AN OBSERVATION CONCERNING WORMHOLES THREADED BY ELECTRIC FIELDS

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Abstract

We examine a simple metric that describes a Reissner-Nordstrom wormhole. Viewed from far away it corresponds to that of a massive charged object. Seen from close up it is a wormhole connecting an $r > 0$ "half-universe" to a separate $r < 0$ half-universe. We integrate T_{00} over space to obtain a total energy for our system that depends on the wormhole's "radius." For $Q^2 >> M^2$ we find that there is a maximal value that defines the wormhole's optimal radius. If we set Q to the electron charge we find this radius to be about 1.93183 X 10^{-36} m. This is almost exactly $\frac{3}{8\pi}$ times the Planck length. It is only 0.133% too large. Equating our expression for the wormhole's optimal radius and $\frac{3}{8\pi}\sqrt{\hbar G / c^3}$ we find the fine-structure constant to be $\frac{9}{16\left(2+\sqrt{34}\right)\pi^2}$. This underestimates α by only 0.266%. We end up with a "particle model" identical, in spirit, to that of Einstein and Rosen.

Keywords: Reissner-Nordstrom Wormhole, Quantum Gravity, Planck Length, Fine-Structure Constant.

Wormholes threaded by electric fields entered the literature in a 1935 paper by Einstein and Rosen (1). They have proven a fertile subject for speculation. (See particularly (2) and (3).) Our metric takes the form:

1) $$ds^2 = \left(1 - \frac{2GM}{c^2\left(r^2+B^2\right)^{1/2}} + \frac{GQ^2}{4\pi\epsilon_0 c^4\left(r^2+B^2\right)}\right) dt^2 - \frac{1}{\left(1-\frac{2GM}{c^2\left(r^2+B^2\right)^{1/2}}+\frac{GQ^2}{4\pi\epsilon_0 c^4\left(r^2+B^2\right)}\right)} dr^2 - \left(r^2 + B^2\right)\left(d\theta^2 + Sin^2(\theta)\, d\varphi^2\right) .$$

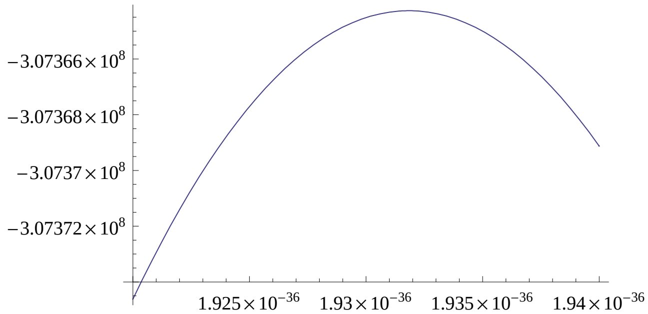
(We work in SI units.) Here B denotes the radius of the wormhole throat. We are mostly interested in the case where Q is the electron charge and M is the electron mass. This metric divides the universe into two half-universes, one having $r > 0$ and the other $r < 0$. Viewing it from far away, an observer in the first half-universe would see the Reissner-Nordstrom metric of a a massive charged particle. An observer in the other half-universe would see that of its antiparticle. The electric field is given by $E_r = -\partial_r \frac{Q}{4\pi\epsilon_0\left(r^2+B^2\right)^{1/2}}$. The Euler characteristic of the manifold is zero.

We note that this metric possesses no horizons or singularities (physical or coordinate) provided $Q^2 >> M^2$ (in m^2). We find, from Einstein's equation, that there is an enormous concentration of negative energy nearby the wormhole, and enormous radial pressure as well. The weak energy condition is violated very flagrantly. We will return to this issue later. The wormhole is gravitationally repulsive to a neutral test particle. But a massless neutral test particle, fired directly at it, will pass through and emerge into the other half-universe.

We calculate $\int_V T_{00}\sqrt{-\mathrm{Det}\, g}\; d^3x$ where we integrate over the whole of 3-space in one or the other half-universe. We will call the result $\mathcal{E}$ – the "total energy" of our wormhole. We find this result to be finite and, of course, negative. It is given by:

2) $\mathcal{E} = 3Mc^2 - \frac{B c^4 \pi}{4G} + \frac{7 G M Q^2}{12 B^2 c^2 \epsilon_0 \pi} - \frac{5 G Q^4}{512 B^3 c^4 \epsilon_0^2 \pi} - \frac{16 \epsilon_0 G M^2 \pi + Q^2}{16 B \epsilon_0}$. Here Q is the electron charge, M the electron mass.

Below we graph the result (in J) as a function of B (in m):



We see that it has a unique maximum which we will call B_{max}. This corresponds not to a minimum of energy but, rather, a minimum of *negative energy*. (Perhaps the system resists violating the weak energy condition to the greatest extent possible.) We make a helpful discovery – for M not much larger than 10^{20} times the electron mass the M-containing terms in 2) are so small as to make no noticeable contribution to the result. This allows us to drop M from our calculations, thereby simplifying the mathematics. We find:

3) $B_{\text{max}} = \frac{1}{4} \sqrt{\frac{2+\sqrt{34}}{\pi}} \sqrt{\frac{G Q^2}{c^4 \epsilon_0}}$.

Using the most accurate estimates available for our physical constants, we obtain a length that is almost exactly $\frac{3}{8\pi} \mathcal{L}_p$ (where $\mathcal{L}_p$ denotes the Planck length). We calculate $\frac{8\pi}{3} B_{\text{max}}$ and find it to be 1.61841 X 10^{-35}m. This number exceeds $\mathcal{L}_p$ by only 0.133%, and the reader will recall that $\hbar$ figures nowhere in our calculation of B_{max}. We do not know where the factor of $\frac{8\pi}{3}$ comes from but it does "look like" something that might come from physics. If we are willing to accept it, we can simply equate $\frac{8\pi}{3}\left(\frac{1}{4} \sqrt{\frac{2+\sqrt{34}}{\pi}} \sqrt{\frac{G Q^2}{c^4 \epsilon_0}}\right)$ and $\sqrt{\hbar G / c^3}$ to obtain a sort of "prediction" of the fine-structure constant:

4) $\alpha = \frac{9}{16\left(2+\sqrt{34}\right)\pi^2}$.

We find α = .00727794 which is .99734 times the experimentally measured value. An estimation of α from something like a physical model is, to the author's knowledge, unprecedented.

We set Q to the electron charge for obvious reasons. But, provided that $M^2 << Q^2$, M can be ignored in our calculations. We might just as well be discussing an electron or a μ or a τ. Quarks would have values of B_{max} that were reduced by 1/3 or 2/3. We are inclined to picture mesons and baryons as bound states of two or three wormhole mouths held together by the strong force but we have no specific details to offer. Gauge bosons might better be regarded as excitations in some higher-dimensional internal space. Elementary particles, as far as we know, are quantized fields not classical geometrical structures (wormholes or otherwise). What we have described can best be regarded a "particle metaphor," not a "particle model." The question 'metaphor for what?' remains unanswered.

What to make of all of this? The skeptic will, of course, say "nothing" – we have been taken in by a strange and meaningless coincidence. He may be entirely right. Still, an agreement to the level of 0.133% is fairly impressive. It is not exact, however, and we wonder what we might do differently. There are many possibilities. For instance, we have taken no account of spin or the possible role of the strong and weak interactions. Some attempts have been made to do so. Unfortunately, the math has, thus far, proven intractable. Neutrinos represent an interesting case. We can set Q to zero in 2) and find that the total energy possesses a unique maximum at $B = 2\,GM/c^2$. $B_{\max} \approx 3 \text{ X } 10^{-64}$m. We still have a wormhole but its radius is very narrow. There is a coordinate singularity at $r = 0$.

Were the other mouths of our wormholes to open into distant parts of our own universe we would expect there to be equal amounts of matter and antimatter. This does not appear to be the case and we, following Einstein and Rosen, suggest taking the half-universe idea at face value. In effect, we are living in a double-universe with all our wormholes opening into a common half-universe identical to ours but with each particle replaced by its antiparticle. (Parity would be reversed there as well.) Since PC is not an exact symmetry of the electroweak interaction we might worry that this could not be the case. But we could imagine the Standard Model (or whatever eventually supersedes it) differing between the two half-universes in such a way that particles in one are treated like antiparticles in the other.

We are left to make sense of the enormous (negative) energy and pressure required to keep our wormhole intact. Absent these it should not exist at all, according to classical General Relativity. Perhaps there is an exotic matter field that would provide what we want but we have no specific suggestions. Or we may be in the same position as physicists a century ago for whom classical hydrogen atoms could not exist. With the advent of Quantum Mechanics it became clear that they can and should exist. Were we in possession of a workable theory of quantum gravity it might be equally clear to us that our wormholes can and should exist. We have, at present, no such theory. But, perhaps, the above observations might provide a clue in the search for one.

References.

1) Einstein, A., Rosen, N. (1935). *The Particle Problem in General Relativity. Phys. Rev.* **48**, 73.

2) Graves, J. C., Brill, D. R. (1960). *Oscillatory Character of Reissner-Nordstrom Metric for an Ideal Charged Wormhole. Phys. Rev.* **120**, 1507.

3) Harris, E. G. (1993). *Wormhole Connecting Two Reisnner-Nordstrom Universes. Am. J. Phys.* **61**, 1140.