A novel approach on Classical-Quantum Systems in Multi-Frame work environment

KOTHAPALLI CHAITANYA DEEPTHI
Assistant Professor
Sir C.R.Reddy College of Engineering, Eluru, Andhra Pradesh, India

Abstract - Classical–quantum systems provide a unifying framework for integrating classical and quantum components at the level of dynamics, information processing, and computation. This paper proposes a novel application-oriented formulation of classical–quantum systems that connects recent rigorous dynamical models with practical hybrid quantum–classical computing architectures. Building on moment-based descriptions of hybrid dynamics and quasifree quantum–classical constructions, a generalized formalism is presented in which classical variables and quantum operators coexist in a single state representation defined by coupled moment hierarchies and hybrid Poisson structures. On top of this, an algorithmic layer is introduced that maps such hybrid states to concrete hybrid computational workflows, including variational quantum algorithms, hybrid solvers for strongly correlated materials, and domain-specific hybrid models in chemistry and materials science. The methods are evaluated conceptually against recent NISQ-era implementations, such as hybrid quantum–classical simulation of the Single-Impurity Anderson Model and variational simulation of correlated electron systems, highlighting how classical–quantum system design can concentrate quantum resources on the most intractable subproblems while offloading regular structure to classical solvers. Results from published case studies indicate that such architectures can reduce classical complexity, improve effective accuracy, and extend the scope of NISQ devices without requiring full fault tolerance. The paper concludes that classical–quantum systems, understood as a joint dynamical and computational paradigm, offer a principled route to scalable hybrid architectures and motivates further research in co-designed models, stability analysis, and application-specific decompositions.

Keywords: classical–quantum systems, hybrid dynamics, hybrid quantum–classical computing, variational quantum algorithms, NISQ, strongly correlated materials.

I. INTRODUCTION

Classical–quantum systems arise whenever classical and quantum degrees of freedom interact, either in physical models (e.g., classical fields coupled to quantum matter) or in computation where classical processors orchestrate quantum hardware. In the NISQ era, this interaction is no longer a theoretical curiosity but a central design principle: practical algorithms rely on classical optimization, control, and error mitigation wrapped around noisy quantum cores.

Traditional quantum information theory assumes fully quantum systems, with classicality emerging only at the measurement interface, while most numerical modeling in physics treats relevant subsystems either fully quantum or fully classical. Recent work has challenged this binary view by developing consistent dynamical formalisms for hybrid systems and by demonstrating experimentally that hybrid quantum–classical approaches can tackle strongly correlated materials and quantum simulation tasks beyond straightforward classical treatments.

This paper advances a **novel classical–quantum system perspective** that explicitly ties together:

- Rigorous hybrid dynamical models in terms of moments and hybrid phase spaces.
- Quasifree formalisms where classical and quantum variables are treated uniformly.
- Practical hybrid quantum–classical algorithms that instantiate these ideas on real hardware.

1.1 Motivation

Several converging trends motivate a systematic treatment of classical–quantum systems:

- **Formal consistency:** New moment-based and quasifree frameworks show that consistent hybrid dynamics can be defined without ad hoc collapse rules, avoiding some no-go arguments against classical–quantum couplings.
- **NISQ constraints:** Hybrid architectures concentrate quantum effort on small, hard sub problems (e.g., correlated subspaces), while classical resources handle the surrounding structure, mitigating depth and noise issues.
- **Emerging applications:** Hybrid schemes have recently been used to decode electron behavior in complex materials (e.g., SIAM and Hubbard models) and to realize variational quantum simulation frameworks with clear application roadmaps in chemistry and condensed-matter physics.

1.2 Contributions

The main contributions are:

- A **unified conceptual model** of classical–quantum systems linking dynamical hybrid formalisms to algorithmic NISQ-era architectures.
- A **methods framework** for decomposing physical and computational problems into classical and quantum sectors, including a mapping from hybrid moment hierarchies to hybrid algorithm building blocks.
- A **results-oriented synthesis** of recent hybrid simulations in strongly correlated materials and chemical systems, interpreted through this classical–quantum lens.

II. LITERATURE SURVEY

2.1 Formal Classical–Quantum Hybrid Dynamics

Recent formal work has revisited the problem of defining consistent dynamics on spaces that include both classical and quantum degrees of freedom.

- **Moment-based hybrid formalism:** A 2024 study introduced a description of hybrid systems in terms of moments of classical phase-space variables and quantum operators, defining a hybrid Poisson bracket and an effective Hamiltonian governing coupled evolution. The hybrid state combines a classical probability distribution and a quantum density matrix into a single object constrained by positivity and a generalized uncertainty principle.
- **Quasifree hybrid systems:** Another line of work considers continuous-variable systems where classical and quantum variables are put on the same footing via canonical operators with scalar commutators. This allows a unified treatment of operations involving measurements, classical control, and classical parameters, framing hybrid channels as special cases of general quantum operations.

These approaches show that classical–quantum hybrids can be modeled with consistent dynamics, albeit with nontrivial constraints to avoid pathological behavior such as unrestricted uncertainty violation.

2.2 Classical–Quantum Hybrid Models in Physics

Hybrid classical–quantum models have long been used heuristically, for example by treating some degrees of freedom (like heavy nuclei or external fields) classically while describing electrons or localized modes quantum mechanically. An arXiv study on classical–quantum hybrid models analyzes systems in which certain variables follow classical equations of motion while others obey Schrödinger-type dynamics, with coupling terms determined by expectation values.

Such models appear in:

- Quantum–classical molecular dynamics where nuclear trajectories are classical but electronic states are quantum.
- Mean-field approximations where a classical field is driven by quantum expectation values in condensed-matter systems.

Formalizing these schemes within the newer moment and quasifree frameworks is a key step toward principled classical–quantum system design.

2.3 Hybrid Quantum–Classical Computing Architectures

On the computational side, NISQ-era algorithms almost universally adopt hybrid architectures.

- **Variational quantum computing:** A recent review focuses on variational quantum computing for simulation, emphasizing parameterized quantum circuits evaluated on quantum hardware with parameters optimized classically. This framework covers VQE for ground states, excited-state extensions, and more general quantum simulation tasks in chemistry and many-body physics.
- **NISQ-era advances:** A 2025 overview of advances in quantum computation highlights hybrid quantum–classical algorithms as a central strategy for making current devices useful, with applications in chemistry, materials science, many-body physics, and machine learning, all relying on classical optimization wrapped around quantum subroutines. These works frame hybrid algorithms as *computational* classical–quantum systems: the quantum part manipulates states in a large Hilbert space, while classical resources manage optimization, control, and error mitigation.

2.4 Recent Hybrid Applications

Several concrete hybrid applications illustrate the power of classical–quantum systems:

- **Strongly correlated materials:** Researchers used a hybrid quantum–classical approach to solve the Single-Impurity Anderson Model (SIAM), with the quantum device computing the Green's function while classical algorithms handled the remaining self-consistent loop. This enabled observation of a quantum phase transition in the Hubbard model and demonstrated that hybrid schemes can tackle strongly correlated electron behavior beyond straightforward classical methods.
- **Variational simulation for chemistry:** A 2025 review on chemical applications of variational quantum eigenvalue-based methods discusses hybrid VQE-style schemes that use quantum devices to evaluate Hamiltonian expectation values while classical optimizers refine wavefunction parameters, targeting realistic chemical systems.
- **Hybrid NISQ strategies:** An analysis of strategic paths for useful NISQ applications argues for co-design of quantum algorithms and classical HPC

infrastructure, emphasizing hybrid workflows where classical simulation and quantum processing are tightly integrated. These case studies serve as empirical anchors for the classical–quantum system concepts developed in this paper.

III. MATERIALS AND METHODS

3.1 Classical–Quantum State Representation

The proposed framework describes a classical–quantum system via a **hybrid state** characterized by:

- A set of classical variables $x = (x_1, \dots, x_n)$ with an associated classical distribution or moment hierarchy (e.g., $\langle x_i \rangle, \langle x_i x_j \rangle$).
- A quantum subsystem represented by a density operator ρ or equivalently by moments of canonical operators (e.g., $\langle \hat{q}^k \hat{p}^\ell \rangle$).
- A coupling structure encoded in a hybrid effective Hamiltonian $H_{\text{eff}}(x, \hat{O})$ and a hybrid Poisson bracket that governs joint dynamics in the combined phase space.

In spirit, this follows the moment-based hybrid formalism and quasifree hybrid systems, but is tailored to computational scenarios where the *quantum* part is realized on hardware and the *classical* part runs on conventional processors.

3.2 Problem Decomposition

Given a target physical or computational problem (e.g., correlated electrons, fluid dynamics, optimization), the methodology proceeds in three steps:

1. **Structural analysis:** Identify substructures that are inherently quantum (e.g., strongly correlated subspace, entangled cluster) and those amenable to classical treatment (e.g., mean-field environment, classical field).
2. **Partitioning:** Map the quantum subspace to a parameterized quantum model (e.g., ansatz state or variational circuit) and the classical part to differential equations, mean-field updates, or optimization variables.
3. **Coupling:** Define interaction terms whereby classical variables enter the quantum Hamiltonian (as parameters) and quantum expectation values feed back into classical equations, implementing a closed-loop hybrid system.

This procedure generalizes both physics-driven hybrid models and algorithmic hybrid workflows.

3.3 Hybrid Algorithmic Layer

On top of the hybrid state and coupling, an **algorithmic layer** instantiates concrete workflows:

- **Variational loop:** A classical optimizer updates parameters of a quantum ansatz based on quantum evaluations of a cost function (e.g., energy, Green's function), as is standard in VQE and related algorithms.
- **Self-consistent loops:** For models like SIAM or Hubbard, the quantum device computes quantities such as Green's functions or self-energies, while classical solvers handle Dyson equations and self-consistency conditions.
- **Dynamic integration:** For software systems, dynamic integration patterns coordinate calls between classical services and quantum back-ends, as explored in hybrid quantum/classical software architectures.

The method emphasizes **co-design**: quantum and classical components are simultaneously specified to match each other's strengths and limitations.

3.4 Evaluation Criteria

Because a general analytical "result" is not a numerical performance table but a conceptual system, evaluation focuses on:

- **Expressiveness and consistency:** Compatibility with formal hybrid dynamics and constraints such as uncertainty bounds and positivity.
- **Computational efficiency:** Reduction in classical complexity relative to fully classical treatments for representative tasks, given existing experimental and numerical evidence.
- **Scalability:** How the quantum subproblem scales with system size and whether the classical–quantum split delays the onset of NISQ limitations like depth and noise.

IV. RESULTS AND DISCUSIONS

4.1 Conceptual Unification of Hybrid Dynamics and Hybrid Computing

Interpreting recent formal results through the lens of NISQ-era algorithms yields a unified view:

- Moment-based and quasifree frameworks supply **mathematically consistent** models for coupled classical–quantum evolution, providing a principled way to describe measurement, feedback, and parameter dependence.
- Variational quantum computing and hybrid simulation schemes naturally implement discrete-time approximations of these continuous hybrid dynamics, with classical optimizers playing the role of effective classical degrees of freedom coupled to quantum states.

This shows that widely-used hybrid algorithms are not ad hoc, but particular discretizations of more general classical–quantum systems.

4.2 Classical–Quantum Decomposition in Strongly Correlated Materials

The hybrid SIAM and Hubbard simulations provide a concrete "result" for classical–quantum decomposition:

- In the SIAM approach, the quantum processor is tasked with evaluating the Green's function of the impurity problem, a step that becomes intractable for classical

methods as correlation strength and system size increase.

- Classical solvers handle the remaining self-consistent loop and embedding, which remain tractable even for large lattices when the most costly part is offloaded.

From the classical–quantum system perspective, this corresponds to assigning the *nonperturbative, highly entangled* sector to the quantum subsystem while letting the classical subsystem manage the surrounding environment and global consistency, thereby reducing overall computational burden.

4.3 Variational Simulation as Classical–Quantum System

Variational quantum computing for quantum simulation can be reinterpreted within this framework:

- The trial state and its parameter evolution define a **quantum subsystem** whose dynamics are driven by classical parameters updated according to gradients or other optimization rules, which constitute a classical subsystem.
- Hybrid performance analyses in the NISQ era confirm that such schemes can capture relevant physics in many-body and chemical systems while keeping circuit depth bounded by NISQ hardware limits.

This mapping clarifies how classical–quantum systems encode not only static states but **learning dynamics** in which classical and quantum parts co-evolve.

4.4 Architectural Implications for Hybrid Software Systems

Work on dynamic integration in hybrid quantum/classical software suggests that future systems will require:

- Well-defined interfaces for classical services to invoke quantum algorithms and to integrate quantum results into larger classical workflows.
- Scheduling and orchestration mechanisms that treat quantum back-ends as specialized accelerators within a broader classical–quantum system topology, consistent with strategic NISQ roadmaps calling for tight HPC-quantum integration.

Within the proposed framework, such software architectures become **implementations of classical–quantum systems at the stack level**, rather than just loose couplings.

V. CONCLUSION

This paper has developed a unified classical–quantum system perspective that connects:

- Formal hybrid dynamics based on moments and quasifree models, which provide consistent mathematical foundations for coupled classical–quantum evolution.
- Hybrid quantum–classical algorithms for simulation, optimization, and materials modeling, which instantiate these ideas in the NISQ computing regime.

The proposed methods outline how to decompose physical and computational problems into classical and quantum sectors, define consistent couplings, and implement these couplings as hybrid algorithms on real hardware.

VI. REFERENCES

[1]. Gouba, L., et al. Hybrid classical–quantum systems in terms of moments. Physical Review A, 109, 032209 (2024).
[2]. Di Lorenzo, A. Classical–quantum hybrid models. arXiv preprint arXiv:2309.05014 (2023).
[3]. Holevo, A., et al. Quantum–classical hybrid systems and their quasifree structures. Quantum 7, 1068 (2023).
[4]. The Quantum Insider. Hybrid quantum–classical computing decodes electron behavior in complex materials (2024).
[5]. Cerezo, M., et al. Variational quantum computing for quantum simulation. arXiv preprint arXiv:2510.25449 (2025).
[6]. Zhang, X., et al. Advances in quantum computation in the NISQ era. PMC Article (2025).
[7]. Lüttgen, S., et al. Chemical applications of variational quantum eigenvalue-based methods. Applied Physics Reviews 12, 031304 (2025).
[8]. Moghaddam, M., et al. Dynamic integration for hybrid quantum/classical software systems. Science of Computer Programming (2024).
[9]. Perdomo-Ortiz, A., et al. Strategic paths for enabling useful NISQ applications. arXiv preprint arXiv:2405.14561.