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A Case on Portfolio Risk and Return

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Abstract

This case deals with the calculation of a stock portfolio's risk as well as its expected return. Students will learn about the meaning of risk and its relation with expected return. First, they will compute a portfolio's expected return under different probable states of economy (i.e. recession, normal economy, and expansion). Then, they will compute the expected return of the overall portfolio considering the probabilities of each state of economy. After estimating the portfolio's expected return, students will estimate the portfolio's risk level. As an additional exercise, in order to estimate the expected return of the overall portfolio, students will compute each stock's expected return first and then they will incorporate the portfolio weights. Students will also rank each stock according to its expected return and see if the riskiest stock has the highest expected return. This case is a hands-on experience for students who want to learn about the relation between risk and expected return.

Keywords: Risk, Return, Portfolio
JEL classifications: G11, G17

Introduction

Jan Boyd has recently retired. Besides her 401k account, she has a regular brokerage account that she uses for stock investments. She knows that diversification is good in reducing risk, therefore she is considering forming a portfolio of different stocks. She has consulted with her friends and she has finally chosen four different stocks to invest in. Before making a final decision, she meets with her best friend, Katie. Katie has worked for an investment firm in the past, and she has agreed to help Jan.

"Katie, you are so experienced… Thank you for agreeing to meet with me today" Jan said.

Katie is very happy that she is helping her best friend. But, she is not sure what Jan wants to know. "Are you planning to invest in stocks, bonds, a combination of stocks and bonds, or something else?"

"I don't have too much money. So, it would be meaningless to buy so many different things. I have decided to invest in four different stocks" Jan answered.

"Have you decided what the weight of each stock in your portfolio will be?" Katie asked.

"Yes. I will invest 40% of my money in one stock, 20% in another one, 10% in the third one, and 30% in the fourth stock. I am good with those weights but I am not sure how the economy will be next year" Jan exclaimed.

"Jan, yes, if you want to estimate your return for next year, you need to think about what will happen in a recession, for example. Or if the economy is just flat, what will happen? You may consider different scenarios and try to estimate the chances of these. For example, what is the probability of a recession next year? And if a recession happens, what is your expectation from each stock? You know, the economy affects companies' performance as well as the mood in the stock market" Katie suggested.

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"I am thinking about doing all of that, but I don't know much about these calculations. Do you know where I can find some information on these calculations?" Jan asked.

"I think you can look at some websites. Nowadays, finance-related websites explain everything in detail. You can first go over them, and then if you have any questions, I can help you."

"O.K. I will do that. Since I am doing my own investments, I want to learn about all of these things. Thank you Katie. If you'd like, we can eat something in our regular place and chat for a while. You are not in a hurry, are you?" Jan asked.

"No, no. I am in my lunch break. We can go there and eat and relax a little bit. I wish I were like you; retired and happy. You are now free!" Katie said.

"For now, retirement is good. I don't know if I will get bored soon though".

After lunch, Jan has started working on her task. She has already found some information on two websites.

Portfolio Risk and Return

Investopedia.com explains portfolio expected return as below:

"Expected return is calculated as the weighted average of the likely profits of the assets in the portfolio, weighted by the likely profits of each asset class. Expected return is calculated by using the following formula:

$$E(R) = \sum_{i=1}^{n} P_i \times R_i$$

Written another way, the same formula is as follows: $E(R) = w_1R_1 + w_2R_q + ...+ w_nR_n$"

Zenwealth.com explains the calculation of portfolio expected return and risk as follows:

"The Expected Return on a Portfolio is computed as the weighted average of the expected returns on the stocks which comprise the portfolio. The weights reflect the proportion of the portfolio invested in the stocks. This can be expressed as follows:

$$E[R_p] = \sum_{i=1}^{N} w_i E[R_i]$$

where

- $E[R_p]$ = the expected return on the portfolio,
- N = the number of stocks in the portfolio,
- w_i = the proportion of the portfolio invested in stock i, and
- $E[R_i]$ = the expected return on stock i.

For a portfolio consisting of two assets, the above equation can be expressed as

$$E[R_p] = w_1 E[R_1] + (1 - w_1) E[R_2]$$

Using either the correlation coefficient or the covariance, the Variance on a Two-Asset Portfolio can be calculated as follows:

$$\sigma_p^2 = (w_1)^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2$$
$$= (w_1)^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\sigma_{12}$$

The standard deviation on the portfolio equals the positive square root of the variance."

In the above formulas:
$E[R_p]$ is the portfolio expected return,
W_1 is the weight of the first stock,
$E[R_1]$ is the expected return on the first asset,
$E[R_2]$ is the expected return on the second asset,
σ_p is the standard deviation of the portfolio returns,
σ_1 is the standard deviation of the first asset's returns,
σ_2 is the standard deviation of the second asset's returns, and
ϱ_{12} is the correlation between the two assets' returns.

Zenwealth.com also explains the calculation of the correlation between the two assets' returns. In order to find the correlation, first the covariance needs to be computed:

"The variance/standard deviation of a portfolio reflects not only the variance/standard deviation of the stocks that make up the portfolio but also how the returns on the stocks which comprise the portfolio vary together. Two measures of how the returns on a pair of stocks vary together are the covariance and the correlation coefficient.

The Covariance between the returns on two stocks can be calculated using the following equation:

$$Cov(R_1,R_2) = \sigma_{12} = \sum_{i=1}^{N} p_i(R_{1i} - E[R_1])(R_{2i} - E[R_2])$$

where

- σ_{12} = the covariance between the returns on stocks 1 and 2,
- N = the number of states,
- p_i = the probability of state i,
- R_{1i} = the return on stock 1 in state i,
- $E[R_1]$ = the expected return on stock 1,
- R_{2i} = the return on stock 2 in state i, and
- $E[R_2]$ = the expected return on stock 2.

The Correlation Coefficient between the returns on two stocks can be calculated using the following equation:

$$Corr(R_1,R_2) = \rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2} = \frac{Cov(R_1,R_2)}{SD(R_1)SD(R_2)}$$

where

- ρ_{12} = the correlation coefficient between the returns on stocks 1 and 2,
- σ_{12} = the covariance between the returns on stocks 1 and 2,
- σ_1 = the standard deviation on stock 1, and
- σ_2 = the standard deviation on stock 2."

Portfolio Risk and Return without the use of Covariance and Correlation
Looking at the above formulas, we can say that while the calculation of portfolio expected return is straightforward, the calculation of portfolio risk is complex. First, standard deviations of each individual stock and covariance needs to be computed. Then, correlation

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needs to be computed. After computing standard deviations, covariance and correlation, then we can compute portfolio risk. There is an easier procedure to estimate portfolio risk. This procedure is explained in an example below:

Let's assume that we expect the economy to be in one of four different states next year (these may be "recession", "normal economy", "strong economy", and "very strong economy", for example). We want to invest 40% of our money in Stock A and 60% of our money in Stock B. We estimated the probabilities of each state economy as follows: Recession 20% chance, Normal economy 40% chance, Strong economy 25% chance, and Very strong economy 15% chance. We also estimated each stock's expected return under each state of economy. Our estimates are summarized in the below table:

State	Probability	Return on Stock A	Return on Stock B
Recession	20%	5%	10%
Normal	40%	10%	15%
Strong	25%	13%	17%
Very Strong	15%	20%	24%

We want to compute our portfolio's expected return and risk. How can we do it?

First, we need to use the weights to estimate our portfolio's expected return under each scenario:

E(Rp) recession = (0.40)(5%) + (0.60)(10%) = 8%

Here, we are investing 40% of our money in A and A's expected return is 5%, therefore we multiply 0.40 and 5% for Stock A. We do the same for B. We are investing 60% of our money in B and B's expected return in 10%, therefore we multiply 0.60 and 10% for Stock B. In total, the expected return on our portfolio in a recession is 8% (as computed).

Then, we do the same for the other states of economy. We compute the expected return on our portfolio under "normal economy", "strong economy", and "very strong economy."

E(Rp) normal = (0.40)(10%) + (0.60)(15%) = 13%
E(Rp) strong = (0.40)(13%) + (0.60)(17%) = 15.4%
E(Rp) very strong = (0.40)(20%) + (0.60)(24%) = 22.4%

After finding the expected portfolio return under each scenario, we can combine them (using the probability of each state of economy) to compute the overall portfolio return as follows:
E(Rp) = [(0.20)(8) + (0.40)(13) + (0.25)(15.4) + (0.15)(22.4) = 14.01%

There is a 20% chance of a recession, 40% of a normal economy, 25% of a strong economy, and 15% of a very strong economy, therefore we used these probabilities in our calculations above.

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After computing the overall portfolio expected return, we can compute the portfolio risk using the following formula:

σ_p = SQRT [p_1 [(E(R_1)-E(R_p)]^2 + p_2 [(E(R_2)-E(R_p)]^2 + p_3 [(E(R_3)-E(R_p)]^2 + p_4 [(E(R_4)-E(R_p)]^2]
Therefore:
σ_p = SQRT [0.20(8-14.01)^2 + 0.40(13-14.01)^2 + 0.25(15.4-14.01)^2 + 0.15(22.4-14.01)^2
= SQRT [7.22 + 0.41 + 0.48 + 10.56]
=SQRT [18.67]
= 4.32

This above formula can be modified to include different number of assets and different number of scenarios.

The Decision
Jan wants to include four different stocks in her portfolio. She calls these stocks stock A, stock B, stock C, and stock D. She wants to invest 40% of her money in Stock A, 20% in Stock B, 10% in Stock C, and 30% in Stock D.

She has estimated each stock's expected return for the next year under different scenarios. Her first scenario is a recession. She thinks that there is a 15% chance of a recession next year. If a recession happens, she thinks that the returns on stocks A, B, C, and D will be -10%, -3%, 0%, and 2%, respectively, next year.

Her second scenario is a normal economy (where the economy is flat). She sees a 40% chance of a normal economy next year. She thinks that, under a normal economy, the returns on stocks A, B, C, and D will be 15%, 12%, 9%, and 7%, respectively, next year.

Her third scenario is an expansion in the economy. She sees a 45% chance of an expanding economy next year. She thinks that, under this scenario, the returns on stocks A, B, C, and D will be 20%, 15%, 12%, and 8%, respectively, next year.

She has prepared a table that summarizes these estimates. The table is shown below.

State	Probability	A	B	C	D
Recession	15%	-10%	-3%	0	2%
Normal	40%	15%	12%	9%	7%
Expansion	45%	20%	15%	12%	8%

Weights in each stock:
Wa=40%, Wb=20%, Wc=10%, Wd=30%

Related to her investments, she wants to answer the following questions:
1. What is the expected return on this portfolio under each state of economy (i.e. recession, normal, and expansion)?

2. What is the expected overall portfolio return? (Hint: Please consider the probabilities of each state of economy)

3. What is the risk (i.e. standard deviation) of this portfolio?

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4. What are the expected returns on individual stocks ($E(R)_A$, $E(R)_B$, $E(R)_C$, $E(R)_D$)?

5. Which stock has the highest expected return? Which one has the highest expected risk?

6. She wants to use the expected returns on individual stocks ($E(R)_A$, $E(R)_B$, $E(R)_C$, $E(R)_D$) in an equation to find the expected return on this portfolio. How does she do that? Will she find the same expected return here as she finds in #2?

Hint: Instead of finding the expected return on the portfolio in each state of the economy first, she needs to first find $E(R)_A$, $E(R)_B$, $E(R)_C$, $E(R)_D$ and then she will find the expected return on the portfolio.

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