete Example 4. Invite students to write practice problems that pairs of students can solve. Observe students as they work to determine where intervention may be necessary. Then walk students through Example 5. Provide additional practice, if necessary.

Reading Skill: Evaluate Arguments

Ask students to define the word *ethical*. Then remind students of your earlier discussion about celebrity-endorsed products. Ask: *What might be unethical about using celebrities to sell products?* Read the text to extend the discussion to include advertisements that rely on the claims of “experts.” Ask: *Why should you be suspicious when you hear claims such as “3 out of 4 dentists,” “2 out of 3 doctors,” “4 out of 6 parents,” and so on? What questions should you ask before accepting arguments?* Invite students to discuss why evaluating claims and arguments critically is important.

MATH LINK

Have students practice using calculators to convert mixed numbers and improper fractions.

THINK ABOUT MATH

ANSWER KEY

1. division; 22,000 pounds

MATH LINK

Have students read the Math Link. Organize students into pairs and have students write three to five mixed numbers, exchange their numbers with their partners, and have the partner rewrite the mixed numbers as improper fractions. Have pairs check each other’s work as you observe.

Engage and Extend

ELL Instruction: Use Visuals Use pictures of real objects to help students understand mixed numbers. For example, show pictures of three pizzas and say: *Suppose we ordered three pizzas. Each pizza had eight slices, and we ate 19 slices. How many pizzas would we have eaten?* Encourage students to find their own pictures, create problems associated with the pictures, and share with the class how to solve those problems.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 402 and 403.

Extension Activity: Compare Mixed Numbers Write the following mixed numbers on the board: $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{2}{5}$, $1\frac{1}{2}$. Using calculators, have students determine which of the mixed numbers, when multiplied by itself, comes closest to but does not exceed 2.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Identify integers
- Compare and order integers
- Find the absolute value of an integer

✓ Determine Student Readiness

In this lesson, students learn that integers are positive and negative whole numbers. They will compare and order them and find their absolute value. To check for readiness, have students mark 0 and the first few positive whole numbers on a number line. To demonstrate readiness for absolute value, have students practice finding the distance between two numbers on the number line.

Key Concept

Identify, compare, and order integers, as well as find their absolute value, in order to better understand the meaning and value of integers.

Concept Background: Explain that integers are the set of whole numbers and their opposites. As the class progresses through the chapter, have them research marine organisms. Students should find three organisms that live at different depths. Have students write the depths from least to greatest negative number on a vertical number line.

Develop Core Skills

Core Skill: Apply Number Sense Concepts

Remind students that the term *number sense* applies to a general understanding of numbers, the quantities they represent, and their relationships with each other. Explain to them that they will extend their number sense with negative numbers. Brainstorm examples of what negative numbers represent with your students, such as below sea level, temperature below 0 degrees, and debt.

Core Skill: Represent Real-World Problems

Explain to students that integers and absolute value have meaning in many real-world situations. Relate back to the discussion about negative numbers to recall applications.

Pre-Teach Vocabulary

Rely on Prior Knowledge

Write the vocabulary words on the board, and read them aloud. Ask students to identify words they have heard or read before. Ask them to recall the contexts in which they heard or read the words. Encourage students to combine prior knowledge to define the words as they understand them.

Tier 2 Words:

infinite (p. 104)
opposite (p. 104)

Tier 3 Words:

absolute value (p. 106)
integer (p. 104)

DURING THE LESSON

PAGE 104

Understand Integers

Read the section as a class, making vocabulary review cards for each boldfaced word. To help students better understand negative numbers, explain that numbers are positive or negative relative to the defined location of zero. For example, if sea level is defined as zero, places below sea level are negative and places above sea level are positive. Since a certain temperature is defined as 0°F, temperatures colder than this are described as negative in degrees Fahrenheit and warmer temperatures are described as positive. Explain to students that they can extend this thinking by defining the top of their desk as zero. Point out that a negative number describes a point below the desk, and a positive number describes a point above. A positive or negative sign before a number simply gives them the location of the number relative to zero.

Guide students through the steps in Example 1. Practice with a few other examples, such as 5 and -3 . After students conceptually understand opposites with respect to distance, you may wish to point out (or confirm) that the opposite of a number is the same digit(s) but with the opposite sign.

Guide students through the steps in Examples 2 and 3. Make sure that students understand that the numbers on a number line always increase from left to right or bottom to top, even when the negative numbers are added to it. Students may automatically think that if the digit is greater, then the number is greater, regardless of the sign. Guide students to realize that all positive numbers are greater than all negative numbers.

MATH LINK

Ask students where they have seen or used ellipses before. Examples may include an unfinished quote, a message where a thought is left incomplete, or as a way to indicate that something continues indefinitely in the same manner.

Core Skill: Apply Number Sense Concepts

PAGE 105

After reading the section, explain to students that they will learn about absolute value on the next page. Tell them to consider what absolute value means and how it could apply to real life as they read the next page.

PAGE 106

THINK ABOUT MATH

ANSWER KEY

- | | |
|-------------------|------|
| 1. -4 | 6. > |
| 2. +19 | 7. < |
| 3. not an integer | 8. > |
| 4. -3 | 9. < |
| 5. not an integer | |

Absolute Value

Read the section as a class. Make sure students understand the difference between the opposite of a number and the absolute value of a number. The absolute value of a number just gives the distance from zero, regardless of what side of zero it is on. The opposite of a number is the number that is the same distance from zero but on the opposite side. Opposite numbers have opposite signs, but the absolute value of any number is always positive.

Guide students through the steps in Example 4, asking them to think about why the absolute values of opposite numbers are the same.

THINK ABOUT MATH

ANSWER KEY

- | | |
|-------|-------|
| 1. 9 | 3. 13 |
| 2. 12 | 4. 25 |

MATH LINK

Explain to students that a number with no sign is exactly the same as a positive number. There is no third type of number that is neither positive nor negative (except for 0).

Core Skill: Represent Real-World Problems

Give students time to read the sidebar. To help them answer the assignment at the end, ask them if they have ever purchased something and then returned it the following month. Discuss reasons why credit card companies use positive numbers to represent the balance of money loaned.

Engage and Extend

ELL Instruction: Interpret a Model Show students a thermometer, or an illustration of a thermometer. Ask students to explain how a thermometer works as a model of positive and negative integers. Encourage students to use the words *integers*, *opposites*, and *absolute value* in their discussions. Provide students with more real-world examples if they are having difficulty with these terms.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student page 404.

Extension Activity: Collect and Display Data Have students use print or online resources to find real-world examples of positive and negative numbers, such as the elevation of landforms relative to sea level or the loss and gain of yards during play in a football game. Ask students to create a printed or digital display of the example and explain its relevance to positive and negative numbers.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Use a number line to find the sum of two integers
- Use integer addition rules to find the sum of two integers

✓ Determine Student Readiness

Draw or have volunteers draw number lines from 0 to positive 25 on the board. Ask students to use the number lines to add positive whole numbers. Observe their work, intervening if necessary.

Key Concept

Two ways to find the sum of two integers include using a number line and using a sequence of rules.

Concept Background: Remind students that a number line continues infinitely in opposite directions, representing both negative and positive integers. Also remind them that absolute value is the distance between 0 and a number on the number line, whether positive or negative.

Develop Core Skills

Core Practice: Reason Abstractly

Explain to students that in mathematics, the word abstract can express a characteristic, feature, or quantity apart from objects. For example, write the problem $2 + 3 = 5$ on the board. The numbers in the problem are abstract representations of real objects. Because students can reason abstractly, that is, by using symbols or representations for objects, they know that $2 + 3 = 5$ without having to actually combine two objects with three objects. Ask students how positive and negative are represented abstractly in mathematics (using the $+$ and $-$ symbols).

Core Skill: Perform Operations

Refer to the problem $2 + 3 = 5$. Remind students that the symbol $+$ means to perform the operation of addition. Have students use a number line to model the operation.

Pre-Teach Vocabulary

Act Them Out

Write the vocabulary words and definitions on separate cards. Also write the words on the board. Ask for six volunteers. Give each volunteer a word card. Have the volunteers act out their words and have students guess which word they are performing by holding up the appropriate definition card.

Tier 2 Words:
negative (p. 108)
positive (p. 110)
sign (p. 108)

Tier 3 Words:
addend (p. 108)

Test Words:
illustrate (p. 110)
sequence (p. 109)

DURING THE LESSON

Use a Number Line to Add Integers

PAGE 108

Remind students that if a number has no sign, it is understood to be positive.

Draw a large number line on the board and work through Example 1 as a class. Ask students which direction they moved on the number line when they added a positive number (right). Point out that when adding a negative number, they move in the opposite direction, or left.

Guide students through the steps of Example 2. Ask students to identify the key word that show \$15 is negative (overdraws). It may also help students understand the example better if they imagine they are Michael. *If you overdraw your bank account by \$15, what would that look like? How would that change if you added money?* If students are fluent in counting by 5s, you can draw a number line from -20 to 100 in increments of 5s, and then count by 5s as you move along the number line. That way, the number line won't need to be so long.

Work through Example 3 as a class. Explain that adding positive integers is the same as adding whole numbers.

Follow a Sequence of Steps

PAGE 109

Read the text leading to the passage as a class. Have students read the passage and order the sentences in their proper sequence. Afterward, discuss the sequence. Next, suggest a familiar activity, such as making a sandwich or assembling a puzzle. As a class, write the sequence of steps required to accomplish the task. Discuss the consequences of following steps that are out of order.

Core Practice: Reason Abstractly

Allow students time to read and think about the sidebar. Ask if they can think of other math examples that are abstract. One example is a number line. It is an abstraction that students can use to manipulate numbers, but it doesn't really exist as an object. Ask students to brainstorm other examples of abstractions they can use to work with numbers and values. Students may say, for example, that coordinate grids don't really exist, but they are abstractions that show the relationships between pairs of numbers.

MATH LINK

Write the word *addends* on the board. Ask one volunteer to write and solve an addition problem on the board. Ask another volunteer to identify the addends in the problem. Repeat the activity until it is clear that students understand the term.

THINK ABOUT MATH

ANSWER KEY

- | | |
|------|------|
| 1. D | 4. B |
| 2. A | 5. C |
| 3. E | 6. F |

Use Rules to Add Integers

Have students write what they know about whole number addition in a list in their notebooks. As they go through the lesson, have them write how each list item is similar to or different from integer addition.

Guide students through the steps of Examples 4–6. Use number lines or counters to support each problem as needed, and to show students that both methods yield the same answer.

To help students understand the rule of when a sum is positive and when it is negative, have them assign one color as positive and another color as negative. Have them circle each addend in the examples and the Think about Math problems that follow in the appropriate color. Then after they have solved the problems, have them circle the sums in the appropriate color. Have them draw conclusions about when the sign of a sum will be positive and when it will be negative.

Core Skill: Perform Operations

Have students read the sidebar independently. Remind them to refer to the number line in Example 3 as they read, to help them understand the text. After students have written their explanations, ask them to share their ideas.

For students who need an alternative to a number line to illustrate integer addition, give students counters in two colors. Let one color represent positive numbers and the other color represent negative numbers. Have students use the counters to practice finding zero sums. You can use small objects such as paper clips and erasers if counters are not available.

Engage and Extend

ELL Instruction: Build Vocabulary Invite students to play a game. Use tape to draw a number line along one edge of a table. Mark the center of the tape 0. Mark numbers to the right as positive integers and numbers to the left as negative integers. Have a volunteer stand in front of the 0 and push a paper clip or counter in a positive direction. Ask the student to announce the distance the counter moved. Then have the volunteer push it back in the opposite direction, again announcing how far the counter moved. Have a volunteer record the measurements on the board. Then ask the player to determine how far the counter moved in all while explaining the problem solving process using the vocabulary words. Invite other students to play.

THINK ABOUT MATH

ANSWER KEY

- | | |
|------|------|
| 1. E | 4. C |
| 2. A | 5. D |
| 3. F | 6. B |

MATH LINK

Zero is the only number that is considered neither positive nor negative.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 404 and 405.

Extension Activity: Classify Concepts Have students research the use of submersibles to explore the layers or zones of the ocean, from the surface, or Sunlight Zone, to the trenches, or Hadalpelagic Zone. Have students use their research to explain the concept of positive and negative integers.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Use a number line to subtract integers
- Use addition of opposite integers to find the difference between two integers

✓ Determine Student Readiness

Give students opportunities to demonstrate proficiency in adding positive and negative integers before starting this lesson.

Key Concept

Subtract two integers by adding the opposite of the integer that is being subtracted.

Concept Background: Help students begin to recognize that subtracting integers is an extension of adding integers. Ask students what the relationship is between addition and subtraction (they are opposite operation). Have them predict how subtracting integers will relate to adding integers.

Develop Core Skills

Core Skill: Perform Operations

Explain to students that they can use a number line to model the operation of subtraction, just as they did with addition in the previous lesson. Have volunteers draw number lines and model simple positive number subtraction problems, such as $6 - 4 = 2$.

Core Skill: Represent Real-World Problems

Explain to students that operations with negative numbers are sometimes used to solve real-world problems. Have students examine lines of latitude and longitude on a real or projected globe. Point out the Equator and Prime Meridian. Explain that latitudes run north and south of the Equator, and longitudes run east and west of the Prime Meridian. When no specific direction is given, meaning N for north or S for south, a latitude north of the equator is identified as positive and a latitude south of the equator is identified as negative. The same holds true for lines of longitude. When no specific East or West direction is attached to a longitude, longitudes to the east of the Prime Meridian are positive, and those that run west are negative. Move your finger different distances and directions from 0 degrees and ask students to describe the distance moved in integers.

Pre-Teach Vocabulary

Define Everyday Words

Write the vocabulary words on the board. Put plenty of space between them. Read the words aloud, and ask students to think of times they have used or heard these words and in what context the words were used. Write students' responses next to each word, recording as many examples as students can think of.

Tier 2 Words:

point (p. 114)
solve (p. 114)

Tier 3 Words:

tic mark (p. 114)

DURING THE LESSON

PAGE 114

Use a Number Line to Subtract Integers

Work through Example 1 as a class. Draw a number line from -10 to 10 on the board. Mark -6 and $+4$ on the number line. Ask: *Which number is greater? How many tic marks on the number line do you need to get from -6 to $+4$? How many tic marks on the number line do you need to move to get from $+4$ to -6 ?* Guide students to understand that the difference between -6 and $+4$ is 10 tic marks whether going from negative to positive or positive to negative.

Review the rules described in the Math Link on page 114. Then ask students to apply those rules as you guide them through the steps of Example 2.

MATH LINK

If needed, work through a few examples with the students to make sure they understand the difference between adding and subtracting using a number line.

PAGE 115

Use Addition to Subtract Integers

Work through Examples 3 and 4 as a class. Make sure students recall and correctly apply the rules for adding integers. Point out that subtracting integers requires the exact same steps as adding integers after the problem is rewritten as an addition problem. The only thing new they learn in these examples is to write the subtraction problem as an addition problem.

Core Skill: Perform Operations

Read the text with students, helping them understand that subtracting one number from another is the same as adding the opposite of the number. For example, write the problems $5 - 2$ and $5 + -2$ on the board. Draw a number line from 0 to 5 beneath the problems. Use the number line to demonstrate how the second problem is the same as the first, and the answer is the same. Next, have students complete the activity presented in the last paragraph of the sidebar. Encourage students to share their number lines and explanations with the class.

MATH LINK

Tell students that this fact is important to remember because it is the key to subtracting integers without using a number line.

PAGE 116

THINK ABOUT MATH

ANSWER KEY

- | | |
|------|------|
| 1. E | 4. D |
| 2. F | 5. A |
| 3. C | 6. B |

Core Skill: Represent Real-World Problems

After reading the sidebar, encourage students to work together to solve the problem. Have them draw a number line from -15 to $+15$ to model the solution. Allow students to find another way to solve the problem, too, such as drawing a picture. Afterward, encourage students to share the model they used to solve the real-world problem.

Engage and Extend

ELL Instruction: Make Connections Pair English language learner with fluent English speakers to discuss how subtracting integers is similar to and different from adding integers and subtracting whole numbers. Encourage students to use lesson vocabulary as much as possible.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 405.

Extension Activity: Explain Phenomena in Terms of Concepts Have students research weather records in any location of interest that has records going back at least five decades. Ask students to determine the record highs and lows for each decade, and calculate the variation between the two. Then have them explain the variations in the context of the principle of positive and negative integers.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Multiply two integers
- Divide two integers

✓ Determine Student Readiness

In this lesson, students multiply and divide positive and negative whole numbers. To demonstrate readiness, have students review multiplying and dividing whole numbers by finding 1-digit by 1-digit products and the corresponding quotients.

Key Concept

Use rules to find products and quotients of integers.

Concept Background: Remind students that multiplication can be thought of as repeated addition. Demonstrate $-2 \times 4 = (-2) + (-2) + (-2) + (-2)$ on a number line. Ask: *What is the sign of the sum? What will be the sign of the product?* Have students predict the sign in the division of negative integers.

Develop Core Skills

Core Skill: Interpret Data Displays

Explain to students that tables can be useful for displaying data, making the data easy to understand or interpret. Ask students to think back to earlier lessons to describe how they have used tables to organize and display data.

Core Practice: Use Appropriate Tools Strategically

Explain to students that diagrams, graphs, and tables are tools, and that like all tools, they are used strategically, meaning for a specific and necessary purpose. At times, a graph is required. At other times, a table serves a specific purpose best. Ask students to imagine teaching the class about a topic they find interesting. Ask them what tools they would use to help clarify or explain the ideas they want to teach.

Pre-Teach Vocabulary

Visualize Words

Explain that when some people attempt to interpret tables, they have difficulty distinguishing between *rows* and *columns*. Draw a table on the board. Label the rows, columns, and give it a title. Have students copy the table in their notebooks as a reminder. Invite students to think of mnemonic devices they might use to distinguish between rows and columns. Perhaps, for example, they may think of columns as the columns of a building (vertical) and rows as the rows in a garden (horizontal).

Tier 2 Words:

repeated (p. 118)

Tier 3 Words:

column (p. 119)

inverse (p. 120)

row (p. 119)

table (p. 119)

Test Words:

title (p. 119)

DURING THE LESSON

Multiply Integers

PAGE 118

Ask a volunteer to remind the class what repeated addition is and give an example.

Encourage students to solve Example 1 independently. Afterward, ask students to explain what they did to find the answer. Next, ask students to solve Example 2 independently. Again, ask them to explain what they did to find the answer. Then ask them to compare the processes for multiplying two positive integers and multiplying one positive and one negative integer. If students struggle, encourage them to draw number lines to model the solution.

Guide students through Examples 3 and 4 on page 120. Explain to students that after they understand the examples, they do not need to apply that reasoning every time they solve an integer multiplication problem. Point out the two rules listed after the examples.

Read a Table

PAGE 119

If possible, project the page on the board. Read the opening text to students, emphasizing the boldfaced words. If you have projected the page, point to the boldfaced words as you read. Ask volunteers to identify the features those words describe in the table. Discuss the value of using a table to organize data, or information. Organize students into three groups. Assign one of the questions found below the table to each group, and have students work together to find the answer. When groups are finished, have volunteers from each group give the answer and explain the solution.

Core Skill: Interpret Data Displays

Give students time to read the text in the sidebar. Then, together, discuss the features of the table. Ask the question that follows, and invite volunteers to answer. Students may say, for example, that it would be helpful to expand the table to include more months, or they may even want to insert more columns to show how many adult men and women also visit the park in the same months.

THINK ABOUT MATH

ANSWER KEY

- | | |
|------|------|
| 1. B | 3. D |
| 2. A | 4. C |

Divide Integers

Guide students through Examples 5 and 6. Example 6 may be tricky if students are not fully comfortable with multiplying integers. Reassure students that their number sense will improve with practice, and that they will be able to apply a rule to divide just as they did with multiplication. At this point you could ask them to predict the rule for dividing integers.

Continue to connect to what students have learned about multiplying integers as you walk through Examples 7 and 8. Ask students to compare the rules for identifying the sign of the products and quotients of integers. (The rules are the same for both products and quotients.) Note that these rules only apply for the product or quotient of two numbers. The product of three negative numbers, for example, is negative. You can generalize the rule for any number of factors: if an odd number of the factors is negative, the product is negative. If an even number of the factors is negative, the product is positive.

Core Practice: Use Appropriate Tools Strategically

Allow students time to read the sidebar and answer the questions that follow. Explain that a table is a way to show information, and just like word problems, it is important to understand the given information first before solving problems based on the table.

Engage and Extend

ELL Instruction: Make a Table Ask students to think about how many minutes they spend watching television or online videos during the school week and on the weekends. Record, tally and average their answers as they observe. Then ask students to explain how you can organize the data you have collected in a table to make it easier to read and understand. Direct students to tell you exactly what to do, one step at a time. Respond to their commands, making changes as students become aware of their necessity. Discuss the features of the final table.

THINK ABOUT MATH

ANSWER KEY

- | | |
|------|------|
| 1. D | 3. B |
| 2. C | 4. A |

MATH LINK

Have students apply what they have learned about tables to identify the title and labels of this table. Note that only the columns and not the rows have labels.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 406.

Extension Activity: Compare Operations Have students work in small groups to create Venn diagrams. Provide them with copies of an empty diagram (see the Graphic Organizer section of the Instructor Resource Binder). Have students label one circle: Adding and Subtracting Integers. Have them label the second circle: Multiplying and Dividing Integers. Have them use the Math Links and text in the chapter to fill in as many details as possible in the diagram.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Plot and identify points on a coordinate grid

✓ Determine Student Readiness

Students will learn about the coordinate grid and how to identify and plot points using ordered pairs. To determine readiness, have students draw and label horizontal and vertical number lines that display both positive and negative integers.

Key Concept

Coordinate grids are a method of locating points in the plane by means of directions and numbers.

Concept Background: Gather some examples of maps that use coordinate grids, or print sample maps from an online search. Show the examples to students, and encourage them to figure out the purpose of the lines that run north and south and east and west. Explain to students that those arrangements of lines form a coordinate grid. By finding where horizontal and vertical lines intersect, they can locate specific features or locations.

Develop Core Skills

Core Skill: Interpret Data Displays

In the previous lesson, students learned to use tables to display and interpret data and information. Tell students that in this lesson, they will display and interpret data (in this case, points) on a grid, or coordinate plane. Refer to the Concept Background and ask students to describe what kinds of data a map usually contains.

Core Practice: Make Sense of Problems

Explain to students that many people find it helpful to draw pictures to make sense of a problem. Share what you do to make sense of the problems you solve. For example, you might show how you draw pictures to help you follow the steps in a building or cooking project. Invite students to describe some of the strategies they use to solve problems. Answers may include creating tables, drawing pictures, using number lines, guessing and then checking, and using prior knowledge.

Pre-Teach Vocabulary

Connect Words to Examples

Use one of the maps you gathered prior to the lesson. Display the map, or if possible, project it on a wall to make it easier to see. Read the vocabulary words to students and explain their meanings. As you define the words for students, explain the relationship between each word and a specific feature on the map.

Tier 2 Words:

origin (p. 124)

Tier 3 Words:

coordinate plane (p. 124)

ordered pair (p. 124)

perpendicular (p. 124)

x-axis (p. 124)

x-coordinate (p. 124)

y-axis (p. 124)

y-coordinate (p. 124)

Test Word:

grid (p. 125)

DURING THE LESSON

PAGE 124

The Coordinate Plane

Read the first paragraph as a class. Have a volunteer identify the two number lines that make up the coordinate plane. Explain that an ordered pair represents a point on the plane. (You may want to refer to the abstract representations students learned about in earlier lessons; an ordered pair is another example.) Make sure students know the format of an ordered pair, and that this is the only correct way to write it. Tell students to remember to include the comma and parentheses when writing ordered pairs. Remind students of the need for precision in math. Without the comma or parentheses, the symbols do not represent a point.

If students have difficulty remembering which coordinate comes first in an ordered pair, point out that x comes before y in the alphabet. Distribute graph paper and allow student to practice plotting points A through D from page 125.

Guide students through the steps in Examples 1–4, taking time to make sure students understand the processes for both plotting and identifying points.

MATH LINK

Have students find or show examples of perpendicular lines in the classroom. Also show them and have them identify non-examples, or lines that are not perpendicular.

MATH LINK

Read the Math Link with the class and ask a volunteer to explain why it is true. Help them understand that it is because of the nature of number lines. Negative numbers appear to the left of and below the origin, or 0.

Core Skill: Interpret Data Displays

Allow students time to read the sidebar. Then give partners graph paper to complete the activity. If any groups are struggling, provide remediation by reviewing how to draw the axes, which axis is which, and how to locate points and write ordered pairs. Or you may wait to assign this activity until students have worked through Examples 1 through 3.

Core Practice: Making Sense of Problems

Read the sidebar as a class. Ask students questions as they draw a coordinate grid to match the problem: *Which axis represents east to west? Which axis represents north to south? Label the directions. In terms of the grid, how can you represent one mile in your drawing?*

THINK ABOUT MATH**ANSWER KEY**

- | | |
|-------------|-------------|
| 1. <i>J</i> | 5. (2, 1) |
| 2. <i>H</i> | 6. (-8, 3) |
| 3. <i>M</i> | 7. (-4, -9) |
| 4. <i>B</i> | 8. (5, -8) |

Engage and Extend

ELL Instruction: Describe a Coordinate Grid Have students refer to the vocabulary words. Then invite students to select a coordinate grid from the lesson and use the vocabulary words to describe its parts.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 406.

Extension Activity: Investigate Descartes Have students research Descartes, inventor of the Cartesian plane. After reading about his life and work in mathematics, have students write one or two questions about Descartes' work that they would like to answer through further research. Afterward, ask students to share their research in a digital presentation.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Translate between verbal and symbolic representations of expressions
- Simplify expressions
- Evaluate expressions

✓ Determine Student Readiness

To simplify and evaluate expressions, students will need to know and apply the order of operations.

Ask a volunteer to remind the class of the order of operations (parentheses, exponents, multiplication, division, addition, subtraction). Then practice a few two- and three-step problems. For example: $5 - 8 \div 2$; $4 + 5 \times -6$; $-2.3 - 4 \times 1.2$

Key Concept

Mathematical and real-world situations can be represented by expressions that can be simplified and evaluated.

Concept Background: Tell students that mathematical expressions are phrases or parts of sentences, just as verbal expressions are phrases and parts of sentences. Write a sentence and an equation on the board, such as *The red fox leaped among the flowers in the meadow.* and $(2 \times 4) + \frac{16}{8} = x$. Circle *among the flowers* and *in the meadow* in the sentence. Circle (2×4) and $\frac{16}{8}$ in the equation. Explain that all four are expressions, and ask students to describe the difference among them. Then explain that the mathematical expressions can be simplified or evaluated. Ask: *How can you simplify $\frac{16}{8}$?*

Develop Core Skills

Core Skill: Evaluate Expressions

Have students refer to the five-step problem solving approach they learned in Lesson 1.7. Ask students how this approach has helped them solve word problems. Students may respond that the approach has helped them organize the given information and know what to do and in what order. Explain that they will also use steps in a specific order when they solve problems using expressions.

Core Practice: Make Sense of Problems

Explain to students that they will be reviewing how to use key words to help them understand word problems in this lesson. Have volunteers identify key words they have learned for each of the four operations.

Pre-Teach Vocabulary

Word Bench

Have students find familiar words among the vocabulary words, or words that seem similar to words they know.

Ask students to use the words they list to suggest meanings for the vocabulary words. Guide them in coming up with accurate definitions.

Tier 2 Word:

variable (p. 134)

Tier 3 Words:

algebraic expression (p. 134)

coefficient (p. 134)

constant term (p. 134)

mathematical expression (p. 134)

symbolic expression (p. 135)

verbal expression (p. 136)

DURING THE LESSON

Verbal and Symbolic Representations of Expressions

PAGE 134

Give students time to read the first paragraph. Then ask students to think of other examples of expressions, both in context and as abstract mathematical expressions.

(*Six apples and four oranges* is in context, while $6 + 4$ is the abstract representation.)

Students may assume that the symbol “=” can also be included in expressions. Explain to students that expressions do not include the equal sign. If an equal sign is used, it is not an expression but an equation, which students will learn about later in the chapter.

Example 1 shows some examples of mathematical expressions. Point out that in the second expression, the parentheses are used to set apart a negative number. In the third expression, they are used to show multiplication: 23 is multiplied by the quantity inside the parentheses.

Complete Example 2 as a class, having students volunteer the correct order of operations.

After reading the paragraph following Example 2, have students identify the differences between an algebraic expression and a mathematical expression. (Unlike a mathematical expression, an algebraic expression contains variables. Because of this, the numbers in algebraic expressions are called coefficients and constant terms.)

Use Example 3 to practice identifying variables, coefficients, and constant terms in algebraic expressions. Write a few more examples for more practice.

Guide students through the steps in Examples 4 and 5 on page 136. At first, it may seem strange to students to assign a variable to an unknown number. Explain that they are choosing a name for the unknown number so they do not have to write “unknown number” or “number of text messages” in the expression. Explain that they can use any letter they want. Point out that including the sentence “Let n represent...” is an important step in the solution process because it explains what the letter stands for in the expression. Be sure students do not skip the step. They will continue this practice as they write equations to represent situations.

Guide students through the steps in Example 6. Have students come up with other real-world situations for $3m + 4$.

Identify Key Words

PAGE 135

Allow students time to read the text and complete the activity. Have students cover up the answers below the box and try to find key words on their own. As they finish, have them compare their key words with a partner and discuss any differences.

MATH LINK

Explain to students that the letter x is often used as a variable, and it is easily confused with the multiplication symbol \times . That is why these three other ways are used to show multiplication. Have students practice writing a product such as $4 \times y$ in the three forms.

PAGE 136

MATH LINK

This is a very significant detail that students may need to be reminded of as they continue to work with expressions and start solving equations. If students have trouble remembering, encourage them to rewrite an expression or equation, placing a 1 in front of the variable.

Core Skill: Evaluate Expressions

After reading the sidebar, discuss with students how these steps for writing an expression are similar to or different from the steps they used in the Five-Step Approach. A similarity is that they need to be completed in order. They need to clearly explain how the problem is solved. Discuss the importance of each step in the examples.

Engage and Extend

ELL Instruction: Write Verbal Statements Have students work in pairs to write verbal situations for the problems in the Think about Math activity on page 137. Have volunteers read them aloud.

PAGE 137

THINK ABOUT MATH

ANSWER KEY

Note: Any variable is acceptable.

1. $p + 12$
 2. $2s - 250$
- Sample answers:
3. four times a number t divided by two
 4. a number c minus nine

Evaluate Expressions

Explain that when you evaluate an expression, you will end up with a number. There should be no variables left. Guide students to understand that when you substitute a given value for the variable, you simply rewrite the expression, replacing the variable with the number. Sometimes, it helps students to put parentheses around the number when they substitute it. Point out that if the variable has a coefficient, the expression needs to show the multiplication when substituting. For example, some students may substitute $x = 4$ in $2x$ and get 24 instead of 8. Make sure students understand that $2x$ means 2 times x so when they substitute, they must multiply.

Core Practice: Make Sense of Problems

After students read the sidebar, lead a class discussion on how they have used key words to help solve problems so far. Then have pairs use key words to write simple math problems for each other, exchange, and solve.

PAGE 138

THINK ABOUT MATH

ANSWER KEY

1. 10.7
2. -6
3. -3
4. 4

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student page 408.

Extension Activity: Interpret Variables Have students use print and online resources to answer the question: *What is a variable?* Have students create a presentation to explain the concept of a variable to students who have never studied the algebraic concept. Encourage students to use free online tools to create their presentations.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand and write equations
- Solve one-step equations

✓ **Determine Student Readiness**

In this lesson, students solve one-step equations.

To determine their readiness, write the expressions $12x + 2x$ and $27 \div 3$ on the board. Ask students to simplify the first expression and evaluate the second. Invite students to suggest more expressions that the class can simplify or evaluate.

Key Concept

Use equations to represent situations, and use inverse operations to solve one-step equations.

Concept Background: Tell students that equations are mathematical sentences. They can be closed, that is have no variables, such as $2 + 10 = 12$, or they can be open sentences that contain one or more variables, such as $12 - x = 17$. While a closed equation is either true or false, an open equation cannot be determined as true or false until it is solved. Point out that students know $2 + 10 = 12$ because $12 = 12$, but they do not know whether $12 - x = 17$ because they do not know what value x is.

Develop Core Skills

Core Practice: Make Sense of Problems

Remind students of when they identified irrelevant information in earlier lessons. Ask students why this is an important skill. A possible response is that real life is not like math problems, and you rarely have just the information you need when solving a problem in daily life.

Core Skill: Represent Real-World Arithmetic Problems

Remind students of how they used expressions to represent verbal or real-world situations in the previous lesson. Ask a volunteer to give an example. Explain that they will learn another way to represent real-world problems in the lesson, this time using equations.

Pre-Teach Vocabulary

Word Map

Have students use the Vocabulary Map (see the Graphic Organizer section of the Instructor Resource Binder for a blackline master) for one or more words. For each they should write a word, the definition, and give an example and a non-example. Assist students in coming up with definitions and allow them to finish the vocabulary maps during the lesson. Make sure students understand the difference between expressions and equations.

Tier 2 Word:

solution (p. 140)

Tier 3 Words:

equal sign (p. 140)

equation (p. 140)

equivalent equation (p. 140)

inverse operations (p. 141)

DURING THE LESSON

PAGE 140

Understand and Write Equations

Understanding, writing, and solving equations may be a bit of a leap from expressions for many students. Before stepping through Example 1, you may want to give students a simpler example, such as *the sum of 5 and a number is 12* ($5 + n = 12$). This will allow them to practice writing equations and become more comfortable with the idea of equations.

Guide students through the steps in Example 1. For step 1, remind students that the variable should represent the unknown quantity in the problem, usually the amount that is being asked for. Since the problem asks for how much Bao earns per hour, the variable should represent that rate.

For step 2, students will need to apply what they know about rates from real-life experiences. The amount earned equals the hourly pay rate times the number of hours worked. Take the time to answer any questions and go through the steps more slowly, giving another example if needed.

For Example 2, point out that substituting into an equation works just like substituting into an expression. Have a volunteer summarize the process for checking a solution: *To check for a solution, substitute it into the equation, replacing the variable. Simplify by doing any operations and then check to make sure both sides are the same number.*

The properties of equality are what allow us to solve equations by performing the same operation on both sides. Tell students that these properties will make more sense as they begin to solve equations.

THINK ABOUT MATH

ANSWER KEY

- $n + 2 = 3$
- $a - 5 = 12$
- $7(8)$ is not equal to 42, so $c = 8$ is not a solution for the equation.
- $-4 + 7 = 3$, so $y = -4$ is a solution for the equation.

Solve Equations

Remind students that they learned about inverse operations (also called opposite operations and reverse operations) in Chapter 1 when learning how to check their work. Explain that to find the value of the variable, they must undo what was done to it. The properties of equality state that you must do the same thing to both sides of the equation to get an equivalent equation.

Guide students through the steps of Examples 3 through 6 making sure they understand how to perform the inverse operation on both sides of the equation. To help students conceptually understand, tell them that to solve the equation, you find the value for the variable that makes it true. For example: *What number plus four equals 21? What number times three equals 24?* Students may be able to solve these in their heads, but explain that using inverse operations allows them to solve more difficult problems, for example with greater numbers or decimals.

Core Practice: Make Sense of Problems

Allow students time to read the sidebar and ask any questions. Then guide students to understand the importance of the question sentence in a problem, usually the last sentence in a word problem. It can act as an anchor or compass when working through the solution to help keep from getting off track or stopping before the final answer is reached. Some students find it helpful to use the question to write an answer sentence with a blank before starting to solve. For example: *Mariska's school spent _____ to buy the lunches.*

Engage and Extend

ELL Instruction: Read Aloud Have students work in groups. Have one person read Skill Practice question 1, 3, or 4 aloud, while the other group members identify key vocabulary that suggest the operations and values needed to write the equation. Have students define key vocabulary in their own words.

THINK ABOUT MATH

ANSWER KEY

- | | |
|-------|------|
| 1. 7 | 3. 6 |
| 2. 11 | 4. 1 |

Core Skill: Represent Real-World Arithmetic Problems

After students read the text, ask volunteers for examples of when they have used math operations to solve problems in their daily lives. Examples may be calculating the cost of purchases, making a budget, and calculating a tip. Then have students solve the problem at the end.

Here is a possible solution:

Let c = the cost of the copies

Multiply the cost of each copy by the total number of copies to get the total cost.

$$c = \$0.10 \times 150$$

$$c = \$15$$

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student pages 408 and 409.

Extension Activity: Formulate Routine Problems

Have students write real-life problems that can be solved by using one-step equations such as using an hourly rate to determine how much a person will earn per day, per week, or per month. Refer students to Example 1 on page 140 for an example. Have students write their equations and solutions on separate pieces of paper, exchange problems with a partner, and solve.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Translate verbal sentences into two-step equations
- Solve two-step equations

✓ Determine Student Readiness

In this lesson, students learn to solve two-step equations. Have students show readiness by applying order of operations to simplify expressions with integers. For example: $18 + 6 \div 6$; $-4 \times -2 + 4$; $-9 - 5 + 3$

Key Concept

Use two inverse operations to solve two-step equations.

Concept Background: Explain to students that two-step equations involve two different operations, such as multiplication and addition. Ask students why it's helpful to take time before beginning the solution process to decide which different operations they must apply to solve an equation.

Develop Core Skills

Core Practice: Make Sense of Problems

Remind students of the importance of making sense of problems before trying to solve them. Ask students what would happen if they used a positive number when they should have used a negative number. (The answer would likely be incorrect.) Then ask students to consider how the answer to a problem is treated differently if it represents money. Responses may include: two decimal places, dollar sign in answer.

Core Skill: Evaluate Expressions

Ask students what it means to *evaluate*. A possible response is "determine the value." Then ask the students what it would mean to evaluate an expression. The meaning of evaluating expressions should flow from the definition they provide of the word evaluate.

Pre-Teach Vocabulary

Affect and Effect

Write the words *affect* and *effect* on the board. Explain the difference to students, and then ask students to explain the difference to you. Invite students to describe situations in which they have *affected* something or experienced an *effect*.

Tier 2 Words:

affect (p. 146)

isolate (p. 145)

Tier 3 Words:

two-step equation (p. 144)

DURING THE LESSON

Translate Verbal Sentences into Two-Step Equations

PAGE 144

Explain to students that the difference between these equations and the ones in the previous lesson is that they will have to perform more than just one inverse operation to find the value of the variable.

Guide students through the steps of Examples 1 and 2. Point out to students that the process for translating words into two-step equations is similar to what they did for one-step equations and expressions. Make sure students check the answers for reasonableness. In Example 1, confirm that the equation does match the words. For Example 2, confirm that the equation matches the situation: $6d$ represents the amount she needs to save, 500 is the amount she has saved so far, and together they add up to the total amount she needs, 2,000.

Solve Two-Step Equations

PAGE 145

Write the word *isolate* on the board. Next to it, write the meaning "to set apart." Explain that the word comes from the Latin word *isolatus*, meaning "made into an island." Tell students that in algebra, they must isolate a variable to solve a problem. A series of steps are done to get the variable alone on one side of the equation; it is as if the variable is on an island.

Point out to students that when they solved one-step equations, they also isolated the variable. Have them look in their notebooks from the previous lesson to confirm that in the answer, the variable was by itself. Here, it will take more than one inverse operation. Have students make a list of what they already know about solving one-step equations. Have a volunteer write the list on the board. The class can refer to it as they learn about solving two-step equations.

Take time to carefully work through each of the steps of Examples 3 through 6. The number of steps may overwhelm some students. Explain that algebra is much less difficult when problems are broken into simpler steps. The problem itself may seem challenging, but each step in the solution is not, allowing the answer to be found without too much difficulty.

Point out how the examples use the reverse of the order of operations; first add or subtract, and then multiply or divide. Explain that this makes the calculations simpler. Tell students that if the solution process to an equation gets messy, make sure the correct order was followed.

For Examples 5 and 6, be sure students understand that the fractions represent division. For example, $\frac{x}{4}$ means $x \div 4$.

MATH LINK

Explain to students that they may use any letter they choose for a variable. Remind them of the importance of writing *Let [letter] =* at the start of every solution to show what their chosen letter represents.

PAGE 146**Core Practice: Make Sense of Problems**

Allow students time to read the sidebar. Then have them complete the activity at the bottom by making a deliberate mistake when solving Example 4. Tell students that dropping negative signs is an extremely common mistake. Explain that this is something to check for if they got the wrong answer, and if they find they make it repeatedly, it can help to circle the number and the negative sign throughout the solution process to make sure it is not dropped.

PAGE 147**MATH LINK**

Review the rules for operations with integers with students. If needed, refer back to Chapter 4.

Core Skill: Evaluate Expressions

Remind students that they can use inverse operations in the opposite order of operations to solve an equation. The first inverse operation would be to subtract 4 to get the equation $5x = 20$. Then divide by 5 to determine that $x = 4$.

PAGE 148**THINK ABOUT MATH****ANSWER KEY**

1. $x = 8$
2. $x = 12$
3. $x = -64$
4. $x = 1$

Engage and Extend

ELL Instruction: Fill in the Blanks Write the following Cloze passage and word choices on the board. Have students fill in the blanks. Tell them that a word can be used more than once. (answer, inverse, isolate, operation, two-step)

In order to solve _____ equations, I must _____ the variable. This involves using _____ operations to undo the _____ on one side. I must perform the correct _____, otherwise I will get the wrong _____.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student page 409.

Extension Activity: Use Concepts to Solve Challenging Problems

Give students equations with three or more steps, such as $\frac{4+x}{5} - 2 = 2$. Have them solve the problems and describe how the order of operations helped them know which operations to do in which order.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Translate verbal statements into inequalities
- Solve one-step inequalities
- Solve two-step inequalities

✓ Determine Student Readiness

To determine student readiness for this lesson, have them demonstrate their ability to solve 1- and 2-step equations. For example: $x + 5 = 9$; $x - 10 = 2$; $2x + 35 = 115$; $\frac{x}{-8} - 1 = 2$

Key Concept

Use inverse operations to solve one- and two-step inequalities.

Concept Background: Tell students that solving one- and two-step inequalities is similar to solving one- and two-step equations. They both use inverse operations and the inverse order of operations. The one difference is when you multiply or divide by a negative number. Explain that equations are relationships showing that two expressions are equal, and inequalities compare expressions that may or may not be equal. Have students think of unequal relationships in real-life situations.

Develop Core Skills

Core Skill: Solve Inequalities

Write the words *equality* and *inequality* on the board. Ask students to explain the difference in meaning. Provide assistance, if necessary. Invite volunteers to draw pictures or use objects found in the classroom to demonstrate the difference. Explain that they can solve for variables in inequalities, much like they did in equations.

Core Practice: Evaluate Reasoning

Discuss with students the importance of reasoning through the solution process. Often times, checking the solution process can result in identifying an error. This is a good way to make sure the answer is correct.

Pre-Teach Vocabulary

I Know Those Words!

This lesson's vocabulary words are commonly used in everyday life. Have students use their own life experiences and prior knowledge to define each vocabulary word in their own words. Challenge students to use the words in original sentences.

Tier 2 Words:

infinite (p. 151)
reverse (p. 152)

Tier 3 Word:

inequality (p. 150)

DURING THE LESSON

Translate Verbal Statements into Inequalities

PAGE 150

Read the text with students before examining the example as a class. Ask a volunteer to read the inequality out loud. Make sure students know how to read the inequality symbols.

To help students understand inequalities conceptually, have them list real-life situations that could be represented by inequalities such as *I need at least 4 potatoes to make this recipe. It will serve no more than 3 people.* Make sure they identify whether the situation should be represented by $<$, $>$, \leq , or \geq . As you work through the examples, have students think back to the situations the class suggested to help understand inequalities. Have students discuss how remembering the real-life situations and knowing how to solve equations helped them solve inequalities. Students should recognize key words in the real-life situations, such as *at least*, *at most*, *more than*, *fewer than*, etc. to write inequalities.

Guide students through the steps of Example 1 on this page and Example 2 on page 151. Point out that the process for translating words into inequalities is the same as for equations and expressions. Students only need to learn the symbols and phrases that represent inequalities.

MATH LINK

Have students test the phrases in problem situations and with numbers and variables.

PAGE 151

THINK ABOUT MATH

ANSWER KEY

1. $n - 8 > 12$
2. $c + \$3 \geq \10
3. $2n + 4 < 25$
4. $3h + \$25 \geq \310

Solve One-Step Inequalities

Read the introduction with students, and then guide them through the steps of Examples 3 and 4 on page 152. Point out how the solutions are checked. With equations, students substituted the value of the variable to confirm that both sides were equal. Explain that similarly, to check the solution set to an inequality, substitute any number from the solution set into the original equation. Then simplify and make sure the relationship is true.

Read the paragraph preceding Example 5 on page 152 as a class. Then work through Example 5, continuing to page 153. Finally, solve Example 6 on page 153.

MATH LINK

Students may recognize these properties as being similar to the Properties of Equality. This is what allows students to solve inequalities similarly to solving equations. The one difference is when multiplying or dividing by a negative number, which students see in Example 5.

PAGE 152**MATH LINK**

Tell students that infinite and inequality have the same prefix, in-, which means “not.” Finite means “something that ends,” and equality means “being equal.” So, infinite means “something that does not end” and inequality means “not equal.” A word related to finite is finish. A word related to equality is equation.

To help students understand, refer back to a number line. Point out that if the solution is $r > 12$, then the solution set contains all the numbers that are greater than 12, and the number line goes on forever.

Core Skill: Solve Inequalities

Allow students time to read the text. Have students write the rule for multiplying and dividing in their notebook. Explain that it is usually simplest to just memorize the rule. Have students solve the problems at the bottom of the sidebar and compare the solution sets.

PAGE 153**THINK ABOUT MATH****ANSWER KEY**

1. $t < -5$
2. $x \leq 2$
3. $a \leq -6$
4. $c < 7$

Solve Two-Step Inequalities

Explain to students that they will follow the same steps for solving two-step inequalities as they did for equations, except flipping the inequality symbol when multiplying or dividing by a negative number.

Guide students through the steps of Examples 7 and 8. Take adequate time to make sure students are comfortable with the concept.

Engage and Extend

ELL Instruction: Restate Examples Invite students to select any example from the lesson to explain in their own words. Invite them to have other students ask questions. Assist them, if necessary, in answering those questions. Encourage discussion of concepts or processes that seem to cause confusion.

Before they start the practice problems at the end of the lesson, have students summarize the steps of solving inequalities. Have them include the similarities and differences between solving inequalities and solving equations.

Core Practice: Evaluate Reasoning

Have students read the sidebar. Explain to students that if they forget to flip the inequality symbol when they check their answer, the inequality will not be true. Explain that like in the story, this is something to look for when looking for a mistake. Emphasize how important it is to check the solution set, as this is one type of error that can be caught.

PAGE 154**THINK ABOUT MATH****ANSWER KEY**

1. $x < -1$
2. $y \leq 45$
3. $m \leq -28$
4. $b < -3$

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student page 410.

Extension Activity: Assess Costs with Inequalities

Have students imagine they are taking a vacation. Their vacation budget is \$7,000. Have them write an inequality that includes the price of a hotel per night, cost of a rental car per day, and an estimate of food expenses per day. Have them use the inequality to decide how many days long their vacation can be. Encourage students to choose a variety of destinations, organizing all of the information and results.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Write expressions to represent patterns
- Write equations to represent patterns

✓ Determine Student Readiness

In this lesson, students learn to identify patterns in sets of data. To determine their readiness, have students use a multiplication chart or mental math to share simple patterns of multiples, such as 2, 4, 6, 8, 10, ... and 5, 10, 15, 20, 25, ...

Key Concept

Identify, represent, and generalize patterns using expressions and equations.

Concept Background: Explain to students that patterns may be visual, auditory, or mathematical. Give them examples such as the beat in a song or the cycle of the moon. Have them identify and explain other the patterns. Ask them how the pattern is repeated, and if they can find a rule that works for each part of the pattern.

Develop Core Skills

Core Skill: Solve Real-World Arithmetic Problems

Remind students of the examples of patterns found in the beat of a song and the phases of the moon. Also remind them of the patterns they can find in a multiplication chart. Explain to students that they can use patterns to make predictions about what will happen next. Ask students to think of ways that they use patterns to make predictions in their lives.

Core Skill: Build Lines of Reasoning

Write the word *reasoning* on the board. Tell students that the base word *reason* has its roots in Latin, and that its original meaning was “discourse,” or discussion. Centuries later, in the 1300s, people used the word *reasoning* to mean “the act of thinking logically,” or an example of logical thought or action. Explain that when students solve problems, they reason or apply logic to the processes they follow. Discuss with students how they use reasoning in their daily lives to solve problems.

Pre-Teach Vocabulary

Opposites

Write the words *input* and *output* on the board. Ask students to explain the words’ meanings in their own words. Challenge students to think of computer-related examples of input and output. Prompt the discussion by sharing an example, such as using a shortcut on a keyboard and seeing the result in a document.

Tier 2 Words:

generalize (p. 156)
term (p. 156)

Tier 3 Words:

common difference (p. 158)
input variable (p. 158)
numerical pattern (p. 156)
output variable (p. 158)

Test Word:

sequence (p. 156)

DURING THE LESSON

PAGE 156

Write Expressions to Represent Patterns

Find pictures of repeating patterns, such as patterns in wallpaper, textiles, and floor tiles. Have students identify the patterns they see. If there are patterns in the classroom, point them out to students.

Explain that patterns can also occur in sets of numbers. Read the text with students, and discuss the boldfaced words to be sure students understand their meanings. Allow students to read the paragraphs before Example 1 near the top of page 156 and answer any questions they have. Walk through the steps in Example 1. Make sure to find the common difference between each pair of consecutive numbers in each row so students grasp the process. Step 3 may be a leap for some students. Refer back to early discussions on abstract representations. Explain that $3n$ is the abstract representation of “find the value of any term n by multiplying its position in the sequence by 3.”

Explain that for the sequence in Example 1, the rule could be described as $y = 3n$, where the input variable, n , is the location of the term and y is the value of the term.

Guide students through the steps of Example 2 on page 158. At first, students may find guessing-and-checking challenging. Allow them time to understand the process.

Walk through Examples 3 and 4 on page 159. Take time with Step 2 of both examples to make sure students understand the process of finding the rule.

MATH LINK

All of the sequences in the examples of this lesson are arithmetic sequences.

PAGE 157**Make a Table**

Have students read the text, up to and including the passage. Have them work in groups to make a table of the information. Then have them read the remaining text and compare the tables. Have them discuss in pairs how well their tables show the information and what might be better shown differently. Then have students explain which apartment they think Catalina should choose and why. If time allows, have student discuss how they use tables in their lives to help them organize information and make decisions.

Core Skill: Solve Real-World Arithmetic Problems

Have students read the text and complete the table. After they have confirmed that there is a common difference of 15 degrees, ask students why this is likely. (It was chance, dependent entirely on the weather.) Ask them if they can use the data in the table to predict temperatures the following week. Explain that weather can change from day to day, making it difficult, even for scientists with the most advanced tools, to predict weather accurately.

PAGE 158**THINK ABOUT MATH****ANSWER KEY**1. $4n$;

Position of Term n	1	2	3	4	5	6
Number in Sequence	4	8	12	16	20	24

2. $3n + 2$;

Position of Term n	1	2	3	4	5	6
Number in Sequence	5	8	11	14	17	20

Core Skill: Build Lines of Reasoning

Have students read the first paragraph and discuss why it's important to understand what makes each step in a solution process important. Help students understand that memorizing a series of steps may be useful in some cases, but the same steps can't apply to all problems. Next, have students work with partners to complete the activity. This is the square number sequence. While students may not be familiar with squares, they may solve it by seeing that the value of the term is the location of the term times itself: 1×1 , 2×2 , 3×3 , etc.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 410.

Engage and Extend

ELL Instruction: Make Connections Organize students into small groups and discuss real-world patterns. Offer an example of wages as an example to start the conversation: For every delivery Joe makes, he earns \$2.00. After his first delivery, he has \$2.00; after his second delivery, he has \$4.00, after his third delivery he has \$6.00, and so on. Have students offer their own examples of real-world patterns, and challenge them to use those patterns to solve problems.

Extension Activity: Construct Concept Maps Challenge students to find real-world examples of input-output relationships, such as those that exist in computer science, mathematics, life science, and physical science. Ask students to construct concept maps to explain the relationships between inputs and outputs in the real world.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Plot a line that represents the linear relationship between two sets of numbers
- Graphically determine the value of the dependent variable
- Determine whether an independent and a dependent variable are linearly related
- Write the equation of a line from a verbal description of the relationship between two sets of numbers

✓ Determine Student Readiness

In this lesson, students learn to identify linear relationships. To determine student readiness, draw or project a coordinate grid on the board. Assign coordinate pairs to volunteers, and have them go to the board to plot them on the grid. Use students' responses to determine if intervention is required.

Key Concept

A variable is something you are trying to measure. There are two kinds of variables, independent and dependent. An **independent variable** has a value that remains the same. That is, it is not affected by a **dependent variable**. A dependent variable is a value that depends on other factors.

Concept Background: Ask students to visualize an arcade filled with pinball machines. Each quarter that players put into a machine gives them three minutes of playing time. The amount of time never changes, but the number of quarters that players put in the machines does. Explain that the value that never changes is the independent variable, and the value that does is the dependent variable. Ask students to identify the variables in the example.

Develop Core Skills

Core Skill: Solve Real-World Arithmetic Problems

Ask students to imagine driving a car that travels 30 miles for each gallon of gas. Have pairs of students draw and complete a data table to show how many miles a driver could travel on 1, 2, 3, 4, and 5 gallons of gas. Afterward, ask students to identify the pattern in the table. Explain that the number of gallons and the number of miles are examples of real-world data that are linearly related. Invite students to suggest other real-world data sets that show a linear relationship.

Core Skill: Solve Linear Equations

Ask students to imagine they are professional painters. One gallon of paint, which costs \$22.50, covers 200 square feet of wall space. Ask students to draw a graph to show the linear relationship between the dependent and independent variables.

Point out to students that the independent variable is the number of gallons, and that the dependent variable is the cost. Tell students that they can express the number of gallons and cost as coordinate pairs, and then plot their coordinates in the coordinate plane. Then ask students to identify the cost of a specific number of gallons of paint, and have students refer to their graphs to find the answer.

Pre-Teach Vocabulary

Guess the Word

Have students focus on vocabulary such as *slope*, *rise*, *run*, *dependent*, and *independent* that may already be familiar to them. Have students share their own definitions of these words. Then explain how they will learn the definitions of those words in a mathematical context.

Tier 2 Words:

dependent variable
(p. 166)
independent
variable (p. 166)
rise (p. 171)
run (p. 171)
slope (p. 170)

Tier 3 Words:

linear relationship
(p. 166)
y-intercept (p. 170)

Test Word:

linear equation
(p. 170)

DURING THE LESSON

Linear Relationships

PAGE 166

Read the first two paragraphs of "Linear Relationships" to the class and set up the problem for them. Explain that you will work through the problem together in the subsequent pages.

Work with the class as a whole, or organize students into small groups, combining students with different abilities. Work through the problem step-by-step, relying on the visuals to understand the text. If students are working in groups, walk among them to listen to their discussions and to determine where intervention might be necessary.

Discuss the coordinate planes that reflect linear relationships.

PAGE 168

Core Skill: Solve Real-World Arithmetic Problems

Before reading the text as a class, explain that mathematical concepts, including linear relationships, are applied to many real-world situations. Read this section as a class and help students understand the data in the data table. Invite a volunteer to answer the problem.

Complete a Data Table

PAGE 169

Have students work in pairs to use the graph to complete the data table. After students complete the table, have them describe the pattern that appears in the data. Ask students to explain why this pattern confirms that the relationship between the number of texts and the monthly charge is linear.

Linear Equations

PAGE 170

Explain to students that they have used graphs to successfully represent linear relationships, and now they are going to use graphs to do the same with linear equations. Guide students through the text, helping them differentiate between the dependent and independent variables. Ask a volunteer to identify the y-intercept and explain what it represents.

Direct students' attention to the graph at the top of page 171. Point out the labels on the axes, the coordinate points, and the solid line that connects the points. Work through the section with students, helping them distinguish between the rise and the run, and assisting them as they use the rise and run to calculate the slope of the line.

21st Century Skills: Critical Thinking and Problem Solving

Read the text as a class to help students understand how linear equations apply to areas other than mathematics. Give students time to graph how often and for how long they use any electronic device throughout an ordinary day. Invite students to share and explain their graphs, and to describe any patterns that might have emerged from the data.

PAGE 171

THINK ABOUT MATH

ANSWER KEY

The slope of the line is 25.

Engage and Extend

ELL Instruction: Restate Definitions Have students look at the graph at the top of page 171 again. Have them underline the words *rise* and *run*. Invite students to share what they know about these words and to model the words in pictures or actions. Then ask students to explain the meaning of each word in the context of the graph.

Core Skill: Solve Linear Equations

PAGE 172

Explain to students that the problem in the main section of the page works hand-in-hand with the sidebar activity. Read the problem for students. Then direct attention to the Core Skill sidebar.

You may want to pair students of different abilities to work through the steps outlined in the activity to calculate the slope of the suspension springs. Otherwise, read each step and give students time to complete it before moving on.

PAGE 173

MATH LINK

In this Math Link, students continue their explorations into compliance. Help students work through the process of predicting the value of the dependent variable, or downward movement of the suspension, given the value of the independent variable, or applied weight.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 412.

Extension Activity: Investigate a Multi-step Problem that Involves Planning and Reasoning Since this lesson contains examples of linear relationships whose slopes are positive, have students investigate, describe, and solve real-world linear relationships for which the slope is negative, so that students appreciate their relevance. Share the example of how the elevation decreases with time as a runner runs downhill.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Use the point-slope form to graph the equation of a line
- Use the slope-intercept form to graph the equation of a line
- Use the two-point form to graph the equation of a line

✓ Determine Student Readiness

In this lesson, students learn to graph linear equations. To determine student readiness, give students an opportunity to explain a graphical representation of a linear relationship. Project the graphs on pages 170–171 onto the board, or have students examine the graphs in their books. Ask questions about the graphs, using the terms *y-intercept*, *slope*, *rise*, and *run* in your questions. Use students' responses to determine whether students need additional practice using graphs to explain linear relationships before beginning this lesson.

Key Concept

There are two ways to graph a linear equation. (1) If two coordinate pairs that lie on the line are known, then the graph of the line can be constructed, or (2) if one coordinate pair that lies on the line and the slope of the line are known, then the graph of the line can be constructed.

Concept Background: Explain to students that the methods for graphing a linear equation on the coordinate plane described in the Key Concept depend on obtaining two coordinate pairs. Those two pairs make it possible to draw a line on which both the points lie. If only one coordinate pair is known, it is possible to use that pair and the slope of the line to find the second coordinate pair.

Develop Core Skills

Core Skill: Perform Operations

Tell students that multiple operations are required to determine the slope of a line, and that sometimes the result is a negative number. Provide problems in which students apply more than one operation, resulting in a negative number. For example, ask students to simplify the expression $\frac{(1.6 - 3.3)}{(9.1 - 4.2)}$. Guide students as they find the solution, $-\frac{1.7}{4.9} = -0.35$.

Core Skill: Interpret Graphs and Functions

To help students interpret graphs and functions, real-world examples are often effective. Ask a volunteer to check the currency conversion rate between the US dollar and British pound, for example. Then, as a class or with students working in pairs, create a graph that reflects the data.

Pre-Teach Vocabulary

Relate Word Parts

List the following words or word parts on the board: *inter*, *intercept*, *point*, *slope*, and *sub*. Next to each word or word part, write its history: *inter*, Latin *inter* meaning “among, between;” *intercept*, Latin *interceptus* meaning “to take or seize between or in passing;” *point*, Old French meaning “dot, smallest amount;” *slope*, Middle English *aslope*, meaning “on the incline;” and *sub*, Latin *sub* meaning “under, close to, up to, toward.” Review the word histories, and ask students to use them to help define their vocabulary words. At the end of the lesson, invite students to revise or add details to their definitions.

Tier 2 Words:

intersect (p. 179)
subscript (p. 181)

Tier 3 Words:

gradient (p. 177)
point-slope form (p. 178)
slope-intercept form (p. 179)
two-point form (p. 180)

DURING THE LESSON

Graphing Linear Equations

PAGE 177

Use the text and the visuals on this page to review the definition of a linear relationship. Ask students to explain in their own words why the relationship between time and distance is linear, but the relationship between temperature and altitude is nonlinear.

Guide students through the text, assisting them as they use the slope of a linear relationship to plot a second coordinate pair on a coordinate plane. Pause occasionally to ask volunteers to relate the visuals to the text to help strengthen their understanding of the text.

Core Skill: Perform Operations

Before reading the text, review the task students completed on the page. Ask them to identify the multiple operations they performed to find a second coordinate pair. Then read the Core Skill sidebar as a class. Ask students to explain the relationship between the following statements:

$$\text{slope} = \frac{\text{rise}}{\text{run}} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}$$

PAGE 178

THINK ABOUT MATH

ANSWER KEY

The rise = $45 - 35 = 10$.
The run = $10 - 5 = 5$.
The slope = $\frac{10}{5} = 2$.

Point-Slope Form

Work through finding the point-slope form with students. As you discuss and solve the problem as a class, refer to the graph frequently to help support student understanding. Have students use a linear equation ($y = mx + b$) to check the point-slope form of the equation of a line.

Core Skill: Interpret Graphs and Functions

The exercise in the Core Skill sidebar gives students an opportunity to apply the point-slope formula to graph a real-world linear relationship. Guide students through the exercise step-by-step, inviting volunteers to create a checklist of important information that they can use to graph and solve the problem.

Slope-Intercept Form

PAGE 179

Before reading and discussing the text as a class, explain that the slope-intercept form is a special case of the point-slope form, where the point also happens to be the y-intercept. Guide students through the text, referring often to the visual solution for support.

21st Century Skills: Critical Thinking and Problem Solving

Read the text as a class, helping students recognize that they can plot real-world data that involves personal behavior, such as spending. Ask students to describe how they can use trends in their past and current personal behavior to predict or alter future behaviors.

Two-Point Form

PAGE 180

Help students use two coordinate pairs to find the equation of a line. Guide students through the text, pausing to examine the graph and determine how it supports the text. Help students perform multiple operations to write a linear equation using the two-point formula. Ask students how they can check their work and invite volunteers to demonstrate.

Engage and Extend

ELL Instruction: Retell with Visuals Invite students to choose one of the forms for finding the equation of a straight line—point-slope form, slope-intercept form, or two-point form. Ask students to explain the form to you in their own words and to include related visuals in their explanations. Offer support if students are struggling to understand these concepts.

PAGE 181

MATH LINK

In this Math Link, students are reminded of the purpose of the subscripts in the point-slope form. Walk students through the sample problem. Then invite volunteers to offer new values for the slope and new points to give students an opportunity to practice using point-slope form to find the equation of a line.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 413.

Extension Activity: Identify Patterns in Data Have students find a stock market chart in a newspaper or online. Explain to students that the market index or price of a particular stock can change from hour to hour or day to day. Ask students to calculate the slope of line segments that indicate these changes. Remind students to include correct units when calculating the slope, for example, dollars per day.

Pairs of Linear Equations

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Solve systems of two linear equations
- Interpret graphs of two linear equations
- Use linear equations to solve problems

✓ Determine Student Readiness

In this lesson, students learn to graph two simultaneous linear equations. To determine student readiness, distribute graph paper and have students draw a coordinate plane. Give them two points on the plane and have students to use the points to determine and graph an equation of the line. Observe students as they work to determine if more practice would be helpful before beginning the lesson.

Key Concept

A pair of linear equations forms a system of two simultaneous linear equations. The solution to a system of two linear equations in two variables corresponds to a point of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Concept Background: Tell students that two variables have a linear relationship if their corresponding points lie on the same line in a coordinate plane. Every pair of coordinate points on a line is a solution to a linear equation that represents the line. Have students graph a line (for example, $y = 3x - 1$). Then have students find a point that is on the line ($(2, 5)$, for example) and have them substitute the value of x and y into the equation to see that the point is a solution to that equation.

Develop Core Skills

Core Skill: Solve Pairs of Linear Equations

Explain to students that there is a relationship between variables in simultaneous equations. Although each line has an infinite number of points, there is only one point at which the two lines will intersect. Write $x + y = 10$ and $2x + y = 14$ on the board. Have students graph both equations to find the point at which the two lines will intersect $(8, 2)$.

Core Skill: Solve Simple Equations by Inspection

Tell students that it is sometimes possible to solve simple equations by inspection, or looking at them and completing mental math. Write the following equations on the board and ask students to solve them by inspection: $x - 4 = 10$; $9x = 63$; $7x = 42$.

Pre-Teach Vocabulary

Use Context Clues

Organize students into groups. Give each group 6 notecards and have students write one vocabulary word on each card. Then provide definitions of the words. Ask students to write the definitions on the backs of their vocabulary word card. Then collect and shuffle the cards. Use the cards in warm-up or exit activities and again as a review for the lesson.

Tier 2 Words:

simultaneous
(p. 184)

Tier 3 Words:

addition method (p. 187)
equilibrium point
(p. 186)
substitution method
(p. 186)
system of simultaneous
linear equations
(p. 184)

Test Words:

eliminate
(p. 187)

DURING THE LESSON

PAGE 184

Simultaneous Linear Equations

Guide students as they read the explanation of simultaneous equations. Prompt their thinking by asking relevant questions (“What does *simultaneous* mean?”, “What does it mean for two equations to be simultaneous?”, etc.). Then explain that the term *system of simultaneous linear equations* implies that the solutions of the equations are dependent on each other. A point of intersection is the solution of simultaneous linear equations.

Assist students as they work through this real-world model of a system of simultaneous linear equations.

Ask them to compare the financial advantages of the two different text-messaging plans.

Tell students to refer to the data table on page 185 to understand the text and the graphical representations of the two text-messaging plans. Point out the term equilibrium point on page 186, and ask students to identify it on the graph and define it in their own words. Help them understand that it is the point at which the two lines intersect, which represents the solution of the simultaneous equations.

THINK ABOUT MATH

ANSWER KEY

Option 1: The total monthly cost would be \$105.00. Your half of the bill would be \$52.50.

Option 2: The total monthly cost would be \$45.00. Your half of the bill would be \$22.50.

Core Skill: Solve Pairs of Linear Equations

Complete the exercise in applying the substitution method for solving a pair of equations without graphing as a class, or have students work through the process independently or in small groups as you observe. Have students apply the substitution method to the equations at the bottom of the sidebar. Then ask them to explain how they applied the method to find the solution to the pair.

Combining Methods to Solve Pairs of Linear Equations

In this section, students learn how to combine the addition method with the substitution method to solve a system without graphing. Walk students through the process, helping them see that by using addition to solve one equation, the solution can be substituted into the remaining equation.

Core Skill: Solve Simple Equations by Inspection

Read the text with students to help them recognize two special cases in which a single solution for a pair of simultaneous equations cannot be obtained. Emphasize that inspection alone is enough to reach this conclusion. In the first example, the left-hand side of both equations is the same ($3p + 2q$), but the right-hand side is different. Plotting these equations would result in parallel lines. Because the lines do not intersect, there is no solution. In the second example, the first equation is a multiple of the second equation, thus making the equations equivalent. This results in identical equations, which have infinite solutions. Guide students through a discussion of the third case.

Engage and Extend

ELL Instruction: Identify Multiple Meanings Write the word *equilibrium* on the board. Ask students to use a print or online dictionary to define the word. Then explain the word's meaning as it applies to children on opposite sides of a seesaw or a circus performer on a high wire. Also explain how equilibrium applies to one's mental health or between two sides of an argument. Finally, ask students to explain the word's meaning as it applies to mathematics.

MATH LINK

Give students an opportunity to think critically about whether the addition method is an appropriate way to eliminate a variable in the given pair of equations. Help students recognize how multiplying one equation by -1 helps them eliminate the variable b , making it possible to solve for a . $a = 1$; $b = \frac{1}{2}$, or 0.5 .

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 413.

Extension Activity: Use Concepts to Solve Non-Routine Problems

Have students work collaboratively to generate examples of when the intersection of two graphs has practical relevance, such as the point at which the orbit of the earth and the orbit of a meteorite intersect, indicating a point at which the meteorite falls to Earth. Encourage students to research their topic and graph lines to indicate an equilibrium point.

BEFORE THE LESSON

Objectives

- After completing the lesson, students will be able to
- Describe the information that a trend line provides about two correlated variables
 - Describe various aspects of the correlation between two variables

✓ Determine Student Readiness

In this lesson, students learn to plot data on a scatter plot and construct trend lines to determine the direction and strength of the correlation between the variables. To determine student readiness for the lesson, write a set of points on the board and have students plot them on a coordinate plane. Observe students as they work to offer guidance, if necessary.

Key Concept

We can use the concept of correlation to describe the relationship between two variables that generally follow a linear pattern but cannot be described by a linear equation. Plotting data on a scatter plot and constructing a trend line can determine the strength and direction of the correlation between such variables.

Concept Background: Tell students that correlation is described in terms of direction and strength. Linearly correlated data that has a trend line with a positive slope is positively correlated, whereas if the trend line has a negative slope, the data are negatively correlated. Data that are strongly correlated are located, on average, close to a trend line. Data that are weakly correlated are located farther away from a trend line.

Develop Core Skills

Core Skill: Represent Real-World Problems

Either in class or before coming to class, have students measure their height and arm span. Draw or project a coordinate plane on the board. Have students plot the measurements on the plane to help students see how real-world data often follow a general pattern, but that there will almost always be variability in the data.

Core Skill: Interpret Data Displays

Discuss the scatter plot of arm-span and height data constructed for the Core Skill: Represent Real-World Problems activity. Ask students to describe the data, including how arm-span and height increase together, but the data are widely spaced.

Pre-Teach Vocabulary

Guess the Word

Provide students with vocabulary cards so that every student receives one. Write one vocabulary word on each card, repeating the words as often as necessary.

Have students holding the same word card form groups. Provide a definition for each word. Have students in each group turn their cards over and write the word's definition and draw a visual to help explain the word's meaning. Collect the cards and redistribute them to give each group a complete set of vocabulary words. Have students in each group read and discuss the words, meanings, and supporting visuals. Then read a definition aloud and challenge students to identify the corresponding word. You may want to repeat this activity again after the lesson.

Tier 2 Words:

cluster (p. 195)
correlation (p. 191)

Tier 3 Words:

outlier (p. 195)
trend line (p. 191)

Test Words:

scatter plot (p. 190)

DURING THE LESSON

Scatter Plots

PAGE 190

Explain to students that correlation describes a general linear pattern in plotted data, but not an exact linear pattern as that of a linear equation. Clarify for students that the variability in the data cannot be described by a mathematical equation.

Have students read the first three paragraphs of Scatter Plots.

Direct students' attention to the table showing daily temperatures and snow cone sales on page 191. Help students recognize that the data in the table have been plotted in the scatter plot. Point out that not all of the data points lie on the same line. However, they are close to a line, called the trend line. The trend line is drawn so that the average of the absolute distance between the points and the line is minimized.

PAGE 191

THINK ABOUT MATH

ANSWER KEY

Give students an opportunity to draw a trend line through the points plotted in the scatter plot. Challenge students to draw a line that minimizes the average distance between the points and the line. Ask students to describe the trend.

Linear Correlations

PAGE 192

Explain to students that correlations can be positive or negative.

Read the first paragraph as a class. Then direct attention to the scatter plot on the previous page. Ask: *What kind of correlation exists between daily temperature and snow cone sales?* Help students identify the line as having a positive, upward slope, meaning there is a positive correlation between temperature and sales.

Continue reading as a class to help students interpret a trend line as representative of a negative correlation. Direct attention to the scatter plot representing the number of hours students watch television and grade-point average. Ask: *What kind of correlation exists between the number of hours a student watches television and the student's grade point average?* Help students interpret the line as having a negative, downward slope.

Nonlinear Correlations

Begin this section by explaining to students that correlated data don't have to be linear. Data that follow a nonlinear pattern can also be correlated. Their trend lines are graphs of nonlinear functions, such as quadratics and exponentials.

Read the introduction to "Quadratic Model: Sizes of Rocks Moved by River Currents" to the class. Then direct their attention to the data table and the corresponding data points on the scatter plot. Before reading the last paragraph as a class, invite a volunteer to explain the curve's appearance and interpret its meaning. Then examine the graph again to note the difference in change between the variables, with the size of carried rocks increasing more quickly than the speed of the water.

Guide students through the explanation of an exponential curve on page 194. Begin by directing their attention to the data table and the corresponding data points on the scatter plot. Invite a volunteer to explain the rate of change in the bacteria population over time, based on the appearance of the trend line. Then have students use the text to confirm that change is rapid at first and diminishes as time increases.

Help students understand that correlation strength describes how closely data are to their trend line. Explain that strongly correlated data are closer to a trend line than are weakly correlated data. Data that follow no observable pattern are said to have no correlation. Read the section "Correlation Strength" as a class and take time to discuss what makes the correlation between height and weight in this example weak.

Before reading the paragraph at the top of page 195, examine the scatter plot as a class. Invite students to identify the variables and then use the data to conclude what kind of relationship exists between them. Next, read the paragraph aloud and give students an opportunity to revise their conclusions, if necessary.

Engage and Extend

ELL Instruction: Retell Invite volunteers to identify the scatter plot in the lesson that they found most interesting. Have them explain the scatter plot's variables and the trend line's strength and direction. Invite them to explain why they found this scatter plot so interesting. Assist students if they are having trouble understanding what a trend line represents.

Core Skill: Represent Real-World Problems

Read this exercise together, emphasizing that data have no value unless they are analyzed to look for relationships among variables. Examine the scatter plot as a class, and ask a volunteer to describe the relationship among the data in terms of correlation strength and direction.

PAGE 193

Core Skill: Interpret Data Displays

Give students time to draw trend lines in the scatter plots and use the lines they draw to explain the relationship between each pair of variables.

PAGE 195

Outliers and Clusters

Read the text together and help students consider both the text and the scatter plot to identify the clusters and the outlier. Point out that data points can form clusters along their trend line, resulting in gaps. These gaps may indicate errors in recording that must be investigated further. Also point out that an outlier is a data point within a correlated data set that is much farther away from the trend line than other data points. Like clusters, outliers can represent errors in data recording, but this must be verified.

THINK ABOUT MATH

ANSWER KEY

In general, as height increases, so does weight. There is a positive linear relationship between the variables.

MATH LINK

Have students read the Math Link. Then invite volunteers to use what they read with what they see in the corresponding scatter plot. Emphasize that although the outlier could have represented an error, further investigation indicated otherwise.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 414 and 415.

Extension Activity: Evaluate Two Sources of Information

Have students use print or online materials to find two real-world examples of scatter plots. Ask students to describe the scatter plots' common characteristics and what conclusions can be drawn from trend lines.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Identify a function
- Determine whether an equation represents a function

✓ Determine Student Readiness

In this lesson, students develop their understanding of the concept of a function. To determine student readiness, write examples of simple equations, such as $2x = 18$, on the board and ask students to solve for the variable. Ask students to work in pairs to generate more simple equations, exchange problems with their partners, and solve for the variables. Observe students as they work, intervening if necessary.

Key Concept

You can look at a function as a set of instructions that tells you what to do with the input, or values you put in. The result of the instructions is called the output. Functions are equations that provide only one output for each input.

Concept Background: A function has both an input and an output. A function can be viewed as a set of instructions in which an input value makes it possible to calculate an output value. Functions can be either linear or nonlinear. The vertical line test can be used to determine whether a graph represents a function. Have students graph capital letters of the alphabet to see if any of the letters can be thought of as functions (only V and W are, possibly M depending on how it is drawn).

Develop Core Skills

Core Skill: Build Lines of Reasoning

Write the equations $y = x^2$ and $y = e^x$ on the board. Ask: *Why is y greater than or equal to zero in each of these equations?* Engage students in a discussion, prompting students to insert values to build lines of reasoning.

Core Skill: Interpret Graphs and Functions

Write the equation $y \times 2 = 2x$ on the board. Ask students to generate data points to draw a graph of the equation. Discuss the properties of the graph as a class, including the direction and value of its slope and the y-intercept.

Pre-Teach Vocabulary

Picture Dictionary

Provide students with definitions of the vocabulary words. Organize students into groups. Ask students to work together to use their book or other resources to select a drawing or diagram to represent the word. Have students share and explain their pictures with the rest of the class, without revealing the words the pictures represent. Have other students reach agreement, based on their own work, about which word should be assigned to each picture. Display the pictures for the duration of the lesson or chapter.

Tier 2 Words:

function (p. 200)
input (p. 200)
output (p. 200)

Tier 3 Words:

linear function (p. 202)
nonlinear function (p. 202)
vertical line test (p. 202)

DURING THE LESSON

PAGE 200

What is a Function?

Refer to the pictures that students drew in the vocabulary activity. Point to the pictures representing *function*, *input*, and *output*. Review the definitions before reading the text as a class. Have students look at the illustration at the bottom of the page, and tell them that they can view a function as a box that has an input and an output. The box contains the instructions for what to do with the input, so that the output can be calculated.

PAGE 201

Is It a Function?

Read the text as a class, referring to the graph of $y = 40x + 2,000$ on page 200 during the discussion. Ask: *Which variable represents the area, or size of the construction project? (x) If $x = 1,500$ square feet, what are the builder's construction costs?*

Next, remind students that a function has only one output for each input. Have students read the rest of the text, and use the graph to help explain why the equation is not a function.

Use the text at the top of page 202 to introduce students to the use of the vertical line test to determine whether a graph represents a function. Return to the graph on page 201. Ask students to explain how the vertical line test tells them that the equation represented by the graph is not a function.

Core Skill: Build Lines of Reasoning

To reinforce the concept of a function as a set of instructions, have students read the text and think through the process of writing instructions for the function that converts temperature from Fahrenheit to Celsius. Next, have students share their instructions and explain their thinking. Discuss students' solutions as a class, ask relevant questions, and form a consensus about the best instructions. Have students test the instructions for accuracy.

Function Categories

PAGE 202

Remind students of the basic form for the equation of a linear function that they applied in earlier lessons: $y = mx + b$. Explain that if a function does not have this form, it is nonlinear.

Perimeter of a Square

Have students read the text and explain why the perimeter of a square represents a linear function. Ask students to include the diagrams and graph in their explanations.

Workplace Connection: Computer Programming Input, Output Values

Read the text together and ask students to summarize the content, emphasizing the function at the core of the software product. You may want to project a search browser and show students other examples of input-output functions, such as converting clothing sizes; monetary currencies; measurements; and speed.

Area of a Square

PAGE 203

Ask students to compare perimeter, which is a linear function, to area, which is a nonlinear function. Ask students to include the graphs in their comparisons.

Engage and Extend

ELL Instruction: Picture Dictionary Review Ask students to show and explain the picture dictionary entries they created before the lesson. As students talk about their entries, prompt them to discuss what revisions they might want to make to better explain the words the pictures represent now that they have learned more about functions.

THINK ABOUT MATH

ANSWER KEY

- A. linear function
- B. nonlinear function
- C. nonlinear function
- D. nonlinear function

Core Skill: Interpret Graphs and Functions

Ask students to read the text and summarize the value of examining the form of a function, that is, whether it is represented by $y = mx + b$, to determine whether a function is linear or nonlinear.

PAGE 204

MATH LINK

Read the text with students and help them use the hint to determine that the equation for the circumference of a circle is a function, because no matter what value is inserted into r , there is only one output, or C . The same is true for the equation for the area of a circle.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 416.

Extension Activity: Interpret Information from a Complex Graph

Have students research and find graphical representations of exponential functions. Have students use two-column charts to calculate data points and then plot the data points on a graph.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand and write ratios
- Understand and find unit rates and unit prices

✓ Determine Student Readiness

To determine student readiness for this lesson, write a variety of fractions on the board and have students use greatest common factors to simplify them. Observe students as they work, and ask questions to determine if intervention is required.

Key Concept

Understand how to write ratios to understand the meaning of a unit rate.

Concept Background: Explain to students that they probably use ratios and unit rates regularly. For example, they may use ratios when they follow a recipe. They may pay attention to unit rates when they purchase food in a grocery store, or pump gas at a gas station. Both ratios and unit rates use the word *per* in their descriptions. For example, a recipe may call for one egg per cup of flour. Gasoline may cost \$3.87 per gallon. Have students suggest other familiar examples of ratios and unit rates.

Develop Core Skills

Core Skill: Understand Ratio Concepts

Explain to students that even complex problems can be restated and broken down into simpler parts. Talking through a complex problem alone or with a partner can help you identify the steps you need to take to solve the problem. Ask students to describe times when they have collaborated in this way to solve complex problems.

Core Skill: Use Ratio and Rate Reasoning

Ask students to think about how they decide whether something is a good buy. They might compare prices at different stores, or compare an individual purchase with buying in bulk. Ask them how they would compare the prices of a 3-pack of an item with an 8-pack of the same item. Lead students to understand that it is easiest to compare the prices of single items. Explain that the price per item is called the unit price.

Pre-Teach Vocabulary

Word Maps

Draw a model of a word map on the board. Begin by drawing a large square. Divide the square into four equal parts. Draw a smaller square over the space where the perpendicular lines intersect. Erase the lines within the smaller square, and write: *A vocabulary word goes here*. Then label the outside corners: *Definition*; *Picture*; *Example*; *Sentence*. Have students copy the model, replacing the directions in the center square with any one of the words from the vocabulary list. Give students time to add information to their maps as you go through the lesson.

Tier 2 Words:

rate (p. 215)
ratio (p. 212)

Tier 3 Words:

unit price (p. 215)
unit rate (p. 215)

Test Word:

paraphrase
(p. 213)

DURING THE LESSON

PAGE 212

Understand Ratios

Read the section with students, and walk through Example 1 as a class. Discuss the three ways to express a ratio. Challenge students to use classroom examples to write ratios. For example, students might say that the ratio of students with brown eyes to blue eyes is 8 to 12.

Invite students to give examples of when ratios might be used in common, real-life situations.

Give students an opportunity to work in pairs to work through Example 2 on page 214. Then ask them to summarize the steps they followed to write a ratio.

PAGE 213

Restate or Paraphrase Information

Read the section with students. Explain that to paraphrase effectively, they must first be sure they understand the purpose of the information. Then they will need to restate the information using their own words. While paraphrasing, it is important to identify key words and important ideas, and then explain them in a clear manner.

Core Skill: Understand Ratio Concepts

Allow students time to read the sidebar. Explain that it is important to read problems carefully to know when two or more steps are needed. Then have them choose partners to complete the activity. Encourage them to talk through the steps they need to take to solve each problem.

THINK ABOUT MATH

ANSWER KEY

1. $6:7, \frac{6}{7}$
2. 1 to 50, $\frac{1}{50}$
3. 10 to 19, 10:19
4. $\frac{1 \text{ can concentrate}}{3 \text{ cans water}}, \frac{1}{3}, 1:3, \text{ or } 1 \text{ to } 3$
5. $31 - 23 = 8$, so the ratio is $\frac{23 \text{ snow days}}{8 \text{ no-snow days}}, \frac{23}{8}, 23:8, \text{ or } 23 \text{ to } 8$

21st Century Skill: Critical Thinking and Problem Solving

Explain to students that being a good problem solver means being willing and able to look at a problem in different ways. Ratios describe relationships. They can describe the relationship between a part and the whole or between two (or more) parts. By looking at different parts of a problem, it is often possible to find more information than seemed obvious at first glance. Give students time to analyze the problem and discuss a reasonable answer. Then have students explain their thinking and revise their answers, if necessary.

MATH LINK

Discuss the text as a class. Then have students practice converting fractions to ratios.

Understand Unit Rates

Read the text as a class. Then help students practice finding unit rates and unit prices for everyday goods and products. Emphasize that being able to solve for unit rates can help students find better values, even when products are not the same price or size. For example, a large box of cereal may cost a bit more, but cost less per ounce. If students need more practice, have them identify unit rates for goods in a local store newspaper circular. Students can practice comparing unit prices of goods and also practice using unit prices to see how much of a product they could purchase with various amounts of money.

Engage and Extend

ELL Instruction: Connect to Real Life Distribute advertisements from a local grocery store or other retail store. Ask students to identify examples of ratios and determine the unit rates for the items they find. Encourage students to use the terms *ratios*, *fractions*, and *unit rates* in their discussion.

THINK ABOUT MATH

ANSWER KEY

1. 22.5 miles per gallon
2. \$6.50 per hour
3. \$0.30
4. \$0.09

Core Skill: Use Ratio and Rate Reasoning

Ask a volunteer to read the sidebar. Point out the example that tells students to calculate the cost of a single shirt by multiplying the numerator and denominator of the fraction $\frac{128}{4}$ by $\frac{1}{4}$. Ask students if they can think of another way to describe how to perform that calculation. Students will probably notice that they would get the same result by dividing by four.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 418.

Extension Activity: Draw Conclusions from Data Give students an opportunity to conduct research, either independently or collaboratively, to find examples of ratios in different areas. For example, students might examine geographic, economic, medical, manufacturing, consumer, entertainment, and sports data. Have students display and explain the ratios they find.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Use unit rates to solve mathematics problems
- Interpret representations of proportional relationships

✓ Determine Student Readiness

This lesson helps students understand the concept of a unit rate, which is a special kind of ratio. To determine student readiness, use words to describe ratios, such as three oranges for every five apples. Have student students translate words into numbers, writing ratios and simplifying them to write them in lowest terms.

Key Concept

A unit rate is a special example of a ratio. When it is expressed in fractional form, the denominator equals one. When expressed verbally, a ratio is an example of a unit rate if the second value being compared is one.

Concept Background: Proportional relationships are related to ratios, which students examined in the previous lesson. Like ratios, unit rates are used to compare two different types of quantities. Write an example of a unit rate in words on the board, such as 33 miles per gallon. Then rewrite the rate in fraction form: $\frac{33 \text{ miles}}{1 \text{ gallon}}$.

Develop Core Skills

Core Skill: Compute Unit Rates Associated with Ratios of Fractions

Explain to students that unit rates are special kinds of ratios, with 1 in the denominator. Like other ratios, they can be written in different ways. For example, tell students to imagine that in a grocery store, apples cost \$2.39 per pound. That is a unit rate. Ask students to write the fraction form of the unit rate. It would be $\frac{2.39}{1}$.

Core Skill: Evaluate Reasoning

Write the word *reason* on the board. Explain that the word has a long history and comes from the Latin word *rerī*, meaning “to think, or reckon.” The word *reasoning* came centuries later, and was used to mean “the act of or an example of thinking logically.” Next, write the word *evaluate* on the board. Explain that this word comes from the French word *évaluer*, meaning “to find the value of.”

Ask student to use the words’ etymology to explain what critical thinkers do when they “evaluate reasoning.” Help students understand that when they evaluate another person’s reasoning, they are determining if a process or a conclusion is logical. They should ask questions, such as *Does the solution the person used to solve a problem make sense? Is the conclusion based on sufficient and sound facts?*

Pre-Teach Vocabulary

Word Study: Relate Word Parts

Write the vocabulary words on the board or a chart. Identify the different parts of each word. Use a dictionary or online etymology website to determine the meanings of the parts of each word. Invite students to define the words using the definition of each of its part. Have volunteers write the definitions on the board or a chart.

Tier 3 Words:

constant of proportionality (p. 220)

proportional relationship (p. 220)

DURING THE LESSON

PAGE 218

What Is a Unit Rate?

In this section, students learn that a unit rate is a special case of a ratio, where the denominator is equal to one unit. Before students begin reading the section, have them draw a two-column chart in their notebooks. The first column should be labeled “Unit Rates,” and the second column should be labeled “Not Unit Rates.” Ask students to record examples of both kinds of ratios as they read the text.

Give students time to read the text on this and the following page. Remind them to add information to their charts as they read. Review the table in the main body of page 219 together, and ask students to explain the arrangement of rates in the table. Ask: *What distinguishes a non-unit rate ratio from a unit rate?*

PAGE 219

Converting Ratios to Unit Rates

In this section, students learn how to convert a ratio into a unit rate. Before guiding students through converting the ratio to a unit rate, remind them of how to write equivalent ratios. Tell students that whenever they find equivalent fractions, they multiply or divide both the numerator and denominator by the same number.

Read the text and example together. Ask a volunteer to explain the steps in the example, and how he or she was able to calculate the unit rate. You may want to show students that they can convert the fraction to a decimal to find the unit rate, too. Remind students that a fraction bar between the numbers means “to divide.” Show students how to divide 3 by 4 and multiply by 100 to find the unit rate per song.

THINK ABOUT MATH

ANSWER KEY

The unit rate is $3 \div \frac{1}{2} = \frac{6 \text{ miles}}{\text{hour}}$

Core Skill: Compute Unit Rates Associated with Ratios of Fractions

Have students read the sidebar. You may want to allow student to use calculators to help them calculate the aspect ratios in the example and as they complete the table. Ask: *What do all unit rates have in common?* Remind students that all unit rates have 1 as a denominator.

Proportional Relationships

PAGE 220

Remind students that in the Core Skill activity on the previous page, they used the proportions of width to height to find aspect ratios for television screens. Guide students through the explanation of proportional relationships. Explain the relationship between the data in the table and the information in the graph. Help students see that points representing a relationship between weight and cost were plotted in the graph. The important concepts for students to understand from this example are:

- The graph of a proportional relationship is a straight line, because as one value changes, the other value changes in a consistent manner (proportionally).
- The slope of the line is equal to the unit rate of the quantities $\frac{x}{y}$ ($\frac{\text{cost}}{\text{weight}}$).
- The slope of the line is also called the constant of proportionality.
- The unit rate, the slope, and the constant of proportionality are all the same amount.

Core Skill: Evaluate Reasoning

Give students time to read the text. Then ask volunteers to explain why it is important to include units when writing a ratio. Explain to students that it is helpful to think about whether the answer in the given context is sensible, or reasonable. If they are unsure, they can check the ratio. The correct ratio for this problem is: $\frac{28 \text{ wheelbarrows}}{3 \text{ hours}}$, because the problem asks for wheelbarrows per hour. Have students use the correct ratio to find the unit rate.

Engage and Extend

ELL Instruction: Multiple Meanings Explain to students that the word *proportion* has many uses in the English language. Have students use a print or online dictionary to find different uses of the word. For example, guide students toward a discussion of proportion as it applies to art and design. You may want to have students apply what they learn about proportion to draw a human face or animal.

Apply Proportional Relationships

PAGE 221

In this section, students learn about the relationship between data and equations, and that proportional relationships are linear relationships. Work through the example together. As you work, remind students that k is the constant of proportionality, which is the unit rate.

Show students the steps for the problem about filling the pool:

Let x = time and y = gallons of water

The table shows the relationship is proportional. The water is flowing at a steady pace. Choose one point to find k .

$$k = \frac{y}{x} = \frac{600}{0.5} = 1,200$$

Since the equation for a proportional relationship is $y = kx$, the equation is $y = 1,200x$.

Have students test the equation by substituting a few of the given values for x and using a calculator to determine if the equation yields the given value for y .

THINK ABOUT MATH

ANSWER KEY

To calculate the time it will take to fill the pool:

$$x = \frac{y}{k} = \frac{12,000 \text{ gallons}}{1,200 \text{ gallons/hour}} = 10 \text{ hours}$$

MATH LINK

This Math Link uses an example to reinforce the relationship between the constant of proportionality—as expressed in an equation and evident in patterns of data—and the slope of the line that represents a proportional relationship.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 418.

Extension Activity: Compare Unit Rates Have students find examples of ratios in online advertisements or in print media and have them convert the ratios into unit rates. In particular, have them look for the same item in different quantities or from different businesses. Then have students use the unit rates to compare values.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand and write proportions
- Solve proportions

✓ Determine Student Readiness

In this lesson, students learn to solve proportions. To determine their readiness, organize students into pairs. Assign the pairs the fractions $\frac{3}{4}$ and $\frac{1}{12}$. Ask students to determine whether or not they are equivalent. Ask students to tell you how they know. Then have them write equivalent fractions for both fractions. Provide more practice problems, if necessary.

Key Concept

Understand how to use proportions to solve problems.

Concept Background: A proportion is a statement of the equality of two ratios. Present a real-life example of a proportion to students. Say: *36 slices of bread will make 12 club sandwiches. How many slices of bread do I need to make 8 club sandwiches?* Tell students that they don't have to solve the problem now, but after completing this lesson, they will be able to write and solve a proportion to find the answer. Revisit the problem at the end of the lesson, and invite a volunteer to write a proportion and solve the problem.

Develop Core Skills

Core Skill: Represent Real-World Problems

Before the lesson, go online and bookmark visual models of proportion, such as proportional drawings found in architecture, car design, art, and among plants and animals. Project the images you find to emphasize how common proportions are in the world. Ask students to come up with other examples.

Core Practice: Build Solution Pathways

Explain to students that there are often many different ways to solve a problem to get the correct answer. Students can choose any method that they prefer. Ask students to think of a time they worked with other students to solve a problem, and discovered that they used different solutions. Invite students to discuss the problem-solving strategies they prefer to use.

Pre-Teach Vocabulary

Prior Knowledge

Write the vocabulary words on the board, and ask students to identify words or word parts that are familiar to them. Ask students to share what they know. Write their responses next to the vocabulary words on the board. Help students use what they already know to make meaning of the words.

Tier 2 Words:

equivalent (p. 224)
proportion (p. 224)
value (p. 226)

Tier 3 Words:

cross-multiplication (p. 225)

DURING THE LESSON

PAGE 224

Understand Proportions

Explain to student that a proportion is an equation showing that two ratios are equal. When the ratios are expressed in fraction form, the fractions are equivalent.

Guide students through the steps in Example 1. Have students find another equivalent fraction, such as $\frac{9}{12}$, and write the proportion: $\frac{3}{4} = \frac{9}{12}$.

Complete Example 2 as a class, having students apply previously learned skills to determine whether the fractions are equivalent. Then work through Example 3. Have students compare and contrast the two methods and decide which one they like better. Finding a common denominator and equivalent fractions may be more familiar, but cross-multiplication may be simpler.

Have students explain each of the bullet points on the bottom of page 225 to a partner in their own words.

MATH LINK

Remind students that equivalent fractions have the same value. In simplest form, they also look the same.

PAGE 226

THINK ABOUT MATH

ANSWER KEY

1. yes, $\frac{50}{20} = \frac{10}{4}$
2. yes, $\frac{8}{3} = \frac{24}{9}$
3. no

Sample Answers:

4. $\frac{12}{13} = \frac{24}{26}$
5. $\frac{6}{3} = \frac{2}{1}$
6. $\frac{25}{20} = \frac{5}{4}$

Solve Proportions

Read the introduction to Example 4 as a class. Then give students time to complete Example 4 independently, given its similarity to the previous example. Ask volunteers to summarize the steps they followed to determine that the proportions are equivalent. Then guide students through Examples 5 and 6 on page 227. Work through the steps together to be sure students understand the process and how to check their answers.

MATH LINK

Allow students time to read and think about the Math Link. Then have them explain the statements in their own words to a partner.

Core Skill: Represent Real-World Problems

Have students read the first paragraph and identify the proportion in the problem ($\frac{\text{width}}{\text{length}} = \frac{10}{19}$). Then organize students into pairs. Have pairs work together to read the problem, and write and solve the proportion. Afterward, ask one student from each pair to write the proportion and its solution on the board. Compare solutions and resolve any discrepancies that may occur.

PAGE 227

THINK ABOUT MATH

ANSWER KEY

1. $a = \frac{1}{30}$
2. $b = 54$
3. $c = 36$
4. $d = 21$

Core Practice: Build Solution Pathways

Read the first paragraph to students. Invite students to share solution strategies they like to use most often, such as creating visuals or listing steps. Work with students to help them solve Example 4 in the two ways the text suggests. Have students discuss in small groups which method they prefer, and why. Then have them choose a method to complete the sidebar activity.

Engage and Extend

ELL Instruction: Restate Examples Invite students to select an example from the lesson to explain to the class, to a group, or to a partner. Afterward, have the partner or a student in the group or class restate or explain the solution process in his or her own words, using simple vocabulary to explain the more complicated processes.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 419.

Extension Activity: Interpret Information Have students go online to find examples of architectural house or room plans. Have student locate the scales used to create the plans. Then have them use proportions to recreate a plan to double the size of the house or room.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand and write percents
- Change fractions to decimals and decimals to fractions
- Change fractions to percents and percents to fractions
- Change decimals to percents and percents to decimals

✓ Determine Student Readiness

In this lesson, students learn what a percent represents. They also learn how to write fractions and decimals as percents. To determine their readiness for the lesson, have students solve simple division problems using whole numbers and decimals.

Key Concept

Percents, like decimals and fractions, represent part of a whole.

Concept Background: Tell students that a percent is a ratio that compares a number to 100. The symbol for percent is %. 10% represents $\frac{10}{100}$ as a fraction, and 0.1 as a decimal. Say: *Suppose 100 people took a survey on exercise habits. If 10 of the 100 people stated they exercised three times each week, then 10%, $\frac{10}{100}$, or 0.1 of the people surveyed exercised three times each week.*

Develop Core Skills

Core Skill: Interpret Data Displays

Have students recall the various visuals, or visual tools, they have seen or used for organizing and displaying data. Responses may include tables, charts, graphs, and tree diagrams. Have students describe how visuals make it easier to interpret, form conclusions, and make predictions about data.

Core Practice: Construct Viable Arguments

Write the word *viable* on the board. Explain that the word comes from the Latin word *vita*, meaning “life,” and the suffix *-able*, meaning “ability or capacity.” The first meaning of the word *viable* was used to describe a newborn organism’s capacity for survival. Explain that today, the word has a broader meaning. In mathematics, the term means “capable of working, or being a reasonable answer.” Ask students to use their new understanding of *viable* with their prior understanding of *argument* to explain the meaning of a “viable argument” in mathematics.

Pre-Teach Vocabulary

Use Prior Experiences

Have students informally define *percent*, *similarity*, and *repeating* based on prior experience with the words. Have students write the definitions down, and then compare them to the descriptions in the text as they complete the lesson.

Tier 2 Words:

percent (p. 230)
similarity (p. 231)

Tier 3 Words:

repeating decimal (p. 234)

DURING THE LESSON

PAGE 230

Understand Percents

Read the first paragraph as a class. Explain to students that another way to think of percents is “out of a hundred.” Guide students through the steps in Example 1. Lead students to understand that because 40 out of the 100 squares are shaded, 40% of the grid is shaded. If there were more or fewer than 100 squares, the number of shaded squares would not equal the percent.

PAGE 231

Compare and Contrast

Allow students time to read the text. Have students work in pairs to compare and contrast fractions and percents. Each pair should find at least two similarities and two differences. Then have students determine if they found the same similarities and differences while reading the section. Discuss with the class why comparing and contrasting is useful. Responses may include that it improves understanding, helps to differentiate two similar concepts, and helps in decision-making.

Core Skill: Interpret Data Displays

Have students read the sidebar, and then discuss the question at the end as a class. While more students rode the bus than any other means of transportation, it was still fewer than half of the total students. Ask students what other questions they can use the circle graph to answer. Examples include: *Which means of transportation is used the least? Do more students take a car or bicycle? About what fraction of the students take a car?*

PAGE 232

Percents as Fractions

Walk through the example as a class. Tell students that another way to remember how to convert a percent to a fraction is to read the percent sign as “hundredths,” since percents are some quantity out of 100. 14% is fourteen hundredths which equals $\frac{14}{100}$.

Percents as Decimals

Read the introduction with the class. Guide students through the steps in Example 3. Again, encourage students to read percent sign as “hundredths,” and then write the decimal with the same name.

THINK ABOUT MATH

- 0.67
- $3\frac{23}{50}$
- 0.03
- $\frac{2}{5}$

Answers should be calculated to the lowest common denominator.

ANSWER KEY

Core Practice: Construct Viable Arguments

Have students read the text. Then organize students into small groups and have them work together to discuss Ginger’s reasoning. Let students work together to determine the correct tip. Since students have not yet learned to find percents of a number, they might use fractions or decimals. One possible solution is using the fact $15\% = 10\% + 5\%$. 10% equals $\frac{1}{10}$, so to find 10% of the bill, calculate $\frac{1}{10}$ of $30 = \$3$. Since 5% is half of 10% , 5% of the bill must be half of $\$3$, or $\$1.50$. So the total tip should be $\$3 + \$1.50 = \$4.50$.

Decimals as Percents and Fractions

PAGE 233

Guide students through the steps in Example 4. Another way to solve the problem is to write 0.6 as hundredths: $0.6 = 0.60 = 60$ hundredths. So, $0.6 = 60\%$.

Walk through Examples 5 and 6 with the class, making sure students understand all of the steps. Provide additional similar examples if needed.

Fractions as Decimals and Percents

Guide students through Example 7. Explain that they can think of a fraction as a division problem, where the fraction bar represents the division symbol: $\frac{5}{8} = 5 \div 8$. This is a useful concept that they can apply in many other types of problems. The first few times students divide, you may need to remind them how to correctly set up the division problem, since it will seem strange to have a greater divisor than dividend. Each time, model attaching the decimal point and zeros.

Engage and Extend

ELL Instruction: Answer Questions Ask students: *What do percents, fractions, and decimals have in common? Which do you find easiest to work with? Why?*

Assist students as they complete Example 8, which models moving the decimal point. Make sure students understand this shortcut. For Example 9, make sure students understand how to express a remainder as a fraction, as shown in the answer.

MATH LINK

Give students a couple of examples, such as 0.25 and 0.763 , to practice using this shortcut. When there are more than two decimal places, the percent will contain a decimal point: $0.763 = 76.3\%$.

MATH LINK

Have students read this Math Link after completing Example 9. Have students practice by writing the answer to Example 9 using bar notation: $\frac{7}{9} = 77.\bar{7}\%$.

PAGE 234

THINK ABOUT MATH

ANSWER KEY

- 25% , 0.25 , $\frac{3}{12} = \frac{1}{4}$
- 66.75% , $0.\bar{6}$, $\frac{2}{3}$
- 6.8% , 0.068 , $\frac{68}{1,000} = \frac{17}{250}$
- 240% , 2.4 , $2\frac{4}{10} = 2\frac{2}{5}$
- 37% , 0.37 , $\frac{37}{100}$
- 0.2% , 0.002 , $\frac{2}{1,000} = \frac{1}{500}$

MATH LINK

Have students read the text and then practice with a few examples, such as 0.2% , 12.5% , and 0.01% .

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lessons page 420.

Extension Activity: Categorize Everyday Examples Have students find examples of real-world situations in which fractions, decimals, or percents are used. Encourage students to look in newspapers and magazines, and also to think of jobs or daily events in which they are used. Then have students create a categorization system for the examples. Ask students to categorize the examples and explain the system.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Write percents as either decimals or fractions to solve problems
- Use proportions to solve percent problems

✓ Determine Student Readiness

In this lesson, students learn how to find the percent of a number, and how to use proportions to solve percent problems. To determine student readiness, write proportions on the board, leaving a numerator or denominator in either fraction empty. Ask students to find the missing values.

Key Concept

Decimals, fractions, and proportions can be used to solve percent problems.

Concept Background: Explain to students that they can use proportions to solve problems involving percents. The basic form of a percent proportion is $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$. Point out that if someone scores 95 points out of 100 points on a quiz, his or her score can be written as $\frac{95}{100}$, or 95%.

Develop Core Skills

Core Skill: Evaluate Reasoning

Remind students that when they evaluate another person's reasoning, they are thinking critically. Ask students to describe some of the informational websites they visit most often or the nonfiction texts they enjoy reading or using as references. Ask students how often they pause in their reading to establish the expertise of the author and the factualness of an author's conclusions. Ask students to share clues they have found in their reading that have made them question the accuracy of the content or the writer's integrity.

Core Skill: Use Percents

Remind students that when they first learned about fractions, they learned to recognize them as comparisons of parts to a whole. Tell them that they can think of percentages the same way, but that all percentages share the same denominator—100. Invite students to give examples of percentages they read or hear about in their daily lives, such as special discounts offered by retailers or interest rates offered by banks.

Pre-Teach Vocabulary

Write Sentences

Write the vocabulary words on the board, and define them for students. Ask students to think about times they have read, heard, or used these words. Then as a class, ask students to help you write a sentence using each word. Write the sentences on the board.

Tier 2 Words:

extremes (p. 238)

means (p. 238)

portion (p. 236)

DURING THE LESSON

Percent of a Number

PAGE 236

Explain to students that they can think of percents as a part of a whole. Read the introduction to students. Explain that in mathematics, the word *of* always means to multiply, so 20% of \$25 is the same as $20\% \times \$25$. Then guide students through the steps of Example 1 on this page, and Examples 2 through 4 on page 237. In Example 3, encourage students to think carefully about the problem. Point out that to solve the problem, they must first translate words into symbols and numbers.

MATH LINK

Make sure students move the decimal point in the correct direction when using this shortcut. Also, assist students as necessary in knowing where to place the decimal in whole numbers.

PAGE 237

THINK ABOUT MATH

ANSWER KEY

1. 87
2. 470
3. 12
4. \$25

Use Proportions to Solve Percent Problems

Read the introduction to students, and have students discuss why the proportion is true. Both ratios show a part-to-whole relationship, since % is part of the whole, 100.

Work through Example 5 as a class, making sure students remember how to use cross-multiplication to solve proportions. Then guide students through the steps in Examples 6 and 7 on page 239. Point out the difference between these problems and those in Examples 1 through 3. Help students see that the problems require different solutions. In Example 6, the percent is missing, and in Example 7, the whole is missing. This is why the proportion is so useful. Have students practice identifying the part, whole, and percent in each problem, giving more examples as needed.

Core Skill: Evaluate Reasoning

Ask students to describe an experience in which they purchased something that came with a discount. Have them explain how they determined what their savings were. Ask: *What did you do to evaluate the problem? What process did you apply to the problem to find your answer? Was your solution reasonable?* Then have students read the text and work with partners to evaluate Silvia's reasoning. Afterward, ask volunteers to share their reactions to Silvia's reasoning.

PAGE 239

THINK ABOUT MATH

ANSWER KEY

1. 9
2. 62.5%
3. 35

Core Skill: Use Percents

Have students read the sidebar and complete the activity. Then have students list the steps they used to solve the percent problems using the proportion. Have them exchange summaries with another student and discuss any differences they find.

Engage and Extend

ELL Instruction: Retell Solutions Have volunteers retell how to identify the part, whole, and percent in a problem. Provide feedback, giving students time to revise their explanations. Allow other students to retell other parts of the lesson as time allows.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 420 and 421.

Extension Activity: Compare Discounts Ask students to imagine being in a department store and being confronted by two possible purchase options: A \$40 sweater is on sale. What is the better discount: 45% off the original price or 25% off the original price and another 20% off the discounted price? Have students write to explain their answers, and encourage them to share their work.

Use Percents in the Real World

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand the interest formula
- Use a formula to find simple interest

✓ Determine Student Readiness

In this lesson, students continue their work with percents. To determine student readiness, write 60%, 6.6%, and 16% in a column on the board. Write 0.066, 0.16, and 0.6 in a second column on the board. Ask a volunteer to match one of the percents to its equivalent decimal. Then give students an opportunity to match the remaining pairs of numbers. Use students' answers to determine if a review is necessary.

Key Concept

Simple interest can be calculated using a formula and percents.

Concept Background: Explain that when you deposit money into a savings account you are actually lending that money to the bank. In return, the bank pays you money, called interest, for the use of your money. Have students discuss the experiences they have had with interest.

Develop Core Skills

Core Skill: Make Sense of Problems

Explain to students that understanding a problem is the first step toward solving it. In other words, students must interpret the problem, or make sense of it. Once they understand a problem, they can begin to plan possible solutions, or strategies for finding the answer. Engage students in a discussion of how they approach word problems. Ask: *What is the first thing you do when you are asked to solve a word problem? What do you do if the problem uses terms or symbols you don't understand? How does taking time to understand a problem before you begin working help you solve the problem successfully?*

Core Skill: Solve Real-World Arithmetic Problems

Engage students in a discussion of real-world percents. Show or project examples of percents found in newspaper, direct mail, or online advertising. Invite students to give examples of times they have found the need to calculate a percent. To prompt discussion, offer your own example, such as a time you calculated the tax on a purchase.

Pre-Teach Vocabulary

Prior Knowledge

Students will probably have seen most of the vocabulary from this lesson in other contexts. Have them list the definitions they already know. As you read through the lesson, relate the students' definitions to the meanings in the text.

Tier 2 Words:

convert (p. 243)
interest (p. 242)
principal (p. 242)
rate (p. 242)
time (p. 242)

Tier 3 Words:

formula (p. 242)

DURING THE LESSON

PAGE 242

Simple Interest Problems

Have students read the first paragraph independently. Ask a volunteer to explain why this type of interest is called *simple*. Point to the interest formula. Give students time to examine the formula.

Work through Example 1 with the class. Remind students of the importance of writing the interest rate as a decimal. Invite students to use a calculator to verify the answer.

Point to the word *convert* on page 243. Explain that numbers of days, months, and weeks can be written as numbers of years. Ensure students understand that one year, 365 days, 12 months, and 52 weeks are equivalent.

Complete Example 2 as a class. Give students time to identify the principal, rate, and time on their own. You may find that students have difficulty understanding the ratio used to convert the units of the time from days to years. Explain that the time, 30 days, is a fraction of a year. Elicit from students why the product of the principal, rate, and time is rounded.

Ask a volunteer to read the last paragraph aloud. Tell students that specific words are used to indicate the units of time. Point to the words *annually* in Example 1 and *annual* in Example 2. Explain that those words are used to point out that the rate is given per year. Point out that the rate is not always given per year. Emphasize that the units for time depends on how the rate is given. Explain that if the rate is given per year, then time should be in years, and that if rate is given per month, then time should be given in months.

Core Skill: Make Sense of Problems

Have a volunteer read the first paragraph aloud. Write the values from the second paragraph on the board. Label each value with the terms used in the interest formula. Ask students to identify the units of each value. Explain that the time units of the values must all be the same in order to calculate interest correctly.

PAGE 243

THINK ABOUT MATH

ANSWER KEY

1. \$4,000
2. \$50
3. \$324,000
4. \$9.04

Core Skill: Solve Real-World Problems

Have students discuss different ways amounts of interest are relevant in their own lives. Ask a volunteer to explain how ratios, percents, multiplication, whole numbers, and fractions, and decimals are applied when calculating the amount of interest in Example 2.

PAGE 244

MATH LINK

Have students think about why an amount of compound interest earned would be greater than an amount of simple interest earned.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 421.

Engage and Extend

ELL Instruction: Make Connections Write the words *principal*, *interest*, *rate*, and *time* on the board. Review the definitions of the words. Explain to students how they can use the Internet to find the annual interest rate for a savings account at a bank. Have students write an interest problem based on the information they learn. Tell students that they will need to determine a principal amount and time on their own. Have students exchange their problem with a partner. Ask students to explain how to solve the problem.

Extension Activity: Collect and Display Interest Terms

Have students research various terms associated with interest, such as *compounded daily* and *APR*. Students should organize the terms in a display with an explanation for each one, using examples to illustrate how these terms affect the amount of interest paid.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Evaluate exponents
- Evaluate arithmetic expressions with exponents

✓ Determine Student Readiness

In this lesson, students learn to evaluate exponents. To determine student readiness, write problems of repeated multiplication, such as $2 \times 2 \times 2$ on the board, and ask students to find the product.

Key Concept

Extend understanding of numbers to exponents and arithmetic expressions that contain exponents.

Concept Background: Exponents are a shorthand way of writing the same factor several times. For example, $3 \times 3 \times 3 \times 3$ can be written as 3^4 . A common error is to multiply the base by the exponent. Make students aware that $3^4 \neq 3 \times 4$.

Develop Core Skills

Core Skill: Evaluate Expressions

Explain to students that in mathematics, an expression is a combination of numbers and symbols. As a group, they represent a value. Write the expressions $12 + 4$; $x - 4$; and $18 \div 3$ on the board. Ask students to identify the values these expressions represent.

Core Skill: Calculate Area and Volume

Draw a cube on the board. Write the following formulae on the board: Area = $s \times s$, or s^2 ; and Volume = $s \times s \times s$, or s^3 . Explain to students that each side (s) of an object like a cube has length and width. The entire object also has height. Each length is a separate measurement, as they can see if they examine the formulae for area and volume. Tell students that they will calculate area and volume in a later chapter. Have them discuss the parts of each formula.

Pre-Teach Vocabulary

Use Visuals

Define each vocabulary word. Organize students into groups. Assign each group a word. Have each group work together to draw a picture that might represent the word. Have students share and explain their pictures to the rest of the class.

Tier 2 Words:

base (p. 250)
power (p. 250)

Tier 3 Words:

exponent (p. 250)

DURING THE LESSON

PAGE 250

Evaluate Exponents

Invite students to identify powers, bases, and exponents in the pictures they drew during the Pre-Teach Vocabulary section. Then read this section together to help students evaluate 2^4 .

Next, complete Examples 1 and 2 and Example 3 on page 251 as a class. Write the powers 4^2 and 7^5 on the board and ask students to evaluate them, first without a calculator and then with a calculator to check their work.

MATH LINK

Read the Math Link to students and ask them to refer to Example 2. Invite students to write other expressions with zero as an exponent, and have other volunteers evaluate the expressions.

MATH LINK

Write the mnemonic device *Super Man Helps Every One* on the board. Explain to students that they can use this memory device to remember the names of the Great Lakes (Superior, Michigan, Huron, Erie, Ontario). Then write PEDMAS on the board. Ask students how this mnemonic device helps them recall the order of operations (Parenthesis, Exponents, Division, Multiplication, Addition, Subtraction).

THINK ABOUT MATH

ANSWER KEY

- | | |
|-------|--------------|
| 1. 64 | 5. 2,097,152 |
| 2. 32 | 6. 7,962,624 |
| 3. 25 | 7. 3,418,801 |
| 4. 27 | 8. 2,985,984 |

Evaluate Arithmetic Expressions with Exponents

Write the mnemonic device PEDMAS on the board, and ask students to make connections between the device and the sequence of steps listed in this section. Next, ask students to complete Example 4 on page 252 independently. Afterward, ask questions to determine student understanding. Invite a volunteer to write a new problem on the board. Challenge students to apply PEDMAS to solve the problem and then compare results. Help students resolve any discrepancies that may occur.

Core Skill: Evaluate Expressions

Write the expression $18 + 2 \times 3 \div 2^2 \times (1 + 5)$ on the board. Ask students to solve it from left to right. Then have them apply PEDMAS and compare the results. Next, read the text together and give students time to write in their notebooks.

THINK ABOUT MATH

ANSWER KEY

- | | |
|-------|-------|
| 1. 36 | 4. 14 |
| 2. 15 | 5. 4 |
| 3. 5 | 6. 18 |

Core Skill: Calculate Area and Volume

Draw or show students a model of a cube. Use the letters l , w , and h to identify the length, width, and height of the cube. Explain that because each face of the cube is a square, each measurement is the same. So, if the length equals 2 inches, the width and height also measure 2 inches. Have students explain the relationship between these measurements and side^2 and side^3 , or s^2 and s^3 . Read the text with students and have them work in pairs or small groups to write their answers to the problem in their notebooks.

Engage and Extend

ELL Instruction: Clarify Exponents Remind students that they cannot multiply a base by an exponent to evaluate an expression. Write some practice problems on the board. Avoid practice problems such as 22, where the exponent and base are the same number. Ask students to identify the base and exponent in an expression before they evaluate it. Have them talk through the process, explaining their thinking as they work.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 423.

Extension Activity: Explain Phenomena in Terms of Concepts Have students research real-life examples in which numbers are squared and cubed. Ask students to present their findings to the class and explain the relationships between the examples and the lesson's concepts.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Find square roots
- Find cube roots

✓ Determine Student Readiness

In this lesson, students learn find roots. To determine student readiness, write the following expression on the board and ask students to find its value: $4 + 3 \times (10 \div 2) - 18$ (1). Ask students to explain how they found the value, encouraging explanations of the order of operations.

Key Concept

Develop and extend understanding of numbers to include the concepts of square roots and cube roots.

Concept Background: Finding square roots and cube roots is the opposite of finding the value of a power. Consider the area of a square and volume of a cube. If those values are known, the square root can be used to find the length of the sides of the square and the cube root can be used to find the length, width, and height of the cube.

Develop Core Skills

Core Skill: Evaluate Reasoning

Explain to students that when they solve problems, they read the problem, think about what it's asking them to do, consider prior experiences solving similar problems, and then select one or more strategies they can use to find the answer. Once they find the answer, they can reflect upon it, evaluating their reasoning. Write the following problem on the board and ask students to work in pairs to read it, choose a solution strategy, find the answer, and then summarize their thinking or reasoning. *Valeria ran 12 miles in 90 minutes. How many miles per hour did she run?* (8)

Core Skill: Interpret Data Displays

Go online to search data displays related to a topic that your students would find interesting, and draw or project the image on the board. Or, ask: *What's your favorite snack?* Record students' answers in a data table. Ask: *Who can tell me what this data display is called? What labels have I included in the data table, and why did I choose them? What is the purpose of a tally mark?* Invite students to examine the data and determine the order of snacks, from most to least favorite.

Pre-Teach Vocabulary

Word Sort

Ask students to define the words *square* and *cube*. Ask a volunteer to explain how they are related. Then have students sort words related to squares and to cubes into two columns in their notebooks.

Tier 2 Words:

square root (p. 254)
squared (p. 254)

Tier 3 Words:

cube root (p. 257)
perfect cube (p. 257)
perfect square (p. 254)
radical sign (p. 254)

Test Word:

cell (p. 255)

DURING THE LESSON

PAGE 254

Find Square Roots

Invite volunteers to read the introductory text aloud, or read the text to students, pausing occasionally to ask students to summarize important ideas. Then walk students through Example 1. Write the problem $n \times n = 25$ on the board. Ask: *How is this problem related to the problem you solved in Example 1?* Have a volunteer write and solve the problem using the radical sign. Then have students use calculators to complete Example 2.

Next, have students approximate a square root in Example 3 on page 256. Ask a volunteer to explain how the class can know that the square root of 150 lies between the square roots of 12 and 13. Then, discuss Example 4 together. Ask: *What makes trying different perfect squares helpful in a problem-solving task like this one? When you squared 17 and found a value less than 324, why did you square 18?*

Finally, remind students of PEMDAS. Point to the letter *E*, and explain that after solving expressions inside parentheses, the next step is to solve exponents and roots. Walk through Example 5 on page 256 as a class. Then write the problem $4 \times \sqrt{4} + \sqrt{16} + (9 - 3)$ on the board, and challenge students to find the answer. (18)

MATH LINK

Read the Math Link to students. Then ask students to explain the contents of the table on page 255. After students have identified the squares in the second column, have them close their books. Call out a number and challenge students to identify its square. Then reverse the process. Call out a square and have students tell you the square root.

Understand a Table

PAGE 255

Have students read the first paragraph in Understand a Table. Then have them examine the table. Ask a volunteer to describe the cells in each column of the table. Next have students read the paragraphs below the table. Have students write a summary of the paragraphs.

Core Skill: Evaluate Reasoning

Have students read the first two paragraphs independently. Afterward, ask students to recall and share experiences in which they worked with others to solve a problem, in math class or elsewhere. Ask students to describe some of the benefits of working with others. Prompt students to recognize the value of talking to others about possible problem-solving strategies, efficient solutions, and whether answers are reasonable. Then read the last paragraph with student. Organize students into pairs, and ask each pair to think of real-world problems that involve square- or cubic-roots. Have students model their problems. Afterward, invite students to show and explain their models and discuss how by working together, they were able to reach sensible and efficient conclusions.

Core Skill: Interpret Data Displays

PAGE 256

Remind students of the data table you constructed to record students' favorite snacks. Then read the Core Skill sidebar together, referring often to the table on page 255. Give students time to write their explanations, and then ask volunteers to share their thinking.

Find Cube Roots

PAGE 257

Model or draw a cube. Point out the shape's dimensions of length, width, and height, and tell students that the length of all edges on a cube are the same, so every length, width, and height is the same. Ask: *What did you learn about the formula for finding the volume of a cube in the previous lesson?* (It can be represented by a power: s^3 .)

Read the introductory text for "Find Cube Root" as a class. Then complete Examples 6 through 8 together as a class before students solve the problems in Think About Math.

Engage and Extend

ELL Instruction: Use a Graphic Organizer As a class, complete a Venn diagram (see the Graphic Organizer section of the Instructor Resource Binder for a blackline master). Label the first circle "Exponents," and the second circle "Roots." Have students suggest items to put in both circles and in the overlapping area. Encourage students to use vocabulary from Lessons 8.1 and 8.2 in the discussion.

THINK ABOUT MATH

ANSWER KEY

- | | |
|-------|-------|
| 1. 2 | 5. 6 |
| 2. 10 | 6. 25 |
| 3. 3 | 7. 4 |
| 4. 15 | 8. 18 |

MATH LINK

Read the Math Link to students. Then write the following problem on the board: $\sqrt{50 - 25}$ and ask students to evaluate the expression. Ask: *What step did you take first to solve the problem? What step did you take second? How would the answer have changed if you had found the square root of each number before you subtracted?*

MATH LINK

Read the Math Link to students. Then invite students to write square roots on the board. Have students use calculators to evaluate each expression. Assist students, if necessary.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 423 and 424.

Extension Activity: Formulate Logical Steps Have students record the steps using a calculator, to determine the square root of a value. Then have students estimate the square roots of the following values. Follow the steps recorded to check their estimates: **1.** 9; **2.** 12; **3.** 16; **4.** 49; **5.** 54

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Translate standard notation to scientific notation
- Translate scientific notation to standard notation

✓ Determine Student Readiness

In this lesson, students learn to write large numbers using scientific notation. To determine student readiness, write the following expressions on the board, and ask students to evaluate them: 10^1 ; 10^2 ; 10^3 .

Key Concept

Develop understanding of large numbers to include scientific notation and how to translate between numbers written in scientific notation and standard notation.

Concept Background: Scientific notation is a convenient way of writing very large and very small numbers, although this lesson teaches only how to use scientific notation with very large numbers. Scientists use scientific notation to express very large and very small numbers in a simpler way. At the end of the lesson, have students list the reasons that scientific notation is convenient to use.

Develop Core Skills

Core Practice: Attend to Precision

Write the word *precision* on the board. Explain that in mathematics, people use different measuring tools, and some are more precise than others. For example, if someone used a metric tape measure to measure the side of a building, they would probably get a measurement close to the true measurement. But their measurement wouldn't be as precise as a measurement taken using a laser tape measure, which is accurate (or close to the true value) to within 3 millimeters. Ask students to give real-world examples of when precision is essential.

Core Skill: Perform Operations

Write the symbols $+$, $-$, \times , and \div on the board. Explain to students that they have used these symbols extensively in this book, as well as in daily problem-solving tasks.

Ask: *What other operations have you learned about in this chapter?* Remind students that they have learned to find square and cube roots, which are also operations. Write the following problems on the board and ask students to perform operations to evaluate them: $\sqrt{49}$; $\sqrt[3]{64}$.

Pre-Teach Vocabulary

Word Maps

Have students use a Vocabulary Map (see the Graphic Organizer section of the Instructor Resource Binder for a blackline master) to explore the lesson vocabulary. Ask students to identify the words they have encountered before. Tell them to share what they know about the words.

Tier 2 Words:

scientific notation (p. 260)
standard notation (p. 260)

Tier 3 Words:

annex zeros (p. 261)
powers of ten (p. 260)

DURING THE LESSON

PAGE 260

Translate Standard Notation to Scientific Notation

Read the introductory text aloud. Write the words *scientific notation*, *powers of ten*, and *standard notation* on the board. Discuss the words' meanings as a class. Point out the examples of scientific notation given in the text. Then work through Example 1 together. Afterward, return to the examples at the end of the introductory text, and ask volunteers to write each number in standard notation (80,000; 21,000,000,000,000,000,000,000; and 527,300)

MATH LINK

Have students read the Math Link and point to specific examples on the page.

MATH LINK

Read the Math Link to students and ask volunteers to find all of the exponents presented on the page. Ask: *Why is there no sign directly in front of each exponent?* Help students understand that if there is no sign, the exponent is understood to be positive.

PAGE 261

THINK ABOUT MATH

ANSWER KEY

1. 18.4×10^4
2. 4.5326×10^8
3. 2×10^7
4. 8.7×10^5
5. 1.265×10^{10}
6. 9.348×10^6

Translate Scientific Notation to Standard Notation

Write the word *annex* on the board. Then next to the word, write: Old French *annexer* (ANN-ex-eh), meaning “to join.” Ask: *What are some synonyms for the word join?* (combine, link, add) Read the introduction to students. Then ask a volunteer to define the term *annex zeros* as it is used in the text.

Next, walk through Examples 2 and 3 with students. Afterward, invite volunteers to write some numbers in scientific notation on the board. Ask other students to work in pairs to write those numbers in standard notation. Compare results as a class. Ask: *What are some examples of numbers that you would find easier to write in scientific notation than in standard notation?*

Core Practice: Attend to Precision

Remind students of the discussion you had at the beginning of the lesson about precision. Then have students read the text and ask: *Why is precision necessary in interpreting numbers written in scientific notation?* Give students time to write their explanations of the pattern they observe in the table. Encourage students to describe what they see.

PAGE 262

THINK ABOUT MATH

ANSWER KEY

1. 310,000
2. 7,000,000,000,000
3. 4,060,000
4. 291,300,000
5. 664,100,000
6. 10,020,000
7. 5,900,000,000
8. 82,200

Core Skill: Perform Operations

Have students to work in pairs to read the text and write both numbers in scientific notation. Ask students to share their work, and analyze any discrepancies that occur with the aim of identifying the source of any error.

Engage and Extend

ELL Instruction: Brainstorm Numbers Organize students into pairs to practice writing numbers using scientific notation. Have each partner write a number using scientific notation, exchange numbers, and rewrite the numbers in standard notation. Ask students to discuss how they reached their answers. Afterward, invite students to brainstorm real-world examples that use numbers so large that they must be written in scientific notation.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 424.

Extension Activity: Collect and Display Data Have students research some very small things in nature that scientists measure, such as the diameter of a red blood cell or the length of a microscopic organism. Have students create a display of some of the examples they find most interesting.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Find the mean, median, and mode
- Find the range
- Understand measures of central tendency

✓ Determine Student Readiness

In this lesson, students learn to identify measures of central tendency in a set of data. To determine student readiness, ask students to collect, sort, and organize data. For example, ask students to generate questions related to student interests, such as favorite Saturday afternoon pastimes, favorite environmental causes, or preferred music styles. Give students time to build data tables, survey students, and record results. Without attempting to analyze the data, invite students to make general observations about what they see, such as which pastime received the most votes.

Key Concept

Understand how data are collected and then analyzed using measures of central tendency and range.

Concept Background: Students can order data to identify trends, or inclinations. They can use data to find patterns, consistencies, and data that lay outside trends. They can use data to come to conclusions and make predictions. For example, weather forecasters use temperature data from previous years as well as the current weather to help make predictions for weather forecasts in the future.

Develop Core Skills

Core Practice: Model with Mathematics

Ask students if they have ever drawn a picture or diagram to explain an idea. Tell them that those pictures and diagrams represent models. Mathematicians use models, too, to support, describe, explain, or extend understanding of a mathematical concept. Just as a picture or diagram can be a model, so can a mathematical equation, a table of values, or a graph. Ask students to describe a model with which they are familiar. Students may say, for example, they graphed linear relationships between hours spent watching television and grade-point average. Or in science class, they drew diagrams to model food webs.

Core Skill: Calculate Mean, Median, and Mode

Write the following data set on the board: {3, 1, 6, 3, 12, 2, 1, 3, 2, 4, 3, 3, 5, 6} Explain that even without knowing what these data represent, it's possible to draw some conclusions. Invite a volunteer to order the numbers. {1, 1, 2, 2, 3, 3, 3, 3, 4, 5, 6, 6, 12}

Ask: *What do you observe about the data in this set? Does one value appear most often? Do any values appear only once? Which values seem to be related? Which value appears to be unrelated?* Explain to students that they will learn to apply measures of central tendency, called the mean, median, and mode, to analyze and form conclusions about what data mean.

Pre-Teach Vocabulary

Use Prior Knowledge

Read the vocabulary words to students. Then ask them to give examples of contexts in which they have read or heard the words before. Prompt students to begin by offering a personal example, such as *I drive to work each day on a highway. A median separates lanes of traffic going in opposite directions. The median is in the middle of the highway.*

Tier 2 Words:

data (p. 270)
range (p. 272)

Tier 3 Words:

mean (p. 270)
measures of central tendency (p. 270)
median (p. 270)
mode (p. 270)

DURING THE LESSON

PAGE 270

Data

Read the text for students to help introduce the concept of data. Help students understand that data are bits of specific information that are often but not always in the form of numbers. Invite students to talk about data as they relate to something of personal interest, such as wins and losses of a sports team.

Measures of Central Tendency

Point out the boldfaced words in the text and remind students of the examples they shared in the Pre-Teach Vocabulary activity. Then assist students as they read the text.

Introduce the text related to Robin's running data. Write Robin's data on the board. Then guide students through the examination of the data in Examples 1 through 4 on page 271. Afterward, ask students to identify the different measures of central tendency and to explain the differences among them. Help students recognize, for example, that the mode in a data set is the item or items that occur most often, while the median is the value in the middle of a data set.

Core Practice: Model with Mathematics

Have students read the text and then explain to you the value of creating charts. Ask: *When you have many data, why is it helpful to “chunk” or group the data in a chart? How does organizing data help you understand what the data mean, or represent?*

MATH LINK

Have students read the text. Write the following data set on the board: {4, 5, 5, 6, 7, 8, 8, 9}. Ask a volunteer to find the median value and to explain the process he or she used.

Range

Introduce the new vocabulary term *range* by reading the introductory text for students. Then walk through the steps of Example 5 with them.

THINK ABOUT MATH**ANSWER KEY**

1. mean: 3; median: 3; mode: 3 and 4
2. mean: 37.6; median: 34; mode: 35
3. \$525
4. 21

Core Skill: Calculate Mean, Median, and Mode

Have students read the first paragraph. Ask: *Why are the mean, median, and mode called measures of central tendency? What makes the word central significant?* Have students work in pairs to identify the measures of central tendency in the data set. Compare results and ask students to explain and resolve discrepancies.

Engage and Extend

ELL Instruction: Analyze Results Take a survey among students. Ask them to vote for their favorite flavor of ice cream. Put the results in a tally table. Then have a volunteer interpret the tallies to build a data set. Ask students to find the range and measures of central tendency among the data, explaining their thinking as they share, and using the correct vocabulary terms. Afterward, ask students to state any observations they made about the data, such as students' most or least favorite flavors. Encourage dialogue to give students multiple opportunities to apply the lesson's vocabulary words.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 426.

Extension Activity: Organize, Represent, and Compare

Data Have students research the age of all US presidents at the time of their inauguration. Then ask students to organize the data they collect in a chart and then in a data set. Ask them to apply measures of central tendency and the range of the data to identify patterns and form conclusions.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Read a bar graph, line graph, and circle graph
- Read a line plot
- Understand different types of graphs

✓ Determine Student Readiness

In this lesson, students learn to use graphs to display data. To determine student readiness, organize students into groups. Assign each group a topic, such as favorite cars, movies, TV shows, books, websites, or desserts. Ask students to collect data among themselves and others in the class. Have them organize the data into data tables that they share with the class. Ask volunteers in each group to identify and explain the labels they used in their tables.

Key Concept

Understand how to analyze data presented in a bar graph, line graph, circle graph, or line plot.

Concept Background: Show examples of print and online graphs that your students are likely to find interesting, such as graphs created by other students. In particular, show examples of bar graphs, line graphs, circle graphs, and line plots. Encourage students to compare the graphs to determine similarities and differences.

Develop Core Skills

Core Skill: Interpret Data Displays

Survey students to ask them to identify their favorite reptile. Begin by generating a list of reptiles, such as crocodiles, alligators, Komodo dragons, sea turtles, tortoises, geckos, and a variety of snakes. Use students' choices to construct a data table. Record tally marks to represent students' votes. Ask questions about the data in the table. Then ask students to suggest other ways to display the data that would make the data easier to read and interpret.

Core Skill: Interpret Graphs

Draw or project a bar graph on the board. Select a graph related to students' interests. Organize students into pairs. Have each pair generate two or three questions whose answers can be found by interpreting the graph. Then collect students' questions and pose them to the class. Invite volunteers to answer the questions.

Pre-Teach Vocabulary

Use Prior Knowledge

Have students find words they recognize in the vocabulary list. They will probably know *bar*, *circle*, *line*, *horizontal*, and *vertical*. Draw connections between the words they know and the new vocabulary in the lesson.

Tier 2 Words:

bar graph (p. 274)
circle graph (p. 278)
graph (p. 274)
line graph (p. 277)
line plot (p. 279)

Tier 3 Words:

horizontal axis
(p. 274)
vertical axis (p. 274)

Test Word:

trend (p. 275)

DURING THE LESSON

Bar Graphs

PAGE 274

Read the text with students, making frequent connections between the text and the visuals. Continue helping students integrate the text with visuals as you guide them through Examples 1 through 3 on page 276 and the top of page 277. Ask students to write a question related to the bar graph on page 276 on a piece of paper. Collect the questions and pose them to the class, inviting volunteers to answer. Ask students to refer to the bar graph to justify their answers.

Make Predictions

PAGE 275

Write the word *predict* on the board. Explain that this word is a verb meaning to foretell in advance, or in other words, to tell what may happen in the future. Emphasize that when people predict events, or make predictions, they use personal observations and experiences or scientific evidence to predict.

Read the text as a class. Have students read the passage and predict what will happen on Thursday, given the weather. After students read the passage, invite volunteers to share their predictions before reading the remaining text.

Core Skill: Interpret Data Displays

Write the words *trend* and *relationship* on the board. Ask students to define the terms, as they understand them, and record their definitions on the board. Then read the first two paragraphs together. Give students an opportunity to revise their definitions, if necessary. Then organize students into two groups. Tell the first group that they are buyers for a large department store with stores all over the country. Tell the other group that they are partners in a travel agency. Ask each group to discuss what kinds of data would be important for them to collect and how that data would affect their business decisions. Ask representatives from each group to summarize their discussions.

21st Century Skill: Global Awareness

PAGE 276

Ask students if they have ever observed people holding clipboards approaching shoppers in a mall, moviegoers outside a theater, or diners at a restaurant, asking them to participate in a survey. Ask: *What kinds of information do you think these people are collecting?* (They could be asking opinions on local, national, or global issues such as climate issues.) *Why is this information valuable to them?* (They may be looking to see what public opinion is and the strength of that opinion and how it affects policy-making.) *How might the information they collect affect your voting choices?* Discuss the varied purposes and consequences of data collection.

MATH LINK

Have students read the text. Have students refer to the graph on the page. Ask students to assume that the average monthly temperature in Los Angeles in June is 66 degrees Fahrenheit. Ask a volunteer to explain how they would determine the height of a bar representing this information.

Line Graphs

PAGE 277

Read the introduction to “Line Graphs” as a class, referring frequently to the data table and the data displayed in a line graph. Invite students to use the graph to ask pertinent questions, such as, *What happened between 2006 and 2007 to cause such a rapid increase in the price of one share?* Record students’ questions on the board, and discuss how displays such as line graphs can lead to more questions and conclusions than simple data tables. Continue examining line graphs as students complete Examples 4 and 5 on page 278.

MATH LINK

Have students read the text. Ask a volunteer to identify examples of points and line segments on the line graph on the page.

Circle Graphs

PAGE 278

Read the text as a class and refer to both the data table and the graph on page 279 during the discussion. Complete Examples 6 and 7 on page 279 together.

Engage and Extend

ELL Instruction: Generate Questions Have students generate questions related to the graphs in the lesson. Then organize students into small groups to find answers to the questions they generated. Answer any remaining questions at the end of the activity.

You may want to invite students to collect original data and make their own circle graphs. For example, survey the class to determine what people ate for breakfast or lunch, or what classes they attend during a certain period of the day. Help students find percentages of the total number of students surveyed to complete a data table and corresponding circle graph. Remind students that circle graphs should always add up to 100 percent.

MATH LINK

Have students read the text. Ask volunteers to identify parts of the line graph, including the title, each axis, labels, value intervals along each axis, points on the graph, and the line segments that connect points.

Line Plots

PAGE 279

Work with students to help them understand the elements of a line plot. Read the introduction that begins on page 279 and continue to page 280 to guide students through Example 8.

If you surveyed students to collect data to make a class circle graph, you may want to have students use the same data to make and explain a line plot. Have student determine the range and measures of central tendency in the data they display.

THINK ABOUT MATH

ANSWER KEY

1. nitrogen and carbon dioxide
2. 78 %

Core Skill: Interpret Graphs

Read the text as a class. Then organize students into groups. Invite each group to use print or online resources to collect data of interest to them. Have students work together to organize their data in a data table and then display the data in a bar graph, circle graph, or line plot. Encourage students to analyze their data to determine the range and measures of central tendency. Invite students to interpret those measures and suggest possible predictions that the data support.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 426.

Extension Activity: Graph Data Have students work in small groups to create graphs related to specific activities in their lives, such as how they use their free time, the things they do to earn money, and how they spend the money they earn. Or, if students in any group find they have a different real-world interest, such as endangered species, energy consumption rates, or occupational options, encourage them to research the topic to collect and then display data.

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Understand stem-and-leaf plots
- Identify misleading displays of data

✓ Determine Student Readiness

In this lesson, students learn to interpret stem-and-leaf plots and to identify misleading graphs. To determine student readiness, display a bar graph from a newspaper, magazine, or online source. Ask students to identify the features of the graph, such as the title, axes, labels, number intervals, and bars. Have students work in small groups to use the graph to find measures of central tendency. Give students time to share their work and discuss and resolve any discrepancies that may occur.

Key Concept

Understand how to analyze stem-and-leaf plots and misleading graphs.

Concept Background: Ask a volunteer to describe the relationship between a plant’s stem and its leaves (the leaves form along the stem; the stem supports the leaves). Explain that the same relationship applies to a stem-and-leaf plot. Data points are “leaves” supported by a stem.

Develop Core Skills

Core Practice: Critique the Reasoning of Others

Write the word *critique* on the board. Explain that the word comes from the Greek *kritike techne*, meaning “the critical art.” Then write the Greek word *kritikos*, and explain that it means “able to make judgments.” Next, write the word *reasoning* on the board. Explain that according to historical records, this word first appeared in the 14th century, and it meant “to think logically, or to work through something in a logical way.” Finally, explain that when students critique the reasoning of others, they are making judgments about the logic of someone’s thinking or actions. Ask: *Why is it important to question the logic of someone’s actions, thinking, and conclusions? What is foremost in your mind when you critique someone else’s reasoning? Why should others critique your reasoning?*

Core Skill: Interpret Data Displays

Draw or project a circle graph or a line plot on the board. Select a graph or plot that reflects a particular interest of your students. Organize students into small groups. Have them record questions about the graph or plot for other students to answer. Collect the questions, share them with the class, and invite volunteers to answer.

Pre-Teach Vocabulary

Study Cards

After you provide students with definitions of the vocabulary words, have them create study cards for the lesson vocabulary. Have students write one word on each card, and then turn the card over to write the word’s definition, an example, and a sentence using the word.

Tier 2 Words:

key (p. 282)
leaf (p. 282)
stem (p. 282)

Tier 3 Words:

outlier (p. 284)
stem-and-leaf plot
(p. 282)

Test Words:

mislead (p. 284)

DURING THE LESSON

PAGE 282

Stem-and-Leaf Plots

Read the explanation of stem-and-leaf plots as a class. Then explore Example 1 on this page and page 283. Afterward, invite volunteers to summarize the correct sequence of steps for constructing a stem-and-leaf plot.



MATH LINK

Ask students to explain the connection between the Math Link and Example 1. Ask: *How would the number 8 appear in the stem-and-leaf plot in Example 1?*

PAGE 283

Understand Persuasive Techniques

Write the word *persuade* on the board. Next to the word, write Latin *persuadere*, meaning “to bring over by talking.” Explain the history of the word *persuade*, and ask students to give examples of times they have attempted to persuade someone to do something or to think a certain way.

Read the text together, and give students time to read the passage and decide which responses the questioner hopes to get. Discuss how simply reading questions like these can influence responses. Invite students to suggest some of the techniques they use when they hope to persuade someone of something and ask them how they can apply these suggestions and techniques to problem solving.

Core Practice: Critique the Reasoning of Others

Read the text as a class. Pause to examine the graphs in Example 3 on page 284 to understand the text. Have students work in groups of twos or threes. Direct students to ask questions that will help judge the reasoning of others.

Misleading Graphs

PAGE 284

Have students read the text and answer the questions that follow the graphs. Next, assist students in answering the questions associated with Examples 4 and 5 on page 285.

As a class, discuss why these graphs emphasize the need for critiquing other peoples' reasoning. Also ask: *How will what you have learned about misleading graphs affect the way you represent data visually?*

MATH LINK

After students read about outliers, ask them to explain why 11 is an outlier in the data set presented in Example 2.

Core Skill: Interpret Data Displays

PAGE 285

Have student read the text. Ask: *What are some reasons people create misleading data displays?* Help students recognize that some people may simply make mistakes or not fully understand that how they present data affects people's interpretations of that data. Other people, however, are well aware of what they are doing. Ask: *What do people who deliberately create misleading graphs hope to accomplish?*

THINK ABOUT MATH

ANSWER KEY

1. A circle graph could be misleading if the percents do not add up to 100%
2. The scale does not increase evenly
3. 16.9

Engage and Extend

ELL Instruction: Collect and Display Data Have students work in groups. Give each group an assorted collection of objects, such as colored markers, colorful sticky notes, desk supplies, or different genres of books. Have students work together to use the objects they have in hand to collect and display data. Ask students to put their data in a stem-and-leaf plot and provide a key. Have students explain their data, their plots, and conclusions about their data.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 427.

Extension Activity: Organize, Represent, and Compare Data Have students work in pairs or small groups. Have each group research online sports statistics, such as a basketball team's scores over one or more seasons. Have students organize the data, put the data in a stem-and-leaf plot, and identify the range and measures of central tendency. Have groups compare their stem-and-leaf plots and critique one another's work for reasonableness or logic.