# Math 1496 – Calc I

## Applied min/max Problems

Now we use calc to solve a variety of applied min/max Probs

**Ex)** Find 2 numbers whose sum is 300 and their product is a max.

Sol$^n$ Let the numbers be $x$ & $y$

so (i) $x+y=300$

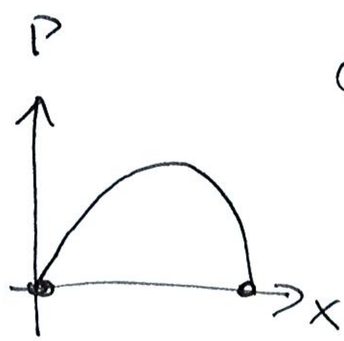
(ii) $P=xy$

we want to max $P$ subject to the constraint

$x+y=300$

so $y=300-x$ & $P=x(300-x)$

$= 300x-x^2$




Calc $P' = 300-2x$

$P'=0$ when $x=150$

to check for a max (or min)

we check the $2^{nd}$ deriv.

$2^{nd}$ Derv. Test

If $f$ is diff & $f'(c)=0$

if $f''(c)>0$ $\cup$ we have a min

$f''(c)<0$ $\cap$ we have a max

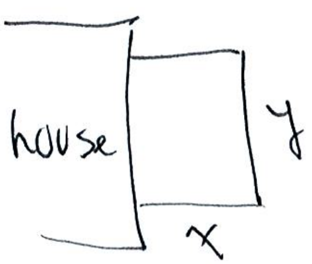
$f''(c)=0$ no conclusion

so $p''=-2<0$ $\cap$ so a max

if $x=150$, $y=300-150=150$

the numbers are 150 & 150

Ex2 A rectangular dog pen is to be built using 100 ft fence. The house is used for 1 side




(i) $2x+y=100$ constraint

(ii) $A=xy$ to max

$y=100-2x$

so $A=x(100-2x)=100x-2x^2$

Calc $A' = 100 - 4x$

$A' = 0$ when $100 - 4x = 0$ $\quad x = 25$

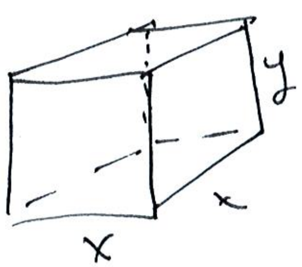
$A'' = -4 < 0$ ∩ max

so $x = 25$ $\quad y = 100 - 2x = 100 - 2(25)$

$= 100 - 50 = 50$

so dem $25' \times 50'$

Ex 3 A manufacture want to design a rect. box (square base) open at the top. Area is 27 sq inch find the dimensions to maximize the volume




$V = x^2 y$

cost $A = x^2 + 4xy = 27$

$y = \frac{27 - x^2}{4x}$ so $V = x^2\left(\frac{27 - x^2}{4x}\right) = \frac{27}{4}x - \frac{x^3}{4}$

$V' = \frac{27}{4} - \frac{3x^2}{4}$ $\quad V' = 0$ $\quad 3x^2 = 27$ $\quad x^2 = 9$

$x = +3$

$V'' = -\frac{6x}{4}$

& when $x=3$ $\quad V'' < 0$ ∩ so a max

with $x=3$ $\quad y = \frac{27-x^2}{4x} = \frac{27-9}{4(3)} = \frac{\cancel{18}^{6}}{4(\cancel{3})} = 3/2$

Dimesin ~~27~~ $3'' \times 3'' \times 3/2''$

In all of these problems, we found max. tomorrow, we will find min.s.

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## Applied Min-Max Prob

You are to min/max something subject to a constraint

(1) draw pic & label

(2) $Eq^n$ of constraint

(3) $eq^n$ of what is to be min/max (target)

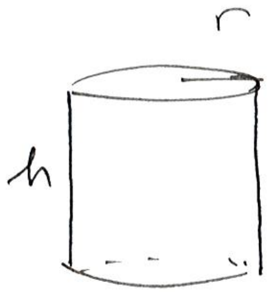
(4) use constraint to get target in terms of 1 variable

5) 1st derv & find CP

6) use 2nd derv test to determine which is max (or min)

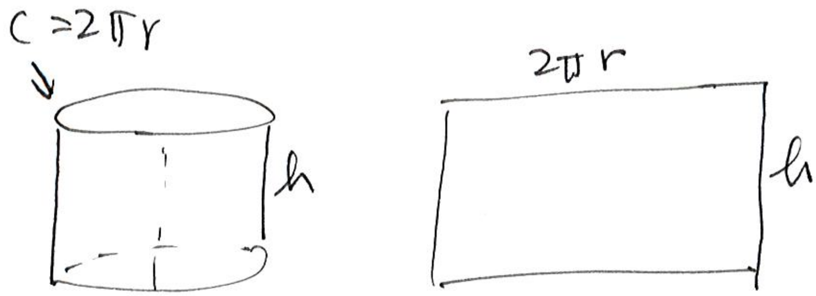
(7) Answer question

Ex 4




$V = \pi r^2 h = 355 \text{ mL}$ $\quad$ $\text{mL} = \text{cm}^3$

$A = 2\pi r^2 + 2\pi r h$ $\quad$ (top, bottom) sides




so $\quad h = \dfrac{355}{\pi r^2}$

$$A = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right) = 2\pi r^2 + \frac{710\cancel{\pi}}{r\cancel{\pi}}$$

$$A' = 4\pi r - \frac{710\cancel{\pi}}{r^2}$$

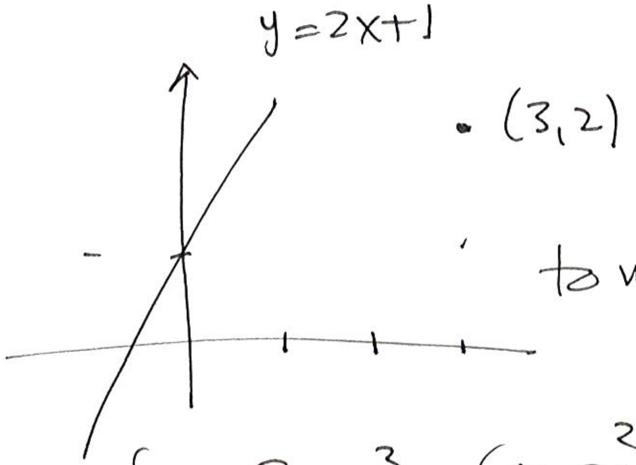
$A' = 0$ when $\quad 4\pi r = \dfrac{710}{r^2} \quad\quad r^3 = \dfrac{710}{4\pi} = \dfrac{355}{2\pi}$

$$r = \sqrt[3]{\frac{355}{2\pi}}$$

$$A'' = 4\pi + \frac{2(710)}{r^3} > 0 \text{ when } r = \sqrt[3]{\frac{355}{2\pi}}$$

$$h = \frac{355}{\pi\left(\frac{355}{2\pi}\right)^{2/3}} = 2 \cdot \sqrt[3]{\frac{355}{2\pi}}$$

## Ex 5 Shortest Distance




$S=\sqrt{(x-3)^2+(y-2)^2}$

to min $S$ is to min $S^2$

let $S = s^2 = (x-3)^2+(y-2)^2$

$= (x-3)^2 + (2x+1-2)^2$

$= (x-3)^2 + (2x-1)^2$

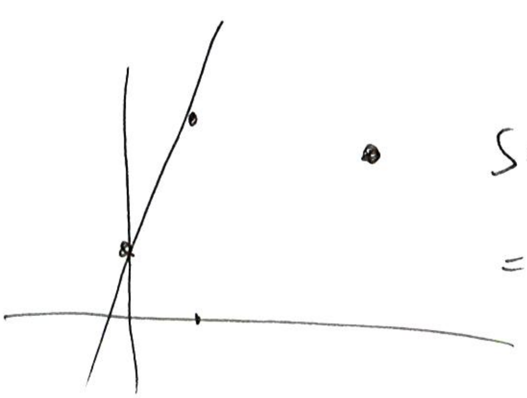
$S' = 2(x-3) + 2(2x-1)(2)$

$S' = 0 \quad 2x-6+8x-4 = 0 \Rightarrow 10x = 10$

$x = 1$

$S'' = 10 > 0$ min

So $x=1,\ y = 2(1)+1$
$= 3$ pt on line $(1,3)$



$S = \sqrt{(1-3)^2 + (3-1)^2}$
$= \sqrt{4+4} = 2\sqrt{2}$