Radar Figure of Merit

A radar figure of merit can be defined based on the received signal to noise ratio of a transmitted radar waveform subsequently reflected from a distant target. First, the 'basic' expression for power reflected from the target and received by the radar can be stated as

$$P_r = \frac{P_t G_t \sigma A_r}{(4\pi R^2)^2} \qquad 1$$

Where:

P_t = Transmitter power, Watts

G_t = Transmit antenna gain

σ = Target "cross section", square meters

A_r = Receive antenna aperture effective Area, square meters

R = Range from the radar to the target, meters

In the above, $P_t G_t$ is transmit EIRP, and when divided by $(4\pi R^2)$ gives the power density at the target, expressed in $Watts/meter^2$. Target cross section σ has units of $meter^2$. By definition, cross section is that equivalent area that intercepts the incident wave and re-radiates it omnidirectionally. Consequently, the product of transmitted power density times cross section determines the power, in Watts, of the target-intercepted portion of the transmitted signal. The target then re-radiates this power, some of which returns to radar receiver. Dividing the re-radiated power by $(4\pi R^2)$ gives the power density at the radar receiving antenna. To obtain the power actually received by the radar, we must multiply the power density by the receive antenna AREA (not the gain), since to get Watts, we need the product of $Watts/meter^2$ times $meter^2$.

Radar Figure of Merit -2

Equation 1 can be restated in terms of receive antenna gain defined in terms of effective collecting area, $G = \frac{4\pi A_e}{\lambda^2}$.

We obtain

$$P_r = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4} \qquad 2$$

Detectability of received reflection depends upon noise in the receiving system, P_n, which is well known to be the product of Boltzmann's constant, system temperature and bandwidth. Noise power is

$$P_n = kT_s B$$

Where
k = Boltzmann's constant
T_s = System noise temperature
B = Radar waveform bandwidth

Returned echo Signal to Noise Ratio is therefore

$$SNR = \frac{P_r}{P_n}$$

$$SNR = \frac{P_t G_t G_r \sigma \lambda^2}{kT_s B(4\pi)^3 R^4} \qquad 3$$

Radar Figure of Merit -3

A radar figure of merit may be defined based on Equation 3. Occasionally only received power or implied or unity B expressions are used as a figure of merit, depending upon specific interests. We will use SNR from Equation 3 as the figure of merit.

Often radar transmitting and receiving antennas are the same. We will make this assumption, though it should be clear how the equations can be modified for bistatic or multistatic operation.

Expressing SNR in an array context is facilitated by identifying EIRP for transmit and G/T for receive. In Eq. 3, we have $EIRP = P_t G_t$ and $G/T = G_r / T_s$. Substituting the array equivalents where we have N nominally identical reflectors with gain G_e, identically pointed, with equal power amplifiers P_a and identical noise temperatures T_a,

$$EIRP_{array} = (NP_a)(NG_e) = N^2 P_a G_e$$

and

$$G/T_{array} = NG_e / T_a$$

Substituting into Equation 3 gives the array equivalent figure of merit,

$$SNR_a = \frac{\left(N^2 P_a G_e\right)\left(NG_e\right)\sigma\lambda^2}{kT_a B(4\pi)^3 R^4} \qquad 4$$

Radar Figure of Merit -4

Since G is frequency dependent, it is convenient to return to the definition of gain, $G = \frac{4\pi A_e}{\lambda^2}$ so that all frequency dependence is explicit. We find

$$SNR_a = \frac{N^3 P_a A_e^2 \sigma}{kT_a B(4\pi\lambda^2)R^4}$$

$$= \frac{N^3 P_a A_e^2 \sigma f^2}{kT_a B(4\pi c^2)R^4} \qquad 5$$

Performances of various array designs as well as single aperture systems can be compared for a given cross section, range, system temperature and radar waveform. In this case a simpler figure of merit can be defined[1]

$$FOM = N^3 P_a A_e^2 f^2 \qquad 6$$

This figure of merit is graphed in Figure 1, showing that an array of about 40 12m reflectors has a higher figure of merit than the Goldstone 70m reflector with a 500kW power amplifier. Only about 20 12m reflectors with 1kW solid state air cooled PA's are required to surpass performance of a 34m reflector with a 500KW water cooled klystron.